REINFORCED-CONCRETE BRIDGES

BY

The Late FREDERICK W. TAYLOR
The Late SANFORD E. THOMPSON

AND

The Late EDWARD SMULSKI

REINFORCED-CONCRETE BRIDGES
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WITH

Formulas Applicable to Structural Steel and Concrete

BY

THE LATE FREDERICK W. TAYLOR

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PREFACE

The prime object of the book is to furnish complete information for logical design of bridges other than trusses and arches. It presents the first comprehensive treatment of all types of bridges that have thus far been developed, such as slab bridges, simple deck and through girder bridges, cantilever and continuous girder bridges, rigid frames, and flat-slab bridges. The treatment includes discussion of the relative economy and usefulness of these types for different conditions and also the procedure to be followed in developing a design best fitted for any specific requirement. Examples of computations and illustrations of design details are also given, making the book of practical value not only to the engineer and builder but also to the student who intends to enter the field of structural engineering.

Although the treatment is specifically for reinforced-concrete bridges, much of the information is of general nature and is applicable also for bridges built of structural steel as well as for other types of structures both of steel and reinforced concrete. Formulas and methods for cantilevers and continuous bridges and for rigid frames furnish means for determining bending moments and shears for loadings carried by a structure. The procedure is the same, irrespective of the material of which the members are to be built. The difference comes only in the determination of the final dimensions of the members.

The treatment of the design of floor construction applies not only to reinforced-concrete but also to structural-steel bridges with concrete floors. The information concerning clearances, loadings, and the treatment of concentrated truck loads is pertinent for bridges of structural steel as well as those of concrete. Furthermore, it is obvious that the formulas and methods are not restricted to bridges, but may be used for building construction or other structures having arrangements of spans similar to those treated in this book. Arrangement of reinforcement and methods of bending bars for diagonal tension are adapted to any reinforced-concrete structure as is the method of comparing the bending moments with the moments of resistance by superimposing one diagram upon the other.

Treatment of certain features and types of design is not available elsewhere. The advantages of cantilevers and of continuous girders, both in reinforced concrete and structural steel, are being more and

more appreciated, so that the practical treatment of these and the illustrations of the effect upon the other spans of the structures of the loads on the cantilevers is particularly timely.

The methods for determining bending moments and shears for continuous slabs, beams, and girders of any number of equal or unequal spans, with constant and variable moments of inertia, provide means for designing continuous structures both in structural steel and concrete safely and economically without resorting to rule-of-thumb methods.

Rigid frames, the use of which often greatly reduces cost of the structure, are treated with the same completeness and applicability to other structures. Multi-span rigid frames and one-span rigid frames with hinged and fixed ends are each awarded an entire chapter, with a third chapter covering special problems in rigid frame design using fixed-point method.

A special feature of the book is the treatment of flat slabs. Arrangements of columns are suggested suitable for bridges. Special formulas are given for the treatment of the unusual types of flat slabs. In developing these formulas the logical methods were used and numerical examples show the procedure to be followed in the use of the formulas and recommendations. These also are of general application.

Effects of temperature changes upon bridges are treated; methods of providing expansion joints are given; expansion bearings are discussed and illustrated and also such details as railings, drainage, and waterproofing. Abutments and piers are treated with due reference to the superstructure.

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Credit for the compilation of the major part of the book must be given to Mr. Smulski. The writer and Mr. Miles N. Clair, Vice President of The Thompson & Lichtner Co., Inc., have carefully reviewed the material from the background of their experience and co-operated in its presentation.

SANFORD E. THOMPSON

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REINFORCED-CONCRETE BRIDGES

CHAPTER I

CHARACTERISTICS AND USE OF REINFORCED-CONCRETE GIRDER BRIDGES

Reinforced concrete is accepted so universally for bridge construction both for highways and railroads and its use is so widespread that a knowledge of the various types of structures that may be built and of the methods of designing them is essential in the experience of every structural engineer and builder.

This volume, then, presents a detailed treatment of the design of all types of reinforced-concrete bridges with the exception of arches and long trusses. It includes slab bridges and concrete pile trestles; simple, cantilever, and continuous girder bridges; and rigid frames both one-span and multi-span. The reinforced-concrete floor systems treated are also useful with many steel truss designs and with arch construction.

The general treatment throughout is adapted both to railway and highway bridges. Illustrations and examples are largely of highway designs and for railway bridges simply require substitution of railroad loadings. General principles of design apply to both railway and highway bridges.

Definition of Girder Bridges. — The term "girder bridge" is used in its broad meaning to cover the various types ranging from simple slabs to rigid frame design.

Use of Reinforced-Concrete Girder Bridges. — Reinforced-concrete slab and girder bridges are widely used for highway bridges, and to a considerable extent for railroad bridges. In structural steel bridges, reinforced-concrete slabs are replacing more and more the other types of floor construction.

For permanent bridges, both reinforced concrete and structural steel give satisfactory results, so that the choice between the two materials depends principally upon the first cost and the cost of maintenance.

Since the relative costs of steel and concrete differ in different countries, and even in different sections of the same country, it is impossible to fix exactly defined limits within which each material is the more economical.

As a general rule, it may be said that in North America reinforced concrete is economical for simply supported bridges for spans up to 70 ft. For continuous and restrained structures, this limit should be raised to 90 ft. In Europe and South American the limits are appreciably greater because of relatively low cost of labor in comparison with material. For longer spans relative economy should be investigated by an engineer thoroughly familiar with the use of both materials.

Examples of Girder Bridges. — Although the most widespread use of reinforced-concrete girder design is in the small- or medium-span bridge or the multi-span viaduct, notable examples are found of structures rivaling steel trusses or masonry arches in length of span.

One of the longest reinforced-concrete girder spans is the three-span continuous bridge across Rio de Peixe in the Santa Katharina State of Brazil, where the length of the center span is 224 ft., and the end spans are each 88 ft. long. (See p. 155.) The center span of the three-span bridge de la Madeleine at Nantes, France, approaches this with a length of 220 ft. Each of the end spans in this bridge is 141.6 ft. long. (See p. 143.) Another long span is in the center of the three-span cantilever bridge across the river South Esk in Scotland, in which the end spans are 150 ft. each. (See p. 143.) A fourth long span is the 202-ft. center span of the three-span continuous bridge across the Danube at Grossmehringen.

One of the longest one-span rigid frames is in the Victoria bridge in Bromberg, Germany, which has a span of 124 ft. (See p. 323.) In North America and Great Britain a number of one-span rigid frames have been built with spans approximately 100 ft. long.

The bridge at Freiburg, Germany, described on p. 127, has the longest span of all the one-span structures with concealed counterweighed cantilevers. The span is 131.2 ft. long.

Such examples indicate the possibilities of reinforced concrete for girder bridges, and the importance of the thorough and illustrative treatment here given.

Requirements for Successful Reinforced-Concrete Structures. — Satisfactory structures of reinforced concrete require that:

1. The structural plans must be prepared by a competent engineer, thoroughly versed in the use of reinforced concrete. The plans should be complete, with details not only of the dimensions of the members and the amounts of reinforcement, but also of the waterproofing, drainage, and provisions for expansion and contraction. These details, though

they appear to be of minor importance, actually affect appreciably the durability and ultimate appearance of the structure.

2. The materials for the concrete should be carefully selected and tested to insure compliance with the adopted specifications. Proper mixtures and consistencies for concrete should be selected on the basis of tests made preliminary to the start of the concreting. It should be kept in mind that not only must the concrete be strong enough to resist the stresses, but also it must be dense and watertight to resist the action of the elements.

3. The construction must be properly supervised. The importance of proper supervision and control of proportioning is apparent when it is considered that the strength and durability of concrete of given proportions depend largely upon proper mixing of materials, proper depositing of concrete, and its proper curing. Reinforcement must be properly placed and kept in place during concreting. It must be protected by a sufficient thickness of concrete. Steel placed too near the surface is liable to rust and spall off the concrete covering, thus contributing to disintegration of the structure. The location of indispensable construction joints should be selected by the engineer and not left to expediency or to the judgment of the contractor.

Selection of Type of Bridge. — In the selection of the proper type of concrete bridge for any particular case, cost is usually the determining factor. Occasionally, however, the problem is complicated by special requirements, such as appearance, restricted headroom, difficult foundations, limited time of construction, or difficulties in formwork caused either by the required height of supports or by the fact that it is necessary to maintain traffic under the bridge during construction.

For bridges having one span, the following types of structures may be used: (1) simply supported deck or through girders; (2) right-angle rigid frames; (3) right-angle frames with concealed cantilevers with or without counterweights; (4) simply supported girders with concealed cantilevers, with or without counterweights; (5) two short concealed spans, one at each side of the opening, each provided with a cantilever extending into the opening and supporting a short center span.

The simply supported structure, type (1), is statically determinate and is the simplest to design, but its cost is the highest. When unyielding foundation is attainable, the rigid frame, types (2) and (3), provide the most economical solution. Girders with cantilevers, as types (4) and (5), should be considered for long spans where small depth of girders is desired.

For a bridge with several spans, the following arrangements should be considered: (1) a number of simply supported girder spans; (2) a

combination of girders provided with cantilevers and short spans supported by these cantilevers; (3) continuous girders supported by independent piers; (4) multi-span rigid frames in which the girders forming the superstructure are rigidly connected with elastic vertical supports.

The first two of the multi-span arrangements are statically determinate; types (3) and (4) are statically indeterminate. Ordinarily, the cost of the structure is highest for simply supported girder spans, type (1), and lowest for rigid frames, type (4). Types (1) and (2), in fact, should be used only where reasonably unyielding foundation is not easily obtainable. Of these two types, the design of type (1) is simpler but more costly than type (2). Where unyielding foundations are available, the continuous girders or rigid frames of type (3) or type (4) should be used. Where heavy piers are required, as in river crossings, or where the structure is to be supported on already existing piers, type (3) with continuous girders is recommended. A rigid connection between heavy piers and the more flexible superstructure should never be attempted. The rigid frame, type (4), is preferable where vertical supports of the bridge are elastic, as in viaducts. In types (2) to (4), the advantage of providing the end spans with cantilevers should be studied particularly with the view of reducing the cost of abutments.

CHAPTER II

CLEARANCES: LIVE LOADS: IMPACT: DISTRIBUTION OF LIVE LOADS BY SLABS

The features of highway bridges treated in this chapter include: (1) width of bridges and required clearances; (2) height and width of curbs; (3) dead loads; (4) live loads and impact; (5) lateral forces and wind pressures; (6) distribution of concentrated wheel loads by the slab.

Width of Roadway. — The effective width of a highway bridge between curbs should be sufficient to accommodate not only present but also future traffic. The width is governed by the number of lanes needed for traffic, being thus a multiple of the width of a single lane plus an added allowance for safety.

In the past, it has been customary to design a highway bridge of the same width as the adjacent roadways. At the present time, with the rapidly accelerating increase in automobile traffic and trucking, it is unwise to build any highway bridge for two-way travel on through traffic routes, or on those which are expected to carry through traffic in the near future, of a less width than three lanes, even though the adjacent roadways at present are only two lanes wide. On three-lane roads of through traffic, which are coming into disrepute because of danger of accidents, it is wise to build a four-lane bridge with the extra lane on the side of the highway which will naturally be widened.

The widening of the highways is inevitable with the increasing demand of the public to build roads to reduce the accident hazard, and avoid the contingent liability of the road builder for accidents due to narrow roads.

With this same criterion of safety paramount, modern design is requiring lanes of a minimum width of 10 ft.[1] This width should be used on the bridge even where adjacent road lanes are only 8 ft. wide. A three-lane bridge would thus be 30 ft. plus an allowance on each side for

[1] Standard Specifications for Highway Bridges adopted by the American Association of State Highway Officials 1935 require a minimum width of only 9 ft. but this is insufficient for high speed travel.

safety — as a continuation of the well-kept shoulders on the highway — of not less than 1 ft., or preferably 2 ft., on each side, thus making a total width between curbs of 32 or 34 ft. Similarly, a four-lane bridge should be 42 or 44 ft. between curbs, and a two-lane bridge — where such is permissible, as for one-way travel — 22 or 24 ft.

Width and clearance diagrams are shown in Figs. 1 and 2, pp. 6 and 7.

At least one sidewalk should be provided on every highway bridge. The width of sidewalk should be governed by the expected pedestrian traffic. A width of 5 ft. would seem to be a fair minimum.

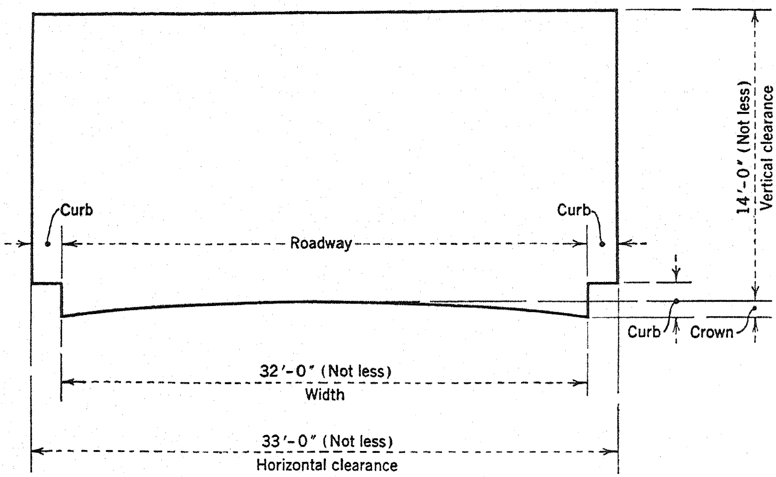


FIG. 1.—Clearance Diagram. *(See p. 6.)*

Width of Highway Bridges Carrying Electric Railway. — A highway bridge carrying a single-track electric railway should have a width of 3 ft. greater between curbs than a bridge without a track. This requires a minimum width of 35 ft. between curbs for a three-lane bridge with 10-ft. lanes. If track is on the side of the bridge, the center of the track should be $6\frac{1}{2}$ ft. from curb. For double tracks, the minimum width should be 44 ft. between curbs.

A bridge for one-way traffic with a single electric railway track for cars running in the same direction may be 24 ft. between curbs, the center of the track being $6\frac{1}{2}$ ft. from the curb. This is in accordance with the Standard Specifications for Highway Bridges. (See Fig. 2, p. 7.)

Bridges on Curves. — The widths specified apply to bridges on a tangent. On curves, the width should properly be increased, depending on the degree of curvature.

Vertical Clearances. — A vertical clearance of not less than 14 ft. is specified for highway traffic and 16 ft. minimum for electric railways, as shown in the diagrams.

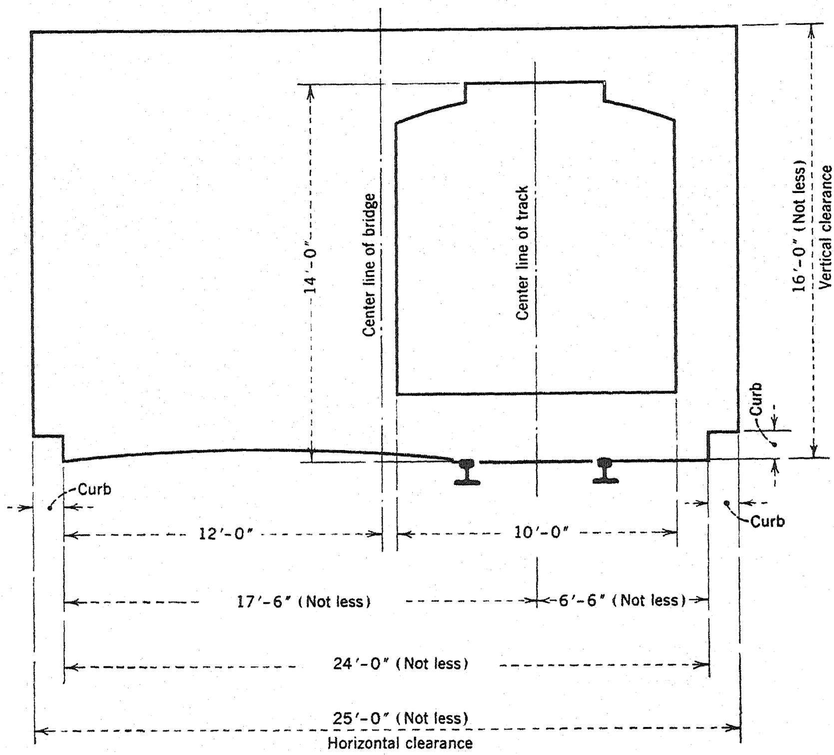


FIG. 2.—Clearance Diagram, Roadway and Electric Railway. (*See p.* 6.)

Curb. — The height of the curb above the adjacent finished roadway should be at least 9 in. to prevent skidding automobiles from leaving the roadway. The width of the curb, i.e., the distance from its face to the inside face of the girder or the railing, should be from 9 to 12 in. Often, this width is increased to form a narrow walk as a protection for occasional pedestrians.

When the curb is not an integral part of the girder, it should be strong enough to resist a lateral force of not less than 500 lb. per lin. ft. applied at the top of the curb.

Dead Load. — The dead load of a bridge consists of the weight of the complete structure, and therefore should include not only the weight of the structural concrete but also that of the pavement, the railings, car tracks, pipes, conduits, and any other utilities carried by the bridge. Snow and ice load is considered as offset by coincident decrease in live load and impact and therefore is not ordinarily included.

Live Loads. — Highway bridges should be designed not for the weight of the heaviest vehicles actually in use in the locality, but for as heavy a loading as can be reasonably expected within the life of the structure. In planning bridges for secondary roads, it is good policy to use the same live loads as for bridges on main highways, because with the increase and shifting of population secondary roads may change into main roads, and also because they may have to be used as detours for primary traffic.

Under separate headings, values for live loads are given as used in American practice.

HIGHWAY LOADINGS

In American practice, the motor truck is universally accepted as a unit for roadway loadings of highway bridges. For the design of slabs, stringers, floor beams, and short girder spans, the concentrated wheel loads of the trucks are used directly. For long spans, the concentrated loads are often replaced by equivalent uniformly distributed loadings with or without load concentration. (See pp. 10 and 27.)

For highway bridges carrying electric railway traffic, the portions of the bridge supporting the tracks are designed either for electric cars or for a combination of electric cars and motor trucks, whichever gives the more unfavorable results.

Traffic Lanes. — For the purpose of design, motor trucks are assumed to travel in trains, each train occupying a traffic lane the width of which is equal to the assumed truck clearance width. In the most modern specifications, this width of lane is accepted as 10 ft., although in some specifications 9 ft. is given.

The following general rules govern the placing of the traffic lanes on the bridge for computation of loading. Traffic lanes may be placed in any position on the bridge. Two adjacent lanes may touch, but they must not overlap. The center line of the lane next to the curb must not be placed nearer the roadway face of the curb than one-half of the lane width. Traffic in adjacent lanes may be considered as headed in the same direction. The lanes should be placed in positions in which they produce maximum stresses in the member under consideration. If the loaded area producing maximum stress in a member is more than two

lanes wide, the intensity of the load may be reduced as explained under " Reduction in Load Intensities " on p. 11.

Truck Loads. — The truck used as a unit for live load in most American specifications is built with two axles, of which the rear axle carries the bulk of the load.

The wheel spacing, weight distribution, and clearance of the trucks that may be used for design purposes are shown in Fig. 3, p. 9.

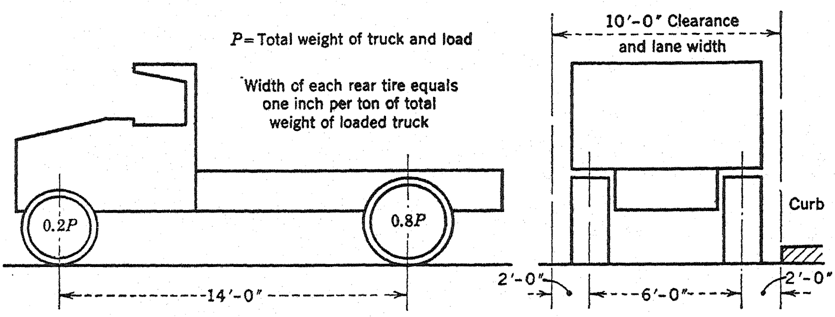


FIG. 3.—Design Truck. (*See p.* 9.)

Loading Specifications. — The loadings here given were agreed upon by the Conference Committee of the American Association of State Highway Officials and the American Railway Engineering Association; they were adopted by these societies in 1929 and readopted in 1935 by the former association. The unit of loading is a truck, the general dimen-

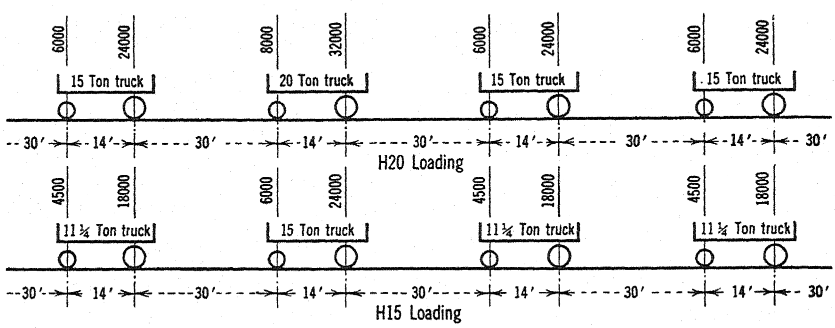


FIG. 4.—Arrangement of Trucks in a Train. (*See p.* 9.)

sions of which are shown in Fig. 3, p. 9. The trucks are arranged in trains in which one heavy truck is preceded and followed by lighter trucks the weight of each of which is 75 per cent of the weight of the heavy truck. The spacing of trucks in a train is shown in Fig. 4, p. 9. Three classes of loadings are specified, namely H20, H15, and H10, of

which the last is intended only for temporary structures. The weights of trucks are given in the following table:

Class	Mark	Weight of Center Truck, tons	Weight of Other Trucks, tons
AA	H20	20	15
A	H15	15	11.25
B	H10	10	7.5

The concentrated truck loadings are specified for loaded lengths of less than 60 ft., although not necessarily confined to this length. For greater loaded lengths, equivalent loadings may be specified. For transverse members such as floor beams, the loaded length is considered as the combined length, longitudinally of the bridge, of the adjacent panels.

Equivalent Loadings. — The equivalent loadings, generally used for loaded lengths greater than 60 ft., consist of uniformly distributed loading combined with a single concentrated load so placed on the span as to produce maximum stress in the member under consideration. The concentrated load should be considered as uniformly distributed across the lane at right angles to the center line.

The equivalent loadings to be used for moments and shears for the three classes are given in the following table:

Class	Mark	Uniform Loading per linear foot of lane, pounds	Concentrated Load	
			For Bending, pounds	For Shear, pounds
AA	H20	640	18 000	26 000
A	H15	480	13 500	19 500
B	H10	320	9 000	13 000

Application of Loadings. — The loadings are applied by that one of the following methods which produces the greater maximum stress in the member considered, due allowance being made for the reduced load intensities hereinafter specified for loaded widths of roadways in excess of two lanes.

1. Each traffic lane loading is considered as a unit, and the number and position of the loaded lanes should be such as will produce maximum stresses.

2. The roadway is considered as loaded over its entire width with a

load per foot of width equal to the load of one traffic lane divided by its width. This applies to both uniform and concentrated loads.

When there are more than two main trusses or girders, as in deck structures of wide roadway, the loaded width for lane loadings is considered as the aggregate width of lanes which may be placed between the trusses or girders adjacent to the one in which the stresses are being computed. The loaded width in the case of a uniformly distributed load is the entire distance, center to center, of the trusses or girders adjacent to the one under consideration.

Reduction in Load Intensity. — If the loaded width of the roadway producing maximum stresses in a member exceeds a two-lane width, the specified intensities of loads are reduced 1 per cent for each additional foot of loaded roadway width with a maximum reduction of 25 per cent. If the loads are lane loads, the loaded width of the roadway is the aggregate width of the lanes considered; if the loads are considered as distributed over the entire width of the roadway, the loaded width of the roadway is as described in the preceding paragraph.

The above provisions for reduction of intensity of load apply to slab spans with main reinforcement parallel to the center line of the roadway only when maximum stresses are produced by loading the entire width of roadway.

The reduction in intensity of floor-beam loads is determined as for main trusses or girders, using the width of roadway which must be loaded to produce maximum stresses in the floor beam.

Heavy-Traffic Loading. — The highway loadings are satisfactory for ordinary country roads. For bridges near large cities and industrial centers, where considerable trucking is to be expected, the authors recommend for live loads the use of trains of 20-ton trucks of the dimensions shown in Fig. 3, but arranged so that the spacing between the rear axle of one truck and the front axle of a truck behind is 19 ft.

Electric-Railway Loadings. — Highway bridges carrying electric railway should be designed (*a*) for highway loading on the entire bridge including the portion of the roadway occupied by the railway; (*b*) electric-railway loading on the railway tracks and the highway loading on the remainder of the roadway. The condition producing largest stress should govern the design.

The loading of the electric railway tracks is governed by the class of traffic expected. Ordinarily the loading consists of two coupled cars, both followed and preceded by uniform loading of the same class as used for the highway loading on the remainder of the roadway. The cars

vary in weight from 20 to 60 tons. The arrangement of wheels of 20-, 40-, and 60-ton cars is shown in Fig. 5, p. 12.

When there is a possibility that the bridge may be required to carry freight cars of steam railroads, their weight may govern the design of the bridge. Arrangement of wheel loads of freight cars is shown in Fig. 6, p. 12.

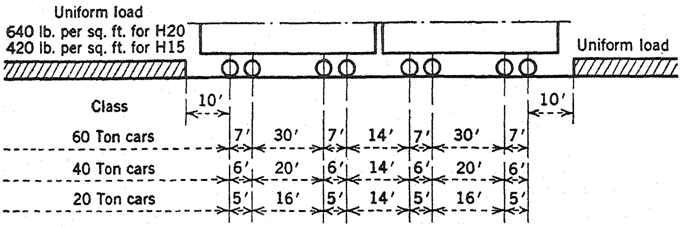


FIG. 5.—Electric Railway Loading. *(See p. 12.)*

Impact. — The effect of the moving loads upon a structure, it is becoming recognized, depends not only upon their magnitude but also upon the ratio of the weight of the structure to the moving load. It is obvious that the effect of a moving load upon a steel structure consisting

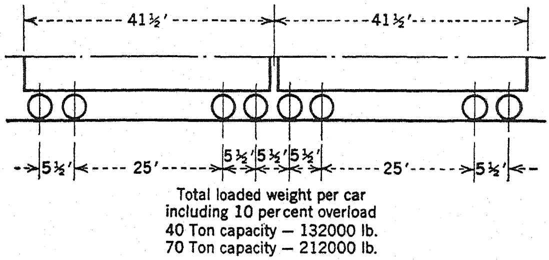


FIG. 6.—Freight Car Loading. *(See p. 12.)*

of a number of thin members, and where the weight of the structure is small in comparison with the live load, is appreciably larger than the effect of the same load upon a solid concrete structure.

In designing bridges, the effect of impact is usually taken care of by increasing in each case the design live load by an amount equal to the design load multiplied by a proper impact ratio.

For impact, let

I = impact ratio.

l = length in feet of that portion of span covered by a moving load to produce maximum stress in that span.

Then, for reinforced-concrete structures, excepting spandrell filled arch bridges for which 50 per cent of the computed value may be used:

Impact ratio:

$$I = \frac{50}{l + 200} \tag{1}$$

LONGITUDINAL, LATERAL AND CENTRIFUGAL FORCES

The following specifications for longitudinal, lateral, and centrifugal forces are from the Standard Specifications mentioned on p. 9.

Longitudinal Force. — Provision should be made for the effect of a longitudinal force of 10 per cent of the live load on the structure, acting 4 ft. above the floor.

Lateral Forces. (*a*) The wind force on the structure is assumed as a moving load equal to 30 lb. per sq. ft. on $1\frac{1}{2}$ times the area of the structure as seen in elevation, including the floor system and the railings, and on one-half the vertical area of all trusses or girders in excess of two in the span.

(*b*) The lateral force due to the wind pressure against the moving live load is considered as acting 6 ft. above the roadway and is as follows: for highway bridges, 200 lb. per lin. ft.; for highway bridges carrying electric railway traffic, 300 lb. per lin. ft.

(*c*) The total assumed wind force is not less than 300 lb. per lin. ft. in the plane of the loaded chord, and 150 lb. per lin. ft. in the plane of the unloaded chord on truss spans; and not less than 300 lb. per lin. ft. on girder spans.

(*d*) In calculating uplift due to the foregoing lateral forces in the posts and anchorages of viaduct towers, highway viaducts are considered as loaded on the leeward traffic lane with the uniform load of 400 lb. per lin. ft. of lane, and viaducts carrying electric railway traffic in addition to the highway traffic are considered as loaded on the leeward track with the uniform load of 800 lb. per lin. ft. of track.

(*e*) A wind pressure of 50 lb. per sq. ft. on the unloaded structure, applied as specified above in paragraph (*a*), is used if it produces greater stresses than the combined wind and lateral forces of paragraphs (*a*) and (*b*).

Centrifugal Force. — If the electric railway track is curved, the structure should be designed to resist a lateral force equal to 10 per cent of the moving railway loading. This lateral force should be considered as acting 4 ft. above the top rail.

Thermal Forces. — Provision should be made for the stresses or movements resulting from the following variations in temperature:

Moderate climate, from 0° to 120° F.
Cold climate, from −30° to 120° F.

The rise and fall in temperature are figured from an assumed temperature at the time of erection.

LIVE LOADS FOR SIDEWALKS AND FOOT BRIDGES

Sidewalks of bridges including floors, stringers, and all supports should be designed for a live load of 100 lb. per sq. ft. of the sidewalk area. Girders or trusses of bridges carrying sidewalks should be designed for a live load on the sidewalk area varying according to a straight line from 100 lb. per sq. ft. for spans of 50 ft. and under, to 70 lb. per sq. ft. for a span of 100 ft. For sidewalks of a width exceeding 5 ft., the unit live loads may be slightly reduced. For a 10-ft. width, nine-tenths of the maximum live load may be used.

For structures with cantilevered sidewalks on both sides, the sidewalks should be considered as loaded on one side or on both sides, whichever condition gives more unfavorable results.

DISTRIBUTION OF CONCENTRATED LOADS BY CONCRETE SLAB

Slabs in a highway bridge may consist of members supported by beams or girders — longitudinal or transverse and, in certain cases, may form the entire bridge.

Effective Width of Slab for Concentrated Loads. — A concentrated load placed upon a wide slab is distributed laterally over a width of slab appreciably greater than the width of the area of contact of the load with the slab. Tests show that, in a slab loaded by a concentrated load, the deflection and the stresses are largest in the portion of the slab directly under the load, and decrease gradually to zero at some distance from the load. To facilitate computations in practice, this variable distribution is replaced by an assumed uniform transverse distribution over a smaller width called the effective width; and this effective width is selected so that the computed maximum stresses in the slab for the assumed distribution are substantially equal to the actual maximum stresses.

The effective width depends upon the ratio of the total width of the slab to its span length; the position of the load in respect to the edge of the slab; the position of the load on the span; and finally upon the condition of restraint of the slab at the supports.

Additional investigations and tests are needed to determine the true distribution of loads by continuous and restrained slabs such as are

commonly used in bridge design, as well as to find the effect of transverse reinforcement upon the distribution.[2]

Formulas for Effective Width of Slab. — The formulas and rules given in the following pages are based on an analysis of various tests. The same unit of length and width must be used throughout.

Let l = span of slab.

l_e = effective width of slab.

g = width of tire.

c = spacing of concentrated loads.

l_1 = distance of center of concentrated load from nearest edge of slab.

D = distance of concentrated load from nearest support.

Concentrated Load in Center of Span of Simply Supported Slab. — The following formulas are for a concentrated load placed in the center of the span. In the first case, covered by formula (2) the load is at an appreciable distance from the edge of the slab; and in the second case, covered by formula (3), the center of the load is placed a distance l_1 from the edge.

Effective Width of Slab for Central Load:

$$l_e = 0.72l + g \tag{2}$$

$$l_e = l_1 + 0.36l + 0.5g \tag{3}$$

Formula (3) applies when l_1 is smaller than $(0.36l + 0.5g)$.

Two Concentrated Loads in Center of Span. — When a simply supported slab is loaded by two concentrated loads placed on the center line of the span and spaced a distance c apart, the effective width should be found for each load separately. If, however, the effective width for one load is larger than the spacing between the loads, then both loads should be treated as one unit, and the effective width for this unit should be taken as equal to the effective width for one load plus the spacing, c, between the loads.

Concentrated Load Not in Center of Span. — When a concentrated load is placed not in the center of the span, but at a distance D from the nearest support, the effective width of slab is based not on the actual span but on a span equal to twice the distance D. For this reason, in formulas (2) and (3) substitute $2D$ for l.

[2] See "Concrete Plain and Reinforced," Vol. I, pp. 69 and 70, also for results of German tests see Bach and Graf in *Deutscher Ausschuss fur Eisenbeton*, Hefte 52 and 54.

When there are several loads on the slab at different distances from the support, it is necessary to find an effective width separately for each load. Then, before computing bending moments, all loads should be reduced to a common effective width.

Effective Width for Continuous and Restrained Slabs. — The formulas and rules for effective widths just given are based on the results of tests of simply supported slabs. Tests of distribution of concentrated loads by continuous or restrained slabs are as yet inconclusive. Therefore, the suggestions in the next paragraph are based on the following reasoning: A loaded span of a continuous slab may be replaced by two end cantilevers, carrying a short simply supported span in between. It is reasonable to assume that the function of distribution is performed by this short simply supported span. Consequently its length should be used in formulas for effective width in place of the actual span of the continuous slab.

It is recommended, therefore, until further tests permit positive conclusion, that the effective widths of continuous spans should be found by substituting in formulas (2) and (3) for the span l (*a*) for interior spans of a continuous slab seven-tenths of the clear span; (*b*) for end spans of continuous slabs, eight-tenths of the theoretical span.

Longitudinal Distribution of Concentrated Loads. — In addition to the lateral distribution of concentrated loads, i.e., the distribution at right angles to the span, there unquestionably takes place also a longitudinal distribution. In American practice usually no longitudinal distribution of concentrated loads is assumed in computations of slabs reinforced in one direction; and the authors approve of this practice. Exception may be made when computing diagonal tension and bond stresses in slabs, because in this case the assumption of concentration of the loads at the ends of the span does not seem reasonable. (See also p. 65.)

When longitudinal distribution is taken into account, it is usually assumed that the loads are distributed at 45°.

CHAPTER III

FORMULAS FOR CONCENTRATED TRUCK LOADS

In modern specifications for highway bridges, concentrated truck loads are considered as units of loading. To facilitate the use of such concentrated loads in design of bridges, formulas are given in this chapter for determining bending moments and shears for several arrangements of truck loads. In the design of simply supported slab and girder bridges, these formulas may be used directly. For statically indeterminate structures, it is usually desirable to use equivalent uniformly distributed loadings. These may be found as explained on p. 27.

The formulas in this chapter are arranged in the following groups:

1. Formulas for longitudinal girders and stringers.
   - *a.* Proportion of each truck carried by one girder or stringer.
   - *b.* Maximum bending moments in longitudinal girders or stringers.
     - Formulas for trucks specified in table on p. 10.
     - General formulas for trains of equal trucks.
     - Formulas for trains of equal trucks spaced 19 ft. apart.
   - *c.* Bending moments at intermediate points.
   - *d.* Maximum external shear diagrams for moving truck loads for the same truck arrangements as under *b*.
2. Formulas for transverse floor beams and slabs spanning between longitudinal through girders.
   - *a.* Bending moments.
   - *b.* External shears.

PROPORTION OF TRUCK LOAD CARRIED BY ONE GIRDER

In a bridge consisting of a number of longitudinal girders, as the first step in determining bending moments and shears in the girders it is necessary to find the portion of the truck train carried by one girder. The same applies to longitudinal stringers.

In general, the loads carried by one girder may be determined by one of the following three methods: (1) by considering the loading on the whole roadway as uniformly distributed in the transverse direction; (2) by placing the loads transversely in the most unfavorable position with respect to the girder under consideration and computing the reaction of the loads upon the girder (see example, p. 288); (3) by combining results from methods (1) and (2).

With truck loads arranged in lanes, as recommended in Chapter II and as is done in the United States and several European countries, repeated computations show that the difference between the results obtained by the three methods is very small. Therefore, no elaborate computations are warranted, and the simplest method, that is, method (1), which assumes uniform distribution, is recommended for use. The proportion of the load carried by one longitudinal girder or stringer may then be found from the following formula.

Portion of Truck Load Carried by One Girder:

$$W = \frac{1}{2} \frac{s + s_1}{c} P \tag{1}$$

where P = truck load used in design, load on both axles.

W = portion of the truck load carried by one girder.

s, s_1 = spacings of center lines of girders on both sides of this girder.

c = clearance width of design truck.

This formula is based on the assumption that each axle load is distributed uniformly across the traffic lane and at right angles to its center line and that the roadway slab is made up of freely supported one-span slabs, so that its reaction upon the girder can be computed by the rules of simple statics.

MAXIMUM BENDING MOMENTS IN LONGITUDINAL GIRDER DUE TO TRUCKS

Maximum bending moments in longitudinal girders due to concentrated truck loads are found for the most unfavorable positions of the loadings on the girder. The truck trains must not only be placed transversely on the roadway so as to produce maximum reactions on the girder under consideration, but also they must be placed in the most unfavorable position longitudinally, that is, so that the center of the span is midway between the resultant of all the axle loads on the girder and the nearest rear axle load. In Fig. 7, p. 19, are shown two trucks placed longitudinally on the girder in the most unfavorable position. The dash line indicates the corresponding bending-moment diagram. The distance between the resultant and the nearest rear axle is here called r.

To facilitate computations of maximum bending moments, formulas are here given for the following arrangements of trucks.

1. Arrangement of trucks as given in the table on p. 10. Heavy truck P preceded and followed by lighter trucks $0.75P$; axles spaced

$l_1 = 14$ ft. and trucks spaced $l_2 = 30$ ft. apart measured between the front axle of one truck and the rear axle of the next truck.

2. Train of trucks of equal weights P spaced any distance l_2 apart.

3. Train of equal trucks P in which distance l_2 is 19 ft., as recommended by the authors on p. 11. Spacing of axles $l_1 = 14$ ft.

In each case the formulas are expressed in terms of W, which is the

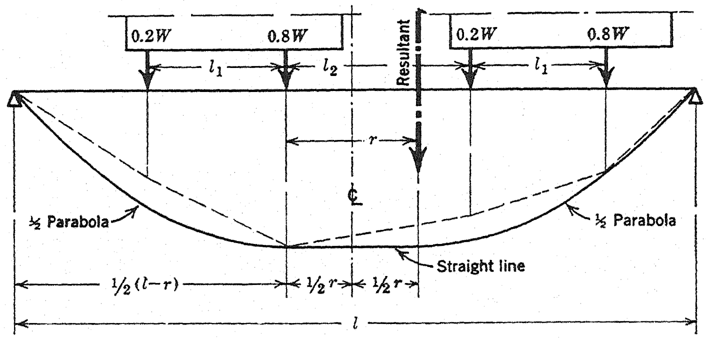


FIG. 7.—Diagram of Largest Possible Bending Moments Due to Truck Loads. *(See p. 18.)*

part of each truck load in the truck train carried by one girder. As for the truck load, $0.8W$ is carried by the rear axle and $0.2W$ by the front axle. See formula (1), p. 18, for value of W in terms of P.

In addition to the notation on p. 18, let

l = span of girder.
l_1 = spacing between front and rear axle in a truck.
l_2 = longitudinal distance between rear axle of one truck and front axle in next truck in a train of trucks.
r = distance between resultant of loads and nearest rear axle.

1. For loading specified in the table on p. 10:

For spans of girder up to 25.2 ft.

$$M_{\text{max.}} = 0.2Wl \tag{2}$$

For spans of girder between 25.2 and 58.5 ft.

$$M_{\text{max.}} = 0.25\left(1 - \frac{2.8}{l}\right)^2 Wl \quad \text{and} \quad r = 2.8 \text{ ft.} \tag{3}$$

For spans of girder between 58.5 ft. and the specified upper limit for the use of concentrated wheel loads.

$$M_{\text{max.}} = \left[0.288\left(1 - \frac{1.48}{l}\right)^2 - \frac{2.8}{l}\right] Wl \quad \text{and} \quad r = 1.48 \text{ ft.} \tag{4}$$

The value W is based on the weight P of the heavy truck in the train.

2. For trains of equal trucks, P, with a longitudinal spacing of axles, l_1, and longitudinal spacing of trucks, l_2:
For spans from $l = 1.8l_1$ to $(0.167l_1 + 1.833l_2)$

$$M_{\text{max.}} = 0.25\left(1 - 0.2\frac{l_1}{l}\right)^2 Wl \quad \text{and} \quad r = 0.2l_1 \tag{5}$$

For spans from $l = (0.167l_1 + 1.833l_2)$ to $(1.7l_1 + 1.5l_2)$

$$M_{\text{max.}} = \left\{\left[0.3\left(1 - \frac{1}{6}\frac{l_2 - l_1}{l}\right)^2\right] - 0.2\frac{l_1}{l}\right\} Wl \quad \text{and} \quad r = \frac{1}{6}(l_2 - l_1) \tag{6}$$

For spans from $l = (1.7l_1 + 1.5l_2)$ to $(3.364l_1 + 1.636l_2)$

$$M_{\text{max.}} = \left\{\left[0.5\left(1 - 0.3\frac{l_1}{l} - 0.5\frac{l_2}{l}\right)^2\right] - 0.2\frac{l_1}{l}\right\} Wl$$

and

$$r = \tfrac{1}{2}(l_2 + 0.6\, l_1) \tag{7}$$

For spans from $l = (3.364l_1 + 1.636l_2)$ to $(3.8l_1 + 2l_2)$

$$M_{\text{max.}} = \left\{\left[0.55\left(1 - 0.364\frac{l_1}{l} - 0.636\frac{l_2}{l}\right)^2\right] - 0.2\frac{l_1}{l}\right\} Wl$$

and

$$r = 0.364l_1 + 0.636l_2 \tag{8}$$

3. For trains of equal trucks in which spacing of axles is $l_1 = 14$ ft., and distance between rear axle of one truck and front axle of next truck is $l_2 = 19$ ft.:

Item	Span Limits, feet	r, feet	$M_{\text{max.}}$, foot-pounds, when W is in pounds, l in feet	Loading
1	under 25.2 ft.	....	$0.2\ Wl$	1 rear axle
2	25.2–37.2	2.8	$0.25\left(1 - \frac{2.8}{l}\right)^2 Wl$	1 truck
3	37.2–52.3	0.833	$\left[\left(0.548 - \frac{0.455}{l}\right)^2 - \frac{2.8}{l}\right] Wl$	1 truck and 1 front axle
4	52.3–88.18	13.7	$\left[\left(0.707 - \frac{9.694}{l}\right)^2 - \frac{2.8}{l}\right] Wl$	2 trucks
5	88.18–91.2	17.18	$\left[\left(0.742 - \frac{12.75}{l}\right)^2 - \frac{2.8}{l}\right] Wl$	2 trucks and front axle
6	over 91.2	2.8	$\left(0.75 - \frac{34.4}{l} + \frac{5.88}{l^2}\right) Wl$	3 trucks

BENDING-MOMENT DIAGRAM AT INTERMEDIATE POINTS DUE TO TRUCKS

To determine points where reinforcement of a girder may be bent up, it is necessary to draw a diagram representing the largest possible bending moments at all points of the girder. This diagram is different from the bending-moment diagram for the loading which produces the absolute maximum bending moment in the girder. In Fig. 7, p. 19, is shown the position of the loads which produces the absolute maximum bending moment in the left half of the girder; the dash lines indicate the bending-moment diagram for this loading, and the heavy lines represent the diagram of largest bending moments for the girder.

To get an exact diagram for the largest bending moments, it would be necessary to find the largest bending moments for a sufficient number of intermediate points; plot them; and finally draw a smooth curve, which then would represent the diagram of the largest bending moments. The most unfavorable loading for any intermediate point occurs when a rear axle is placed directly over that point, and the balance of the girder is loaded with as many loads as can be accommodated.

The amount of work required for preparing a diagram in the manner just described may be greatly reduced, without any appreciable loss in accuracy, by proceeding in the following manner: At both sides of the center of the girder locate the points of absolute maximum bending moments, which are distant $\frac{r}{2}$ from the center, and at these points plot the maximum bending moment as ordinates. Draw one half of a parabola[1] at each end of the girder between the support and the point $\frac{1}{2}(l - r)$ distant from the support, using the maximum bending moment as the maximum ordinate. Connect the two curves by a straight line; and the composite curve, shown in Fig. 7, p. 19, by a heavy line, may be accepted as the largest bending-moment diagram for the girder.

EXTERNAL SHEARS IN LONGITUDINAL GIRDERS FOR MOVING LIVE LOADS

Diagonal tension and diagonal tension reinforcement in a girder are governed by external shears. To solve the problem of diagonal tension, it is necessary to find not only maximum end shears for the moving live loads, but also the largest possible external shears at intermediate points. For this purpose external-shear diagrams may be prepared in the manner here explained.

Maximum end shears for moving loads are produced by a loading in

[1] See Fig. 95, p. 213, for simple method of drawing a parabola.

which a heavy rear axle is placed on the girder next to the support, and the balance of the span is loaded with as many loads as can be accommodated longitudinally on the girder.

Largest possible positive external shears at any intermediate point of a girder are produced when the rear axle of a truck is placed directly above the point under consideration, the balance of the span to the right of the point is fully loaded, and no load is placed between the point and the left support.

External-Shear Diagram. — External-shear diagram is obtained by plotting on the span the maximum end shear and the largest shears at intermediate points. The preparation of the diagram is facilitated by the fact that the shear diagram for moving loads is a straight line as long as all loads remain on the span. When, in order to fulfil the requirement for most unfavorable position, a load of the train leaves the span, a break occurs in the diagram. The easiest way of drawing an external-shear diagram is to find the maximum end shear and the largest shears at the points at which these breaks occur. A positive shear diagram for only one half of the span needs to be drawn, because the other half is governed by a shear diagram of opposite sign produced by loads moving in the opposite direction. The two diagrams are symmetrical about the center of the span. (See Fig. 8, p. 22, for shear diagram for loading arrangement 2, case *c*, below.)

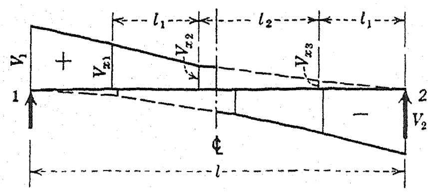


FIG. 8.—Shear Diagram for Moving Loads. *(See p. 22.)*

Formulas are given for maximum end shear and for shears at the breaks for the same arrangements of trucks as described under 1 to 3 on p. 19 in connection with maximum bending-moment formulas.

In addition to notation on p. 19, let

V_1 = maximum positive end shear.

V_{x1}, V_{x2}, V_{x3} = largest external shears at points x_1, x_2, x_3 respectively from left support.

1. For loading specified in table on p. 10:

a. Spans of girder from l = 14 to 44 ft.:

$$V_1 = \left(1 - \frac{2.8}{l}\right)W \tag{9}$$

$$V_{x1} = \frac{11.2}{l}W; \quad \text{for } x_1 = l - 14 \tag{10}$$

b. Spans of girder from $l = 44$ ft. to the limit for which concentrated loads are specified:

$$V_1 = \left(1.6 - \frac{29.2}{l}\right)W \tag{11}$$

$$V_{x1} = \frac{41.2}{l}W; \quad \text{for } x_1 = l - 44 \tag{12}$$

$$V_{x2} = \frac{11.2}{l}W; \quad \text{for } x_2 = l - 14 \tag{13}$$

2. External shears for trains of equal trucks, P; longitudinal spacing of axles l_1; longitudinal spacing of trucks measured between axles, l_2.

a. Spans between $l = l_1$ and $(l_1 + l_2)$:

$$V_1 = \left(1 - 0.2\frac{l_1}{l}\right)W \tag{14}$$

$$V_{x1} = 0.8\frac{l_1}{l}W; \quad \text{for } x_1 = l - l_1 \tag{15}$$

b. Spans from $l = (l_1 + l_2)$ to $(2l_1 + l_2)$:

$$V_1 = \left(1.8 - \frac{l_1}{l} - 0.8\frac{l_2}{l}\right)W \tag{16}$$

$$V_{x1} = \left(0.8\frac{l_1}{l} + \frac{l_2}{l}\right)W; \quad \text{for } x_1 = l - (l_1 + l_2) \tag{17}$$

$$V_{x2} = 0.8\frac{l_1}{l}W; \quad \text{for } x_2 = l - l_1 \tag{18}$$

c. Spans from $l = (2l_1 + l_2)$ to $2(l_1 + l_2)$:

$$V_1 = \left(2 - 1.4\frac{l_1}{l} - \frac{l_2}{l}\right)W \tag{19}$$

$$V_{x1} = \left(2.6\frac{l_1}{l} + \frac{l_2}{l}\right)W; \quad \text{for } x_1 = l - (2l_1 + l_2) \tag{20}$$

$$V_{x2} = \left(0.8\frac{l_1}{l} + \frac{l_2}{l}\right)W; \quad \text{for } x_2 = l - (l_1 + l_2) \tag{21}$$

$$V_{x3} = 0.8\frac{l_1}{l}W; \quad \text{for } x_3 = l - l_1 \tag{22}$$

d. Spans from $l = 2(l_1 + l_2)$ to $(3l_1 + 2l_2)$

$$V_1 = \left(2.8 - 3\frac{l_1}{l} - 2.6\frac{l_2}{l}\right)W \tag{23}$$

$$V_{x1} = \left(2.6\frac{l_1}{l} + 3\frac{l_2}{l}\right)W; \quad \text{for } x_1 = l - 2(l_1 + l_2) \tag{24}$$

$$V_{x2} = \left(2.6\frac{l_1}{l} + \frac{l_2}{l}\right)W; \quad \text{for } x_2 = l - (2l_1 + l_2) \tag{25}$$

$$V_{x3} = \left(0.8\frac{l_1}{l} + \frac{l_2}{l}\right)W; \quad \text{for } x_3 = l - (l_1 + l_2) \tag{26}$$

3. External shears for trains of equal trucks, P; axle spacing of truck $l_1 = 14$ ft.; distance between trucks, $l_2 = 19$ ft.

Item	Span Length, feet	V_1	V_x at Points Distant from Left Support				
			$l - 80$ ft.	$l - 66$ ft.	$l - 47$ ft.	$l - 33$ ft.	$l - 14$ ft.
1	Under 14 ft.	$0.8W$	....	....	....	....	....
2	14 to 33	$\left(1 - \frac{2.8}{l}\right)W$	....	....	....	....	$\frac{11.2}{l}W$
3	33 to 47	$\left(1.8 - \frac{29.2}{l}\right)W$	....	....	....	$\frac{30.2}{l}W$	$\frac{11.2}{l}W$
4	47 to 66	$\left(2 - \frac{38.6}{l}\right)W$	....	....	$\frac{55.4}{l}W$	$\frac{30.2}{l}W$	$\frac{11.2}{l}W$
5	66 to 80	$\left(2.8 - \frac{91.4}{l}\right)W$	....	$\frac{93.4}{l}W$	$\frac{55.4}{l}W$	$\frac{30.2}{l}W$	$\frac{11.2}{l}W$
6	80 to 99	$3\left(1 - \frac{35.8}{l}\right)W$	$\frac{132.6}{l}W$	$\frac{93.4}{l}W$	$\frac{55.4}{l}W$	$\frac{30.2}{l}W$	$\frac{11.2}{l}W$

W is part of truck loads carried by one girder. See formula (1), p. 18.

BENDING MOMENTS AND SHEARS IN TRANSVERSE FLOOR BEAMS

The following formulas may be used to determine bending moments and shears in floor beams and slabs of through bridges, when the floor beams and slabs span between longitudinal girders. In these formulas the beams and slabs are considered as simply supported. See p. 74 for the effect of the restraint at the ends.

Formulas are given only for the conditions where the live load con-

sists of two lines of trucks, i.e., of two traffic lanes. If more than two lanes form the loading, the concentrated wheel loads should be replaced by a loading uniformly distributed transversly, and obtained by considering each axle load as uniformly distributed over the whole width of the traffic lane at right angles to its center line.

Bending moments and shears in the formulas are expressed in terms of W, which is the part of the truck load transmitted to the floor beam. When these formulas are used for designing slabs of through bridges, W should be taken as the load on the rear axle of one truck, and the effect of the bending moment or shear should be considered as distributed longitudinally with the bridge over the effective width of the slab found as given on p. 15.

In addition to notation on p. 19, let

s = longitudinal spacing of floor beams.

b = transverse spacing of wheels of a truck.

$e = c - b$ = truck clearance width minus spacing of wheels.

f = distance from support of floor beam or slab to center line of nearest wheel load, not less than $\frac{e}{2}$.

W = proportion of truck load carried by floor beam.

Value of W for Floor Beams. — The reaction of truck loads upon a floor beam depends upon the longitudinal spacing of floor beams, s.

For spacings of floor beams of less than 6 ft., the rear axle load even if placed directly over a floor beam is distributed by the slab to the adjoining floor beams. The loadings of the floor beam may be taken as consisting of fractions of the loads on rear axle. Then $W = \frac{s}{6.0} \times 0.8P$.

For spacings of floor beams larger than 6 ft. and smaller than the spacing of axles in a truck, l_1, $W = 0.8P$.

For spacings of floor beam larger than the axle spacing l_1 and smaller than the spacing of trucks, l_2, $W = \left(0.8 + 0.2\frac{s - l_1}{s}\right)P$.

For spacings of floor beam larger than the spacing of trucks, l_2, $W = \left(0.8 + 0.2\frac{2s - (l_1 + l_2)}{s}\right)P$.

Maximum Bending Moment in Floor Beam. — Maximum bending moment in a floor beam due to two lines of trucks are given by the following formula:

Maximum bending moment:

$$M_{\text{max.}} = \frac{1}{2}\left[\left(1 - \frac{e}{2l}\right)^2 - \frac{b}{l}\right]Wl \tag{27}$$

Point of maximum bending moment from support:

$$x = \tfrac{1}{2}(l - \tfrac{1}{2}e) \tag{28}$$

The maximum bending-moment diagram for floor beam is clearly shown in Fig. 9, p. 26.

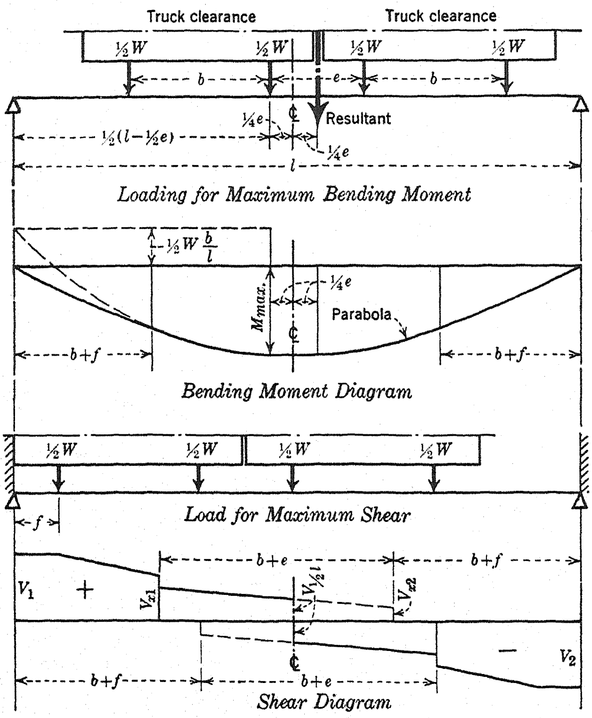


FIG. 9.—Bending Moments and Shears in Floor Beam for Truck Loads. (*See p.* 26.)

External Shears in Floor Beam for Moving Live Load. — Maximum end shear and largest shears at intermediate points may be found from the following formulas. In the formulas let f be the smallest possible

distance between the support of the floor beam and the center line of the nearest wheel load.

Maximum end shear. Wheel distance f from support:

$$V_1 = 2\left[1 - \frac{2b + e + 2f}{2l}\right]W \tag{29}$$

Shear at $x_1 = l - (2b + e + f)$ *from left support:*

$$V_{x1} = \left(\frac{2b + e + 2f}{l}\right)W \text{ left} \tag{30}$$

$$V_{x1} = \left(\frac{1.5b + e + f}{l}\right)W \text{ right} \tag{31}$$

Shear at $x_2 = l - (b + f)$:

$$V_{x2} = \left(\frac{b + 2f}{2l}\right)W \tag{32}$$

Shear at center of span:

$$V_{\frac{1}{2}l} = \frac{1}{2}\left(\frac{l - b}{l}\right)W \tag{32}$$

EQUIVALENT UNIFORMLY DISTRIBUTED LIVE LOAD IN LONGITUDINAL DIRECTION

In designing statically indeterminate structures, it is often desirable to replace the concentrated wheel loads already treated by an equivalent uniformly distributed loading.

The simplest method of determining the equivalent uniformly distributed loading is to compute for concentrated loads the maximum positive bending moment for the span under consideration, assuming it to be statically determinate; to equate the result to the formula for maximum positive bending moment for uniform loading, $M = \frac{1}{8}wl^2$; and from the equation to find the value w. This value may then be used as equivalent uniformly distributed loading for the statically indeterminate structure.

For external shears, the equivalent loading may be different from the equivalent loading determined for bending moments. It should be computed by finding for a simply supported span the maximum end shear for concentrated loadings. This is equated to $V_1 = \frac{1}{2}wl$, and from this equation the value of w is found.

Shear Diagram for Uniformly Distributed Moving Loading. — To get a shear diagram for a simply supported span subjected to uniformly distributed loading, find the end shear and the shears at several inter-

mediate points. The curve connecting the plots at these points is the shear diagram. The left half of the girder is governed by positive shears, and the right half by negative shears which are symmetrical with the positive shears about the center of the span. Fig. 10, p. 28, shows the shear diagram. There are also indicated the values of the end shear and of the shears at several intermediate points.

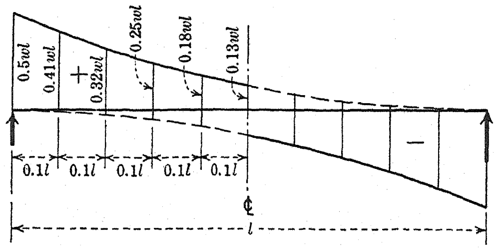


FIG. 10.—Shear Diagram. Uniformly Distributed Moving Loads. (See p. 28.)

Equivalent Uniformly Distributed Loading for Cantilevers. — Equivalent uniformly distributed loading for cantilevers is always much larger than the loading for the main spans of the girder. The shorter the cantilever, the larger is the difference. For instance, a cantilever 20 ft. long would have a unit load three times larger than a 60-ft. main span. A 10-ft.-long cantilever would require a unit load six times larger than that of the main span.

It is recommended that, in computing bending moments and shears in cantilevers, the concentrated wheel loads be used even when the main spans are designed for equivalent uniformly distributed loading. In no case should the same unit live load be used for the cantilevers as for the main spans.

CHAPTER IV

SLAB BRIDGES

This chapter, devoted to simply supported slab bridges, gives description of the slab structures; design details; methods of design; and a numerical example of slab bridge design. Slab highway bridges and railroad bridges are treated under separate headings.

General Characteristics of Slab Bridges. — Slab bridges may be simply supported structures, in which case they consist of one or several one-span slabs simply supported by vertical supports; or they may be continuous structures, in which the slab extends over several spans. If the slab forming the superstructure is rigidly connected with the abutments, the structure becomes a rigid frame which is treated in Chapter XIII.

Slab bridges require more concrete and steel than girder bridges of the same spans; but, since their formwork is simpler and less expensive, they are economical when the saving in cost of formwork is greater than the additional cost of materials. The limits within which slab bridges are economical, therefore, depend upon the relation between the unit cost of materials and the unit cost of formwork. Where the cost of formwork is large in comparison with the cost of materials, as it is in the United States, the limiting span is larger than where the formwork is relatively cheap. Thus, in the United States, slab highway bridges have been found economical for spans up to 30 ft., but in Europe, on account of the relatively low cost of formwork, simply supported slab bridges are seldom used for spans longer than 18 ft.

The smaller headroom with slab bridges may also increase the limit within which slab bridges are economical. See also the discussion on p. 35.

For railroad bridges, simply supported slabs are used extensively, especially in the precast form. Owing to the special conditions governing railroad work, railroad slab bridges are used in the United States for longer spans than would be economical in highway construction.

HIGHWAY SLAB BRIDGES

Details of Design. — A highway slab bridge consists in cross section of a slab bordered at both sides by curbs, which either form an integral part of the structural slab, or are poured separately after the slab has

hardened and the formwork has been removed. Sidewalks, when used, may be supported on thinner slabs and separated from the roadway slab by longitudinal joints, or the slabs may be joined as shown in Fig. 11 (*b*), p. 30. This design may be modified by making the underside of the

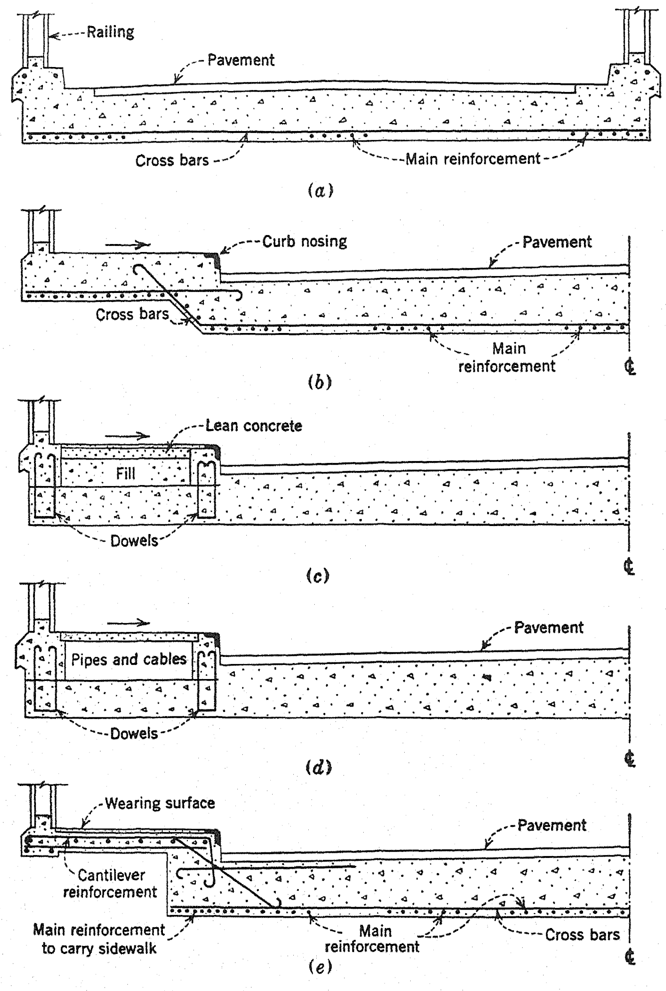


Fig. 11.—Typical Cross Sections of Slab Bridges. (*See p.* 30.)

structural slab level and resting the sidewalk slab on fill. In a further modification, clear space for pipes and ducts is provided between the lower structural slab and the sidewalk slab. Finally the sidewalk slab may be cantilevered out from the main slab. The cross sections of the slab for all these modifications are shown in Fig. 11 (*a*) to (*e*), p. 30.

Provision for Expansion and Temperature Reinforcement. — Expansion bearings for simply supported slab bridges have not been found by experience to be effective. A simple arrangement, where they are used, has been to place, at one end of each span, several thicknesses of roofing felt upon the bridge seat between the support and the slab.[1] But in modern designs expansion bearings are often dispensed with, and the slab is anchored to the supports by vertical steel dowels. These prevent horizontal movements of the slab, but are not effective to transmit bending movements from the slab to the supports, and vice versa.

When a slab is doweled to the supports, shrinkage and drop of temperature produce tension in the slab. This should be resisted by longitudinal reinforcement placed in the bottom of the slab with, and in addition to, the main reinforcement. The cross section of the temperature reinforcement should not be less than 0.2 per cent of the effective cross section of the slab.

In designing abutments to resist earth pressure, advantage may be taken, at the top, of the resistance of the slab to which they are doweled.

Drainage. — The slab should be provided with drains spaced along each curb not more than 10 ft. apart, and arranged to avoid discharge of drainage water against part of any surface. At least two drains should be used at each side.

To facilitate drainage, the top of the roadway is crowned transversely by thickening the slab in the center. When pavement is superimposed, the top of the structural slab may be level and the crown provided in the pavement.

Design of Slab. — The design of the slab is simple. Bending moments are found separately for dead load and for live load. The sum of the maximum positive bending moments governs the thickness of the slab and the amount of reinforcement.

In a simply supported slab, only the maximum positive bending moment needs to be computed. In a continuous slab, bending moments must be computed at all critical sections in the manner explained in Chapter IX or X.

For simply supported slabs, concentrated wheel loads should be used in computing bending moments and external shears. No longitudinal distribution should be assumed, except in computing end shear. (See p. 65.) Maximum bending moments are found in the manner explained on p. 18, and formulas there given may be used. They should be computed for one truck line and assumed to be uniformly distributed laterally over the width of a traffic lane. The span length for a simply supported slab should be the distance between the center lines of the

[1] Other more expensive expansion bearings are described in Chapter XV.

bearings, but it should not exceed the clear span plus the depth of the slab.

Slabs for highway bridges, when properly designed for bending moments, are usually strong enough to resist diagonal tension and bond stresses but should be checked for these. It is good practice to form hooks at the ends of the bottom bars, and also to bend up some of the bars at the ends to act as diagonal tension reinforcement.

Design for Skew Crossings. — For small skews, the main reinforcement of the slab is placed longitudinally with the direction of the road, and not at right angles to the supports. Then the span length is also measured along the same line.

When the angle of the skew is large, however, it is more economical to place the reinforcement at right angles to the supports and use in design the span measured at right angles to the supports. The triangular portion of the slab at each side of the slab, which with this arrangement has no support along its edge, should be supported by a parapet girder which may be extended above the roadway to obtain sufficient depth.

Slab with Cantilevered Sidewalks. — When the sidewalks are cantilevered out as shown in Fig. 11 (*e*), p. 30, the cantilever slab should be reinforced by transverse bars placed near the top of the slab and properly anchored in the main slab. Cantilever loads produce also in the roadway slab transverse negative bending moments which should be resisted by transverse reinforcement. The main slab next to the cantilever should be made strong enough to carry in addition to the loads directly coming upon it the total load on the cantilever plus the value $\frac{M_c}{l}$, where M_c is the cantilever bending moment for live load or for any other unsymmetrical loading, and l is the width of the main span. In Fig. 11 (*e*) the main reinforcement next to the cantilever is spaced much closer than in the rest of the slab.

Cross Reinforcement of Slab Bridges. — To distribute the concentrated loads transversely, and also to prevent shrinkage and temperature cracks, the slab should be provided with reinforcement placed at right angles to the main reinforcement, except in skew bridges where this reinforcement should be placed parallel to the supports. The cross bars should be placed on top of the main reinforcement and be tied to it. Total cross section of cross bars should be from 0.15 to 0.20 per cent of the effective cross section of the slab.

Numerical Example of Slab Design. — The design of a slab bridge is illustrated by the following numerical example. (See Fig. 12, p. 33.)

In solving this and similar problems, the accuracy obtained by the use of a 10 in. slide rule is sufficient.

Example.—Design a slab highway bridge supported by 12.-in-wide abutment walls. Clear span, 25 ft.; width of roadway between curbs, 20 ft.; traffic lanes, 10 ft. wide each.

Live load, 20-ton trucks, $P = 40\,000$ lb., of dimensions shown in Fig. 3, p. 9; pavement, 40 lb. per square feet; impact ratio as given in formula (1), p. 18.

Allowable stresses in pounds per square inch: $f_c = 800$, $f_s = 16\,000$, $v = 40$, $u = 100$.

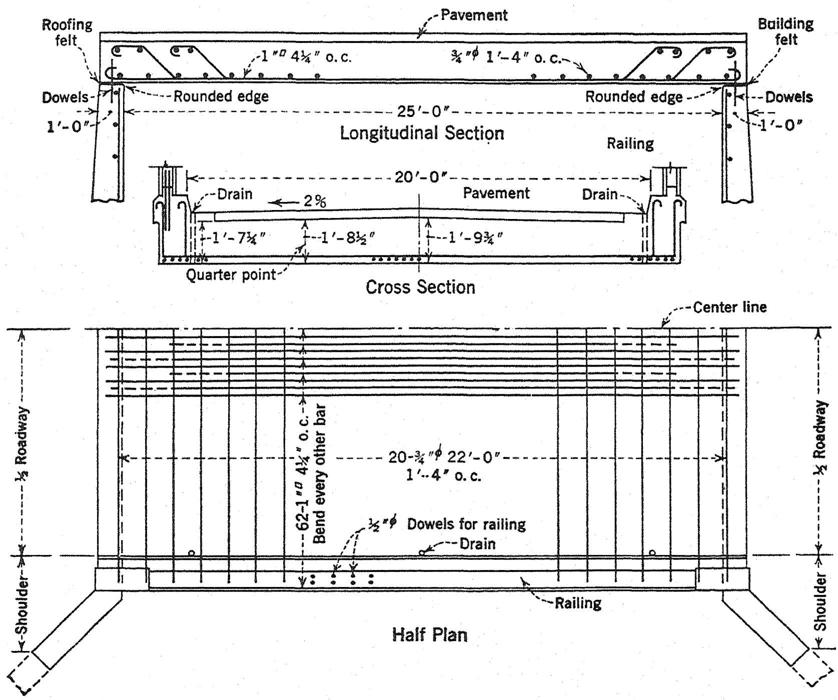


FIG. 12.—Example of Slab Design. (*See p.* 32.)

Constants: $p = 0.0107$, $j = 0.857$, $R = 146.9$.

Solution. — Theoretical span, $l = 25 + 1 = 26$ ft.

Dead Load.	Structural slab	$\frac{21}{12} \times 150 = 262$ (assumed)
	Pavement	40
	Total	$w = 302$ lb. per sq. ft.

Maximum bending moment for dead load $M_D = \frac{1}{8} \times 302 \times 26^2 \times 12 = 306\,000$ in.-lb. per ft. of width.

Live Load. Impact ratio is $I = \dfrac{50}{26 + 200} = 0.22$. Therefore, to take care of impact, multiply the design truck load by 1.22. Thus $P = 1.22 \times 40\,000 = 48\,800$

lb. For maximum bending moment for live load use the formula from the table on p. 20, item 2. Substitute $W = P = 48\,800$ lb.

$$M_{L+I} = 0.25\left(1 - \frac{2.8}{26}\right)^2 \times 48\,800 \times 26 \times 12 = 3\,026\,000 \text{ in.-lb. per 10-ft. lane}$$

and

$$M_{L+I} = \tfrac{1}{10} \times 3\,026\,000 = 302\,600 \text{ in.-lb. per ft. of width}$$

Total maximum bending moment

$$M = 302\,600 + 306\,000 = 608\,600 \text{ in.-lb. per ft. of width of slab}$$

Consequently $d = \sqrt{\dfrac{M}{bR}} = \sqrt{\dfrac{608\,600}{12 \times 146.9}} = 18.5$ in. Use $h = 20.5$ in. and

$A_s = pbd = 0.010\,7 \times 12 \times 18.5 = 2.38$

For temperature $0.002 \times 12 \times 18.5 = \underline{0.44}$

Total $A_s = 2.82$ sq. in. per ft. of width of slab

Use 1-in. square bars spaced $4\frac{1}{4}$ in. on centers. This gives $A_s = 1.0 \times \dfrac{12}{4.25} =$ 2.83 sq. in. The same spacing is used throughout the whole width of slab. Every other bar is bent up at the ends. All bars are provided with hooks at both ends.

The details of the slab are shown in Fig. 12, p. 33.

Diagonal Tension and Bond Stresses. —

End shear for dead load: $V_D = 302 \times 13 = 3\,930$ lb. per ft. of width.

End shear for live load[2] plus impact is found from the formula in the table on p. 24, item 2. $W = 48\,800$ lb., including impact.

$$V_{L+I} = \left(1 - \frac{2.8}{26}\right) \times 48\,800 = 43\,500 \text{ lb. per 10-ft. lane}$$

$$= 4\,350 \text{ lb. per ft. of width}$$

Total end shear

$$V_T = 3\,930 + 4\,350 = 8\,280 \text{ lb. per ft. of width}$$

Unit diagonal tension stress

$$v = \frac{8\,280}{12 \times 0.857 \times 18.5} = 44 \text{ lb. per sq. in.}$$

This exceeds the allowable 40 lb. per sq. in. The design, however, is satisfactory because horizontal bars are hooked at ends, and every other bar is bent up. Both these factors increase the diagonal tension resistance of the slab.

Unit bond stresses are computed at the supports where the bottom reinforcement consists of 1-in. square bars spaced $8\frac{1}{2}$ in. on centers

$$i = 4 \times 1.0 \times \frac{12}{8.5} = 5.64 \text{ in.}$$

$$u = \frac{8\,280}{5.64 \times 0.857 \times 18.5} = 93 \text{ lb. per sq. in.}$$

Bond stresses are satisfactory.

[2] As explained on p. 65, in computing shearing stresses, concentrated wheel loads could be considered as distributed longitudinally, which would give smaller values.

Cross Reinforcement. — Using for cross-section area of transverse reinforcement 0.15 per cent of the effective cross section of the slab, the area of cross bars is $A_s = 0.0015 \times 12 \times 18.5 = 0.33$ sq. in. per ft. of width of slab.

Use $\frac{3}{4}$-in. round bars spaced 1 ft. 4 in. on centers. $A_s = \dfrac{0.44}{16} \times 12 = 0.33$ sq. in.

Dowels for Railing. — Two $\frac{1}{2}$-in. round dowels spaced 1 ft. on centers are arbitrarily provided to tie the concrete railings to the slab.

CONTINUOUS SLAB BRIDGES

In a bridge consisting of a number of spans, when reasonably unyielding foundations are easily obtainable, a superstructure consisting of a continuous slab extending over several spans may be used instead of several simply supported slab spans. The supports also may be rigidly connected with the slabs, thus changing the construction to a multi-span rigid frame. In either case, the resulting design is cheaper and more rigid than one consisting of a number of simply supported spans.

For longer spans, when the headroom below is not limited, a continuous girder design may be more economical than a continuous slab. However, where the headroom is limited, or where the length and the height of the approaches depend upon the depth of the structure, a continuous slab may be economical even for long spans. Structures have been built with spans of slab up to 70 ft. with a ratio of thickness of slab to span of $\frac{1}{32}$.

Continuous slabs have the following advantages over continuous girders:

Smaller thickness of construction, which may mean reduced height of fill for approaches.

Cheaper and simpler formwork.

Simpler arrangement of reinforcement. Main reinforcement is evenly distributed over the whole width of bridge instead of being concentrated at the girders, and one layer of longitudinal bars may be sufficient instead of the several layers required for girders. Absence of stirrups and of the additional slab reinforcement needed in the girder design.

Placing of concrete in thick slabs simpler than in girders and thin slabs. Much less spading required.

Smaller areas of exposed concrete surfaces, and therefore smaller cost of surface finish. Less chance of honeycomb and porous concrete at the surface.

Fewer critical sections in design.

Better distribution of loading both laterally and longitudinally.

The disadvantage of the slab type of bridge, on the other hand, except for short spans are:

Greater cost of materials.

Larger dead loads.

In most cases, the extra cost overbalances the advantages. Comparative cost estimates are essential.

Bending Moments and Shears in Continuous Slabs. — Bending moments and shears in continuous slabs should be computed in exactly the same manner as explained later in connection with continuous girder bridges.

For equal spans, bending-moment coefficients in Chapter IX should be used. For unequal spans, use the fixed point method described in Chapter X. When slab is rigidly connected with the supports, use the method given in Chapter XI for multi-span rigid frames. In no case should arbitrary bending-moment coefficients be used.

Thickness of slab in each span and the required bottom reinforcement are governed by the maximum positive bending moments. At the supports negative bending moments are usually larger than the maximum positive bending moments. A larger effective depth may be provided at the supports by haunches, but if desired uniform thickness may be maintained, because the allowable compression unit stresses at the supports are greater than in the center of the span; and also the bottom reinforcement at the supports may be utilized as compression reinforcement.

Rules given in connection with continuous girders apply also to the design of continuous slabs. (See pp. 165 and 194.)

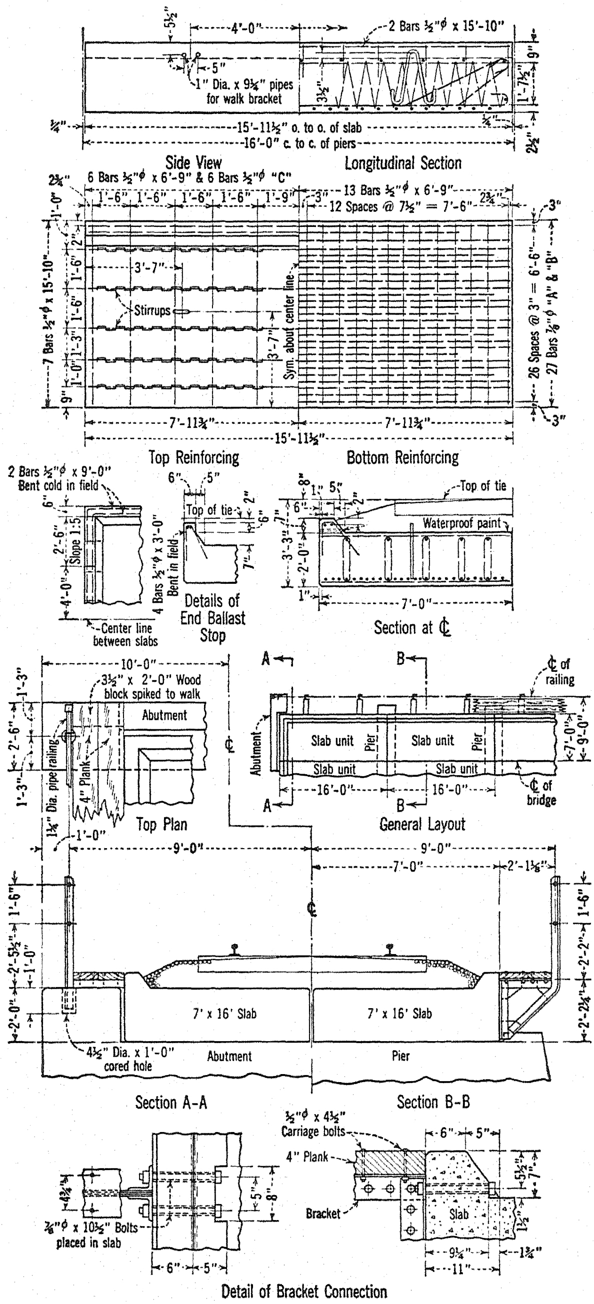
SLABS FOR RAILROAD BRIDGES

In railroad construction, slab bridges are used to a large extent for pier and pile trestles, as explained on p. 428, and for undercrossings, as described on pp. 42 and 43. The deck of these structures may consist of precast slabs or slabs built in place.

Precast slabs may be used when the bridge to be erected replaces an old structure on a railroad line in active operation and it is desired to interfere as little as possible with the traffic. In such cases, the slabs are cast in a conveniently located yard; when sufficiently cured, they are conveyed to the bridge site and put in place by derrick cars. The erection at the bridge site takes very little time and causes comparatively little interference with the traffic.

Slabs built in place are used where it is not necessary to take into consideration the maintenance of traffic, as when the bridge is on a new line, when an old line is being relocated, or when the bridge is intended for future tracks.

General Description of Railroad Slab Bridges. — Slab bridges supporting railroad tracks consist of slabs spanning between supports and reinforced by main longitudinal bars placed at the bottom, cross bars, and stirrups. At the edges the slab is provided with parapets



H. L. Loeffler, Bridge Engineer.

FIG. 13.—Typical Design of Slab Units. Great Northern Railway. (*See p.* 38.)

to retain the ballast. For one-track structures the minimum width of the slab is 13 ft.; for multi-track structures it is a multiple of 13 ft.

When the slabs are precast, the deck is divided into a number of longitudinal units each running the full length of the span. The most desirable arrangement consists of two precast units per span, per track; but when, for longer spans, the units become too heavy for handling, it is necessary to use three or even four longitudinal units per track. Each precast unit should be poured in one continuous operation. Exposed edges should be bevelled by 1-in. V strips.

A typical arrangement of precast slabs in a multi-span crossing supported on abutments and piers is shown in Fig. 13, p. 37. In each span the 16-ft.-long slab is divided longitudinally into two precast units, the details of which are shown in the figure. The slabs were designed

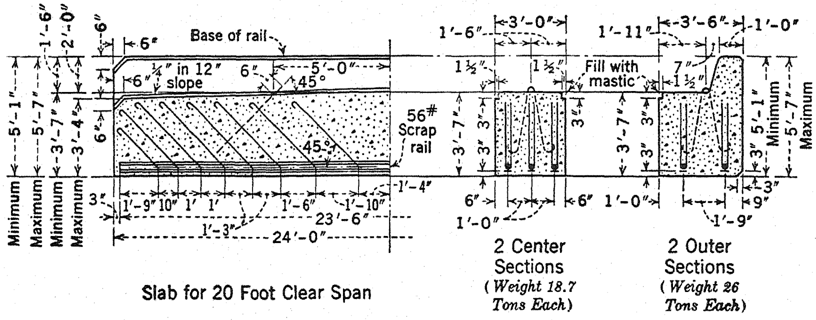


FIG. 14.—Standard Slab Designs. Canadian National Railways. (*See p.* 38.)

by the Great Northern Railway for Cooper's E 65 loading, with an impact allowance $I = 0.75 \times \dfrac{30\,000}{30\,000 + L^2}$, assuming concrete of an ultimate strength of 2 500 lb. per sq. in., and using an allowable unit stress f_c = 800 lb. per sq. in.

Of interest is the design of the plank walk supported on steel brackets which are bolted to the concrete slab. It should be noted that at the abutment the slabs are provided with end ballast stops, the details of which are shown in the figure.

The standards of the Canadian National Railways, shown in Fig. 14, p. 38, use three precast slab units per track for clear spans of slab up to 18 ft., and four units per track for clear distances of 20 ft., thereby limiting the weight of the heaviest unit to about 26 tons.

The total depth of the slab designed for Cooper's E 60 for different spans is given in the following table.

Clear Span	Depth	Clear Span	Depth
14 ft.	2 ft. 7 in.	18 ft.	3 ft. $2\frac{1}{2}$ in.
16 ft.	2 ft. 10 in.	20 ft.	3 ft. 7 in.

In the design shown in Fig. 15, p. 39, and used by the D.L. & W.R.R., the width of the two-track bridge is larger than the standard 13 ft. per track. There, in addition to the 6-ft.-6-in.-wide units under the tracks, special parapet units are used which do not carry any appreciable amount

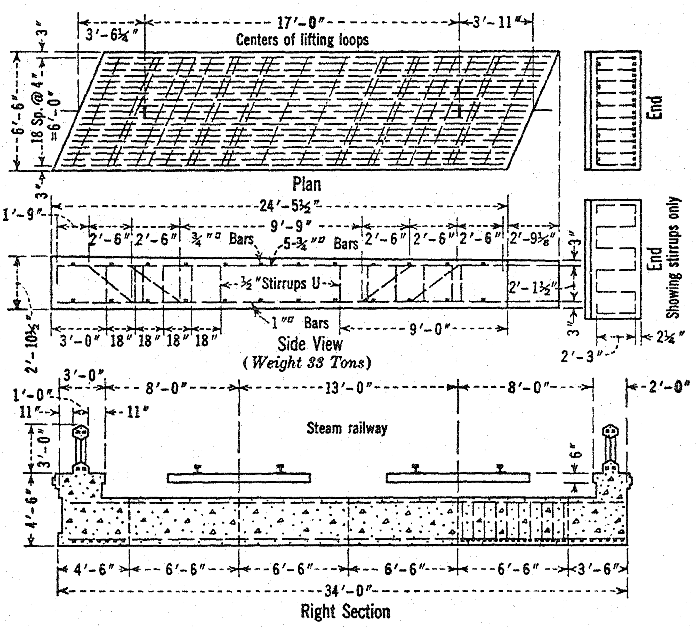


Fig. 15.—Punch Bowl Crossing, Convent, New Jersey. Delaware, Lakawanna & Western R. R. (*See p.* 39.)

of the railroad loading and which have a smaller amount of reinforcement than the interior units. This structure was designed for Cooper's E 60 with an impact ratio equal to the live load divided by the sum of live plus dead load.

Lifting Loops. — To facilitate lifting of the heavy precast slab units, each unit is provided with lifting loops of proper strength imbedded in, and properly anchored to, the slab. Ordinarily two loops per unit are placed on the axis of symmetry of the unit. The loops may extend above the top of the concrete slab (as shown in Fig. 14, p. 38), in which case they are hidden in the ballast. In some designs the loops are placed in

pockets in the slab with their tops below the surface of the slab. After the slabs are in place the pockets are filled with concrete which protects the loops from rusting. In designing the slabs account must be taken of the reversal of stress when they are being lifted.

Design of Slabs. — The design of slabs is simple. The bending moments and shears for the live loads plus the impact are computed, using the concentrated wheel loads. The bending moments per track are then divided by the effective width of the slab, which is usually from 13 to 14 ft., depending upon the width of the bridge; and the unit bending moments are added to the bending moments for dead load for a slab 1 ft. wide.

The depth and the amount of reinforcement are found for the maximum bending moment. The slab should be investigated for diagonal tension and for bond stresses. Diagonal tension reinforcement is usually required, and may consist of bent-up bars and stirrups. In Fig. 13, p. 37, stirrups extend longitudinally and loop about a bottom bar and a top bar placed directly above the bottom bar. In Fig. 15, p. 39, the stirrups are placed crosswise and they also are tied to the top and bottom reinforcement.

Cross bars are usually provided in cast-in-place slabs and in wide precast slab units. In narrow precast units cross bars are often omitted.

Reinforcement of Slabs. — The main bars of slabs usually consist of deformed bars of large diameter. Occasionally old rails are used for longitudinal reinforcement, as shown in Fig. 14, p. 38. The rails must have no defective sections. To increase their bond resistance, as well as to provide diagonal reinforcement, inclined stirrups consisting of small-diameter bars are fastened to the rail in the manner shown in the figure.

Reduction of Depth of Slab. — If required on account of headroom, the depth of the slab may be reduced by compression reinforcement. This method increases appreciably the cost of the slab.

A more economical method of reducing the depth of the slab is by using concrete of high strength.

Continuous Slabs. — The use of continuous slabs for undercrossings is still comparatively rare, owing largely to a baseless prejudice against statically indeterminate designs. In many cases continuous slabs may be used to great economical advantage. One of the advantages also is a material reduction in the depth of construction. In a two-span slab, for example, the largest bending moments act at the center support where larger compression stresses are permitted in American practice than in the center of the span; and where the bottom bars extending across the support may be used as compression reinforcement. The depth of the slab is then governed by the positive bending moments, which are

appreciably smaller than the bending moments in a simply supported span of the same length.

The design of continuous slabs is discussed on p. 35. It should be treated in the same manner as designs of continuous girder bridges. No arbitrary bending-moment coefficients should be used.

A good example of the use of continuous slabs for undercrossing is furnished by the bridge built by D. L. & W. R. R. over Tarrytown Road at Montville, New Jersey. The crossing consists of two skew spans. The length of the continuous slab is 46 ft. $2\frac{3}{8}$ in. The details of reinforcement are shown in Fig. 16, p. 41. As indicated in the cross section, waterproofing consists of membrane protected by a layer of asphalt blocks.

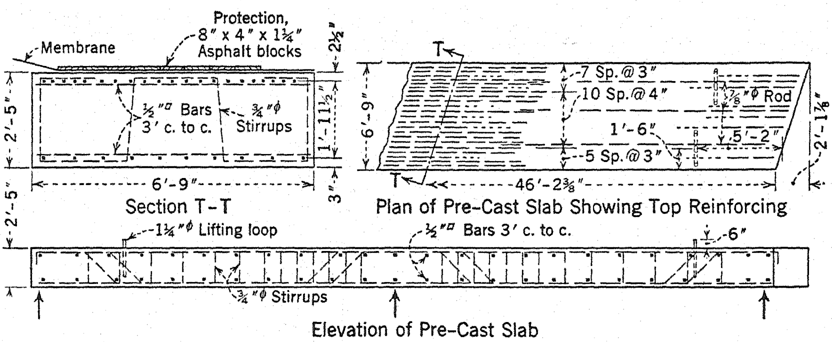


FIG. 16.—Continuous Slab at Montville, New Jersey. D. L. & W. R. R. (*See p.* 41.)

Slabs for Large Skews. — In skew bridges with small skew angle the main reinforcement of the slab is placed parallel to the track, and the design span is measured along the track.

When the skew is large, it is more economical to place the reinforcement at right angles to the supports and to measure the span along a line normal to the supports. Such designs require, along the edges of the slab, parapet girders to carry one end of the triangular portions of the slab, the other end of which rests on the vertical support. Each parapet girder is made flush with the slab at the bottom, and it is extended the required height above the slab.

In the structure used by the D.L. & W.R.R. at Athenia, New Jersey, over the tracks of the Erie Railroad the clear normal distance between the abutments is 29 ft. 9 in., while the distance measured along the track is about 61 ft. The thickness of slab based on the normal span is 2 ft. 10 in. The parapet girders are 71 ft. $3\frac{1}{2}$ in. and 69 ft. 7 in. long overall; and their total depth is 8 ft. 7 in., so that they extend 6 ft. 1 in.

above the top of the slab. The details of the parapet girders are shown in Fig. 17, p. 42.

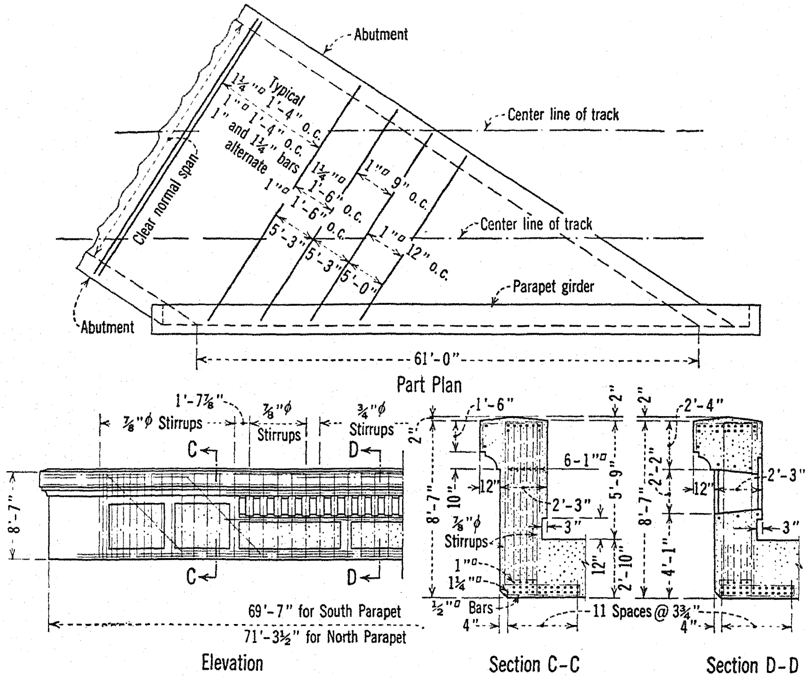


FIG. 17.—Slabs for Large Skews. (*See p.* 42.)

Waterproofing and Drainage. — The superstructure must be properly drained, and the top of the slab should be waterproofed. The most effective method of waterproofing consists of membrane pasted to the top of the slab and protected by a layer of bricks laid on the top of the membrane.

In a cheaper method, the top of the slab and the sides of the parapet are coated with waterproof paint, or with a waterproofing mixture consisting of Portland cement and coal tar, thoroughly mixed before applying.

Example of Railroad Undercrossing. — The Lawrence Ave. Subway in Chicago under the tracks of the Chicago, Milwaukee & St. Paul Railroad is a good example of the use of precast slabs for undercrossings.

This structure carries three railroad tracks across Lawrence Avenue with a skew of 69° 39′. The width of the street measured along the bridge is 66 feet.

The substructure consists of two gravity abutments, two sidewalk bents and one center bent, arranged so as to give a roadway clearance of 23 feet on each side of the bent and a vertical clearance of 13 feet 6 in. The foundations were designed for a pressure of 2.5 tons per square foot.

The superstructure designed for Cooper's loading E60 consists of pre-cast reinforced concrete units each unit poured in one continuous operation using $1:2\frac{1}{2}$ mix in bottom layer and $1:2\frac{1}{2}:3$ mix in rest of slab. The slabs are arranged in six longitudinal rows. Each row consists of two long center units spanning the roadway, each 26 ft. 1 in. long and two short units, each 12 ft. 1 in. long and placed one at each end above the sidewalks. The width of all interior units is 6 ft. $11\frac{1}{2}$ in. with a $\frac{1}{2}$ in. joint between them. The exterior slab units are 8 ft. $5\frac{3}{4}$ in. wide. The depth of the long units is 2 ft. 11 in. at the center of the span. All exterior units are provided along the outside edge with parapets 4 ft. 2 in. deep integral with the units.

Each roadway unit is fixed at the sidewalk bent and at the center bent has a sliding expansion bearing consisting of a zinc plate fastened to the bearing area of the slab. Sidewalk units are fixed at both ends.

CHAPTER V

DECK GIRDER BRIDGES

The design of concrete floor construction as treated in this chapter is adaptable to all types of bridges. The general treatment applies to bridge floor construction irrespective of whether the longitudinal girders are simply supported, provided with cantilevers or statically indeterminate. Preferred floor arrangements are outlined and methods of design of floor slabs are illustrated by two numerical examples.

The design of simply supported longitudinal girders also is discussed in this chapter. Several examples from practice of simply supported deck girder bridges are described and illustrated. Methods of design and examples of other types of girders are given in subsequent chapters.

Floor Arrangements for Deck Girder Bridges. — For the purpose of discussion, the floor arrangements for deck girder bridges are divided into three types: (1) Girder and slab type, in which concrete slabs span between longitudinal girders. (2) Girder, floor beam, and slab type, in which the slab is supported by floor beams spanning between the longitudinal girders. (3) Girder, floor beam, and two-way slab type, in which the panels of the floor slab are supported along four edges by the floor beams and the longitudinal girders, and are reinforced by bars placed in two directions at right angles to each other. The arrangement of type (3) is more economical than the other two arrangements, and its use is recommended, where possible. (See also p. 75.)

1. GIRDER AND SLAB TYPE OF FLOOR

In this group are included bridges consisting of longitudinal main girders with concrete slabs spanning between the girders. Cross beams, when used, do not carry any loads directly, but only serve to stiffen the structure laterally, as explained on p. 48.

Decks without Sidewalks. — The simplest cross section of a bridge of this type without sidewalks is shown in Fig. 18 (*a*), p. 45, in which all girders are practically of the same depth. Since the exterior girders carry smaller loads than the interior girders, the width of their stem and the amount of reinforcement are usually proportionally smaller than for the interior girders.

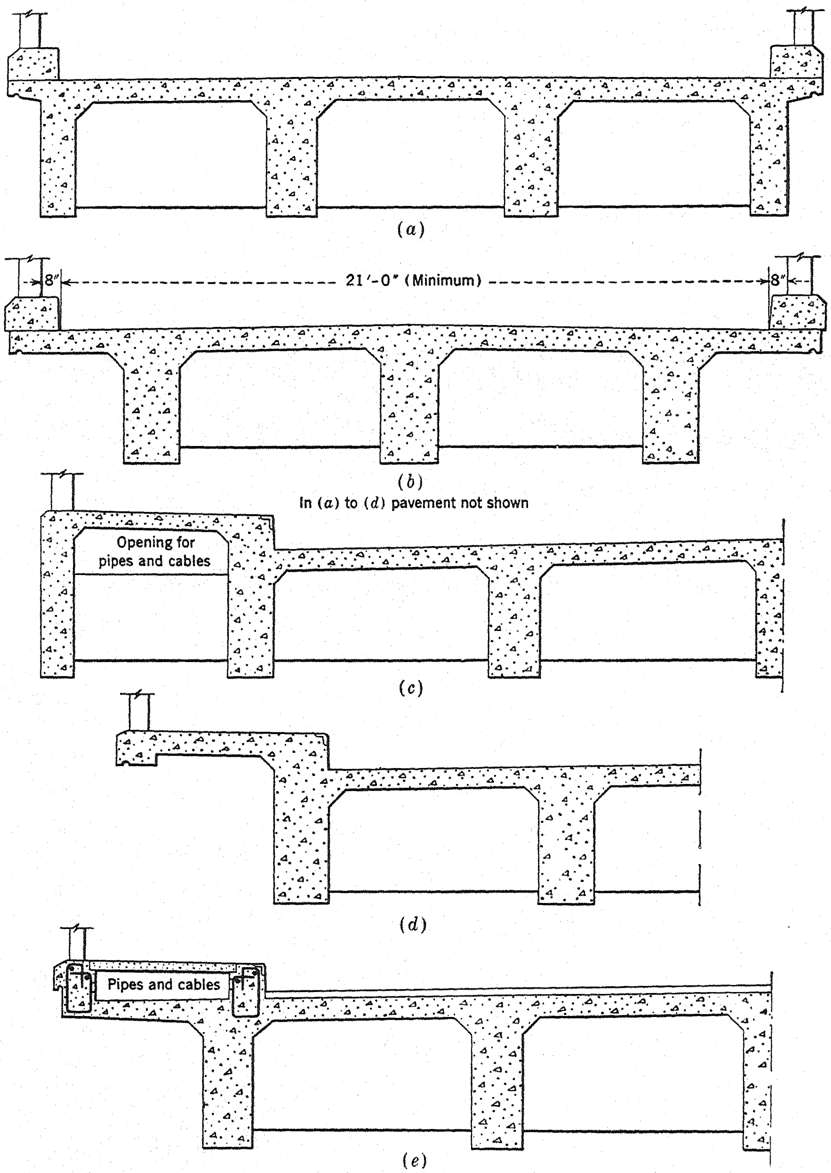


FIG. 18.—Typical Cross Sections of Deck Girder Bridges. (See p. 44.)

The number of main girders in the cross section of a bridge may be reduced without increasing their spacing by cantilevering the roadway slab on both sides, as shown in Fig. 18 (*b*), p. 45. The length of the cantilevers should be selected so as to equalize the loadings on all girders, but it should not be so large as to cause appreciable deflections of the cantilevers under heavy moving loads. The Truckee River bridge described and illustrated on p. 89 is an example of a bridge with cantilevered roadway slab. A general appearance of such a design is evident from Fig. 19, p. 46.

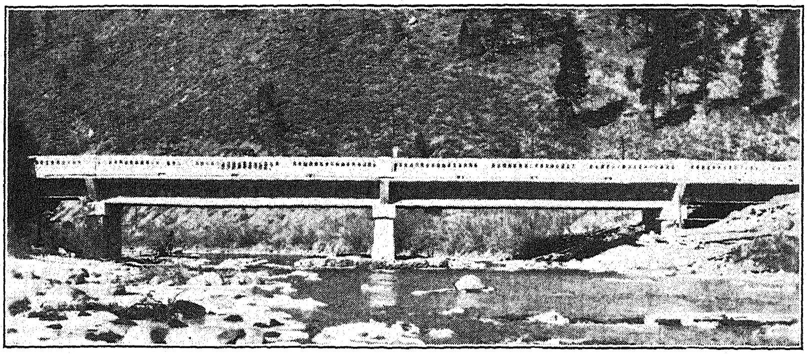


FIG. 19.—Deck Girder Bridge across Truckee River, California. (*See p.* 46.)

Decks with Sidewalks. — A simple cross section of a bridge with sidewalks is shown in Fig. 18 (*c*), p. 45, in which the exterior girders in elevation conceal all the other girders, an effect often desired for the sake of appearance.

Since the exterior girders usually carry appreciably smaller loads than the interior girders, a smaller depth may be used with economy. The cost of the construction sometimes may be still farther reduced by supporting the exterior girders on cantilever extensions of the cross beams, in which case no seats on the piers and abutments are required for the exterior girders. An example of such an arrangement is furnished by the San Gabriel River bridge shown in Fig. 20, p. 47. The position of the girders on the pier is shown in the figure. The outside girders do not appear in the photograph.

In the design shown in Fig. 18 (*d*), p. 45, exterior girders are omitted, and the sidewalk slab is cantilevered out.

Provision for Pipes and Ducts. — In the simplest arrangement, pipes and ducts are placed under the slab, and are either suspended from the slab or carried on brackets, as shown in Fig. 60, p. 142. Openings may have to be provided in cross beams as shown in Fig. 37, p. 91.

A convenient arrangement for pipes and ducts is shown in Fig. 18 (*e*), where the sidewalk slab is a separate thin slab, which may be made removable in whole or in part. Manholes may be provided in the sidewalk slab to give access to the pipes.

In addition to the examples previously noted, provisions for pipes and ducts are shown in Figs. 62, p. 146; 64, p. 153; and 70, p. 160.

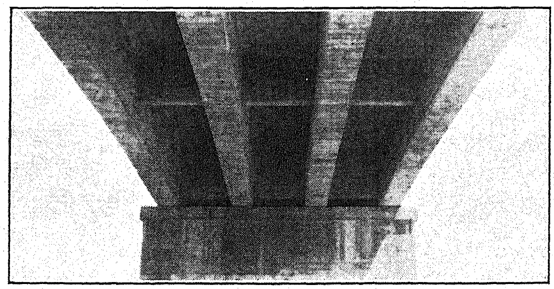


FIG. 20.—Arrangement of Girders on Pier. (*See p.* 46.)

Spacing of Main Girders. — Lateral spacing of the longitudinal main girders affects to a large extent the cost of the bridge; therefore comparative estimates of several arrangements of girders should be made before the final arrangement is adopted. Close spacing of girders means thinner slabs and a larger number of main girders; wide spacing of girders, on the other hand, means thicker slabs, but a smaller number of girders.

The economical spacing of main girders is the spacing which gives the least possible total sum of the cost of formwork, including its vertical supports and bracing, and the cost of materials in the slab and the girders.

Comparing two designs with different spacings of main girders shows that in the design with the closer spacing of girders the quantities of concrete and steel are smaller, because the slabs are thinner, and also because the consequent reduction of the dead load makes possible the use of smaller dimensions of the girders. Closer spacing of girders, therefore, means saving in the cost of materials.

Conversely, with close girder spacing a larger number of expensive girder forms must be used, including their vertical supports and bracings. Closer spacing of girders, therefore, means additional cost of formwork.

Relative economy of the two arrangements depends upon which of the two values is larger; and this, in turn, depends upon the relation between the unit cost of materials and the unit cost of formwork. Where formwork is expensive and materials are relatively cheap, a wide spacing

of girders is economical. The opposite is true where the formwork is cheap and the materials are expensive.

The peculiarities at the bridge site may also influence the economical spacing of the main girders. Where, for instance, vertical supports for the formwork are difficult and expensive, it may be economical to use as few main girders as possible to reduce thereby the number of vertical supports for the formwork, even if under normal conditions in the same locality a closer spacing of girders would be more economical.

In the United States, the cost of labor for formwork is large in comparison with the cost of materials, so that comparatively wide spacings of girders are economical. In examples from practice studied by the authors, spacings between center lines of main girders varying from 8 to 10 ft. predominate. Even larger spacings of girders have been found economical. For example, the California Highway Commission in 1925 found by comparative estimates, for a bridge without sidewalks with a 21-ft.-wide roadway, that an arrangement with two longitudinal girders spaced 12 ft. 9 in. on centers, and with the floor slab cantilevered on both sides, is more economical than the arrangement with four longitudinal girders previously used as a standard. The arrangement with two girders is shown in Fig. 36, p. 90.

In Europe, where the cost of formwork is small in comparison with the cost of materials, close spacing of girders, varying from 4 to 6 ft., has been common practice until recently. Now this arrangement is steadily giving way to wide spacings of girders with floor beams and slabs reinforced in two directions, as described on p. 74 under type (3).

Cross Beams. — In the floor designs of the type here described, cross beams are used to stiffen the girders laterally and to prevent torsion in the outside girders. With properly spaced cross beams, it is permissible to consider the slab as restrained along the outside girders, while in the interior panels a condition approaching that of fixed slab may be assumed.

Another function performed by cross beams is to equalize in a partly loaded bridge the deflections of the girders carrying heavy loading with those of the girders carrying partial loading. This function is of particular importance when the design loading of the bridge consists of one heavy load, such as a steam roller, to be placed in the most unfavorable position and surrounded by a uniformly distributed loading. In such case, instead of making each girder of the bridge strong enough to carry a major part of the roller, the girders may be designed for an average between the most unfavorable loading, and a loading in which the

total live load on the bridge, i.e., the roller and the uniformly distributed loading, is considered as uniformly distributed laterally. Exterior girders are not so favorably affected by the cross beams as the interior girders; therefore, they must be designed for the live load in the most unfavorable position.

When the loading of the bridge consists of trains of trucks, as is common in American practice, the difference between the reaction upon a longitudinal girder due to the most unfavorable lateral position of the truck trains, and the reaction obtained by considering the loading as uniformly distributed laterally, is negligible.

In continental Europe, the requirements as to the spacing of cross beams are more definite than in the United States. In the standard German practice, the spacing of cross beams is not more than one-quarter of the span length of the girder, nor more than 2.5 times the lateral spacing of the girder center lines.

In the United States, there is no uniformity in practice as far as the use and the spacing of cross beams is concerned. Girders of considerable lengths have been built without any cross beams. In the design shown in Fig. 36, p. 90, the end span, 34 ft. long overall, has cross beams at the ends only, while the interior span, 60-ft. overall length, has two intermediate cross beams.

The authors recommend that for short spans one intermediate cross beam in the center of the span should be used; and for long spans cross beams should be spaced not more than 20 ft. on centers. With such arrangement of cross beams, the slab in the exterior panels may be considered as semi-fixed by the outside girders.

Reinforcement of Cross Beams. — When cross beams are intended to distribute the loading laterally, they should be made strong enough to perform the desired function.

Where cross beams are considered only as stiffening struts, they should be provided with bottom reinforcement equal in cross section to about 0.3 per cent of the effective cross section of the cross beam. No top reinforcement is needed because the slab reinforcement directly above the cross beam is sufficient to resist any tension there developed.

DESIGN OF SLABS FOR FLOOR ARRANGEMENT IN FIRST TYPE

The method of design here given applies where the slab extends between parallel longitudinal girders and is entirely supported by them. It may be used not only when the longitudinal girders are simply supported, but also when they are cantilevered or continuous, or when with

their vertical supports they form rigid frames. Typical arrangements of component parts are shown in Fig. 21, p. 50.

Problem to be Solved. — To design the floor slab for a bridge, it is necessary to determine the required slab thicknesses in the different panels, the amounts of the main reinforcement to be placed at right angles to the supporting girders, and the amount of the distributing reinforcement placed parallel to the girders. For this purpose, it is necessary to compute bending moments at the critical sections separately for the dead load and for the concentrated wheel load, and to add the corresponding values. The procedure is explained in the succeeding paragraphs under proper headings.

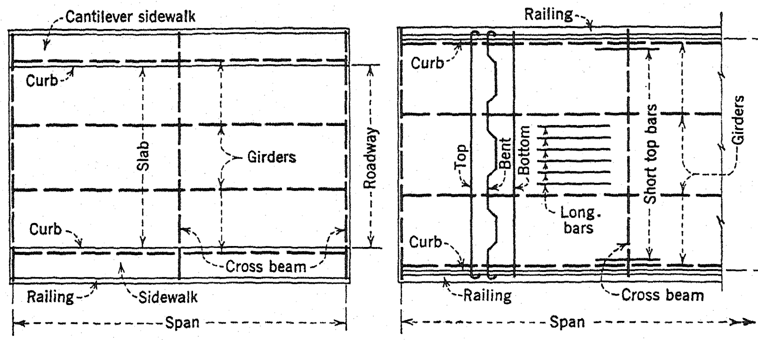


FIG. 21.—Floor Plans of Girder and Slab Type Bridges. (*See p.* 50.)

Theoretical Span Lengths of Slab. — First it is necessary to find the theoretical span lengths of slab to be used in the computations. In accordance with American practice, the theoretical span of continuous and restrained slabs, when built monolithic with the supporting girders, may be taken as the clear distance between the faces of the girders. The bending moments, as well as the effective width for concentrated wheel loads, are based upon this span. Maximum negative bending moments are considered as acting along the edges of the girders.

In Germany and other European countries, the theoretical span is taken as the distance between the center lines of adjacent girders, and the effective width is also based upon this distance. Since the width of the supporting girders varies, the results of this assumption are likely to be erratic and incongruous. For instance, computed on this basis, bending moments in the center of the span for a slab spanning between massive girders would be appreciably larger than for a slab with the same clear span but supported by narrow girders. This is obviously

incorrect. Also, since a large part of the negative bending-moment diagram for this assumption of the span length is within the supporting girder, the negative bending-moment reinforcement arranged in accordance with this plan may be insufficient.

Critical Sections. — For continuous and restrained slabs, bending moments should be computed for the following critical sections: Positive bending moments for the center of each slab panel, and negative bending moments at the edge of the supporting girders. Provision, also, should be made for negative bending moments in the center of each panel which are produced when the panel under consideration is not loaded while the adjoining panels are loaded. These negative bending moments usually are not computed, but instead an arbitrary percentage of reinforcement based on the amount of the bottom reinforcement is used. (See p. 52.)

Bending Moments in Slabs. — When the slab is built monolithic with the girders, the connection between the slab and the girders exerts a restraining effect upon the slab in addition to the effect of continuity. The extent of this effect depends upon the relation between the dimensions of the girder and the thickness of the slab, and also upon the number and spacing of the stiffening cross beams. The actual condition in any such case is intermediate between the condition in a continuous slab not connected with the girders, and that in a slab fixed along the edges of the supporting girders.

To facilitate computations, bending-moment coefficients are given in Tables I and II, which are more fully described on p. 52. For live loads, a concentrated wheel load is assumed as placed in the center of the span. This condition of loading produces maximum results for clear spans up to 10 ft., which is practically the limit for which this type of floor design should be used.

To compute bending moments in a slab proceed as follows:

Find, for each span l, the values of wl^2 for the dead load, and the value of Pl for the concentrated rear wheel load, P. The wheel load should be considered as distributed longitudinally with the girder over an effective width of the slab determined as explained on p. 15.

For each critical section, and separately for dead load and for live load, find the appropriate bending-moment coefficient from Table I and also one from Table II, p. 53.

Estimate from the existing conditions the expected degree of restraint of the slab by the girders, and accordingly adopt for each critical section a value of the coefficient intermediate between the two values taken from the tables. Using the adopted coefficients, compute bending moments for dead load and for live load, including impact, and add the two sets of

values. This procedure is clearly illustrated in the numerical examples on pp. 64 and 85.

Depth of Slab. — The depth of slab in any panel of the slab is determined by the maximum total positive bending moment in that panel. Often, the same thickness of slab is used in all panels, in which case the thickness corresponding to the largest value of the maximum positive bending moments is used. When negative bending moments at the supports are appreciably larger than the positive bending moments, it may be necessary to provide the slab with haunches at the girders, the length of which to be effective must be not less than three times their depth below the slab.

Main Reinforcement. — Main reinforcement of slabs consists of top and bottom bars placed in the direction of the span of the slab. Complicated arrangements of reinforcement should be avoided. Simplicity of arrangement is preferable even if it should entail a small increase in tonnage of reinforcement.

For the accepted thicknesses of slab, the required amounts of reinforcement should be computed for each critical section. The size and spacing of bars for bottom reinforcement should then be selected for the largest positive bending moment. From one-half to two-thirds of the bottom bars may be bent up and extended into the adjoining panel to serve there as negative bending-moment reinforcement. The difference between the area of the bent bars effective at a support of the slab and the amount of reinforcement there needed should be supplied by straight top bars. Negative bending-moment reinforcement is also required in the central part of each span, and the amount of this reinforcement should not be less than one-third of the bottom reinforcement in that span. This is usually provided by extending the required number of top bars continuously along all spans of the slab.

A further discussion of this subject is given in connection with the numerical example on p. 64.

Tables I and II. — In Tables I and II, p. 53, bending-moment coefficients are given at critical sections for two extreme conditions. In Table I it is assumed that the slab is continuous, resting on but not connected with the supporting girders. Values are given for slabs of two to five equal spans. For slabs with more than five spans, the coefficients for the five-span slab may be used.

In Table II, bending-moment coefficients are given for the assumption that the slab in each panel is fixed at both ends by heavy girders.

In both tables, bending-moment coefficients are given for uniformly distributed dead load; for uniformly distributed live load; and for concentrated wheel load P placed in the center of the span.

TABLE I

BENDING-MOMENT COEFFICIENTS α FOR CONTINUOUS SLABS IN FORMULA $M = \alpha wl^2$ FOR UNIFORMLY DISTRIBUTED LOADING, AND IN FORMULA $M = \alpha Pl$ FOR CONCENTRATED LOAD P IN CENTER OF SPAN

Number of Equal Spans	Maximum Positive Bending Moments						Negative Bending Moments					
	End Span		Second Span		Third Span		End Support		Second Support		Third Support	
	Free Ends	Fixed Ends	Free Ends	Fixed Ends	Free Ends	Fixed Ends	Free Ends	Fixed Ends	Free Ends	Fixed Ends	Free Ends	Fixed Ends
	Dead Load w_d $M = \alpha w_d l^2$ ft-lb.											
2	0.07	0.042	......	......	.....	.....	0	−0.083	−0.125	−0.083	....	....
3	0.08	0.042	0.025	0.042	.....	.....	0	−0.083	−0.1	−0.083	....	....
4	0.077	0.042	0.036	0.042	.....	.....	0	−0.083	−0.107	−0.083	−0.071	−0.083
5	0.078	0.042	0.033	0.042	0.046	0.042	0	−0.083	−0.105	−0.083	−0.079	−0.083
	Uniformly Distributed Live Load w_l $M = \alpha w_l l^2$ ft-lb.											
2	0.096	0.054	......	......	.....	.....	0	−0.104	−0.125	−0.083	....	....
3	0.101	0.059	0.075	0.069	.....	.....	0	−0.111	−0.117	−0.094	....	....
4	0.099	0.060	0.080	0.074	.....	.....	0	−0.113	−0.121	−0.098	−0.107	−0.104
5	0.10	0.061	0.079	0.076	0.085	0.08	0	−0.113	−0.120	−0.099	−0.111	−0.107
	Concentrated Live Load P in Center of Span $M = \alpha Pl$ ft-lb.											
2	0.203	0.140	......	......	.....	.....	0	−0.155	−0.188	−0.125	....	....
3	0.212	0.145	0.175	0.168	.....	.....	0	−0.168	−0.175	−0.145	....	....
4	0.210	0.15	0.183	0.175	.....	.....	0	−0.170	−0.180	−0.148	−0.163	−0.155
5	0.210	0.148	0.182	0.178	0.190	0.183	0	−0.170	−0.180	−0.148	−0.168	−0.163

w_d and w_l in pounds per square foot; P in pounds; l in feet.

TABLE II

BENDING MOMENTS. SLAB SPAN FIXED AT BOTH ENDS

	Maximum Positive Bending Moment	Maximum Negative Bending Moment
Uniform loading	$0.042wl^2$	$-0.083wl^2$
Concentrated load P	$0.125Pl$	$-0.125Pl$

Slabs Provided with Cantilevers. — In many designs, the slabs are provided with cantilevers as shown in Fig. 18 (*b*) and (*c*), p. 45, which serve the following purposes: (1) They reduce the number of longitudinal girders in the cross section without increasing their spacing. (2) They equalize the loading on all girders. (3) They reduce positive bending moments in the exterior spans of the slab, and make the distribution of bending moments there similar to that in the interior spans. (4) They reduce the length of the supporting piers, because in

a design with cantilevers the length of the piers is governed by the distance between the outside girders, and not by the total width of the bridge.

In bridges with sidewalks, the natural position for the outside girder is under the curb, in which case most of the sidewalk slab is cantilevered out. If the sidewalk is wide, such arrangement is not advisable, because this would make the length of the cantilever excessive. (See Fig. 18 (*d*), p. 45.)

In bridges without sidewalks, it is advisable to try several arrangements of girders to get the most economical result. The cantilevers must not be too long, else their deflection under heavy loads may have an undesirable effect upon the roadway.

Bending Moments in Slabs with Cantilevers. — In a floor slab provided with cantilevers, the cantilever loads produce bending moments not only in the cantilevers but also in all spans of the slab.

When the slab is not connected with the supporting girders, or when the rigidity of the girders is small, bending moments in the slab due to cantilever loads may be computed by means of the formulas and tables given on p. 182 for continuous beams with cantilevers. For dead loads, all slab spans, as well as the cantilevers, should be considered as loaded simultaneously. For live loads, the cantilevers should be assumed as loaded only when this condition produces an unfavorable effect upon the section of the slab under consideration. See diagrams on pp. 184 to 186.

Usually, the slabs are monolithic with heavy girders, so that the cantilever bending moments, instead of being transmitted in full to the slab, as in the preceding case, are partly resisted by the torsional resistance of the girder, and only the balance of the bending moments is transmitted to the slab. The extent of the effect of the girders depends upon their torsional rigidity and upon the number of cross beams in the span of the bridge. The rigidity of the girder obviously increases with the number of cross beams.

In computing bending moments in the slab due to the loads on the main spans, the effect of the restraint of the slabs by the girders should be determined in the same manner as explained on p. 51 for slabs without cantilevers.

Cross Reinforcement of Slabs. — Cross reinforcement of slabs, i.e., reinforcement at right angles to the main reinforcement, is necessary to resist any tensile stresses produced in the slab in distributing the concentrated loads laterally. This reinforcement also prevents the formation of open cracks due to temperature changes and shrinkage. It is obvious that open cracks in the slab, irrespective of their cause, would seriously interfere with the distributing action of the slab. Also by admitting moisture they might contribute toward the disintegration of the slab.

In American practice, there is no uniformity as far as the amount and the disposition of cross reinforcement are concerned. In designs of erected structures examined by the authors the cross section of the cross bars varied from 0.1 to 0.3 per cent of the effective cross section of the slab at right angles to the cross bars. It is recommended that cross bars amounting to 0.2 per cent of this effective cross section of the slab should be used.

In foreign practice, the amount of cross reinforcement is based upon its function of distributing the concentrated loads. An elaborate theory has been developed by M. Pigeaud[1] for determining the amount of the main and the cross reinforcement for concentrated loads. His method is being used in France and to some extent in Great Britain. The basis of this method does not appear to the authors to be sound, particularly as applied to continuous slabs; therefore it is not recommended for use.

In German practice, the amount of cross reinforcement is a function of the main positive reinforcement for live load. The cross section of cross bars per unit of length of the slab span is equal to the cross section of the main positive reinforcement for live load per unit of width of the slab, multiplied by the ratio c found from the following formula:

$$c = 0.10 + 0.10[l_c - (g + 2p)] \tag{1}$$

where l_c is the effective width of the slab (usually two-thirds of the distance between center lines of the girders); g is width of tire in contact with the slab, usually 0.1 meter; and p is the thickness of the pavement. All values must be in meters.

Reinforcement of Slab over Cross Beams. — To prevent cracks on the top of the slab along the edges of cross beams due to secondary bending moments, top reinforcement is needed in the slab over and at right angles to the cross beams. It is recommended that the amount of this reinforcement should not be less than 0.3 per cent of the effective cross section of the slab and that the bars should extend on each side of the cross beam at least 3 ft. At end cross beams the bars should be anchored in the cross beams.

Skew Crossings

In skew crossings, main girders are usually placed parallel to the direction of the road and not at right angles to the piers. The design span of the girders is then measured along the same direction.

For comparatively small skew angles, the slab may be reinforced by

[1] See *Annales des ponts et chaussées français*, February, 1921.

bars placed parallel to the piers, and not at right angles to the girders; their span must then be measured similarly. For large skews it is advisable to design the slab for a span at right angles to the girders and to place the reinforcement in that direction. This gives a smaller thickness of slab and a smaller amount of reinforcement, but the placing of steel is more complicated because at each end of each bridge span there is a triangular portion of the slab where odd lengths of bars must be used.

Arrangement of Girders for Large Skews. — Occasionally, for wide crossings with large skew angles it may be more desirable to place the girders at right angles to the supports, as shown in Fig. 22, p. 56. Then,

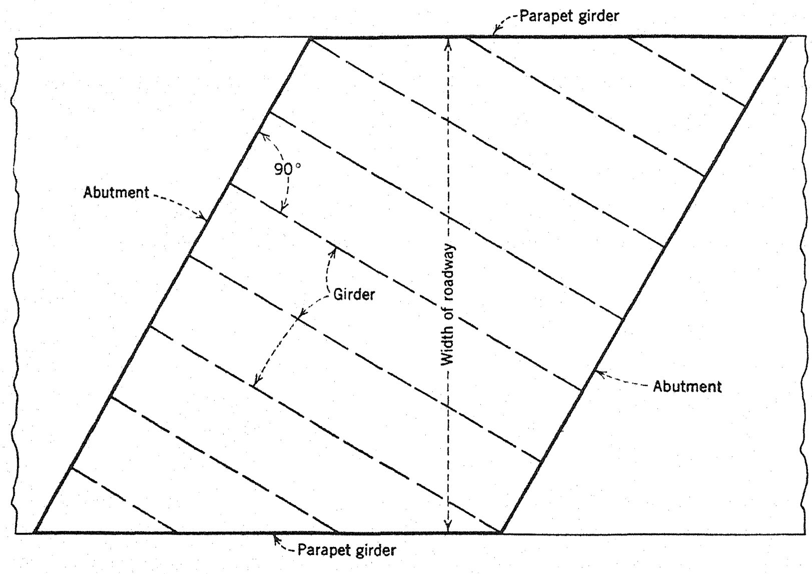


Fig. 22.—Arrangement of Girders in Wide Skew Crossing. *(See p. 56.)*

at each side of the crossing there is a triangular section in which the girders rest at one end on the pier and on the other end on the parapet girder. The parapet girders carry heavy loads, and to increase their depth it is often desirable to extend them above the roadway.

Design of Main Girders. Bridges of Girder and Slab Type

To design the main girders of a bridge of the girder and slab type it is necessary to determine their concrete dimensions, the amount of longitudinal reinforcement, and the amount and spacing of the web reinforcement. For this purpose maximum bending moments and shears must

be computed for dead load, live load, and impact, and also bending moment and shear diagrams prepared for combined dead and live load.

Dimensions of Girder. — The depth and the breadth of stem of a simply supported girder are usually made constant throughout its length. Occasionally, however, the cross section at the supports is increased to keep the diagonal tension within working limits by increasing there either the depth or the breadth of stem.

To prevent excessive deflection, the depth of the girder must be kept within proper limits. In existing simply supported girder bridges, the ratio of the depth of the girders to the span lengths varies from $\frac{1}{10}$ to $\frac{1}{14}$. Since the girders are usually T-beams, for an adopted slab thickness the minimum depth in the center is governed by compression stresses in the flange of the T-beam. Usually it is economical to use a much larger depth than the minimum value.

The breadth of the stem of the girder must be sufficient to keep the shearing unit stresses at the supports within working limits. Also, the girder must be wide enough to accommodate all longitudinal reinforcement without crowding, and preferably without using more than three layers of longitudinal bars. Deep narrow girders should be avoided because their torsional resistance is small, and also because the placing of reinforcement and depositing of concrete are difficult.

Theoretical Span of Girders. — The theoretical span of simply supported girders is the distance between the centers of bearings.

Dead Load Carried by Girder. — In bridges consisting of two longitudinal girders, as shown in Fig. 36, p. 90, the dead load carried by each girder is equal to one-half of the total dead load on the bridge. However, where the bridge consists of more than two girders, an exact determination of the proportion of the weight of the slabs and of the pavement carried by each girder is not feasible.

If the continuous slab were not connected with the girders, the dead loads transmitted by them to each girder could be determined from the continuous slab formulas given on p. 166. Actually, the slabs are monolithic with the girders; and, besides, the bridge is stiffened by cross beams. These two factors restrain the slab along the girders to an unknown degree, so that the actual reaction of the dead load of the slab and of the pavement are intermediate between the reactions for a continuous slab and those for slabs fixed at each support.

The problem of determining the exact dead load transferred by the slab to the girders is further complicated when the girders in a bridge are not all of the same stiffness. For instance, in Fig. 24, p. 61, the girders under the curbs are deeper, and therefore stiffer, than the other girders. The deflection of the stiffer girders is smaller than that of the other

girders; and unequal deflection affects the distribution of the slab loads in the following manner: When the deflections of two adjoining girders are different, the slab between them bends, thereby exerting, in addition to the usual reactions, negative reactions upon the girder with the larger deflection and positive reactions upon the other girder. Therefore, the stiffer girder receives an increase in the dead-load reaction while the other girder is relieved of a load of the same magnitude.

From the above discussion, it is evident that no simple rational formulas can be developed for determining the exact proportion of the dead loads transferred by the slab to the girders. The following simple suggestions may be used as a guide as follows:

Exterior girders in all cases should be assumed to carry one-half of the slab load in the end panel.

In a bridge with two slab spans in cross section, the interior girder should be assumed to carry six-tenths of the total slab dead load.

In all other cases, the first interior girder should be assumed to carry 0.55 of the slab dead loads on the panels supported by it. All other interior girders should be designed for one-half of the slab dead loads. The slab dead load on a girder of greater stiffness than the adjoining girders should be 10 per cent larger than required by the foregoing suggestions.

These suggestions apply not only to simply supported girder bridges, but also to all statically indeterminate girder designs.

Reactions on Girders Due to Cantilever Slabs. — When the bridge slab in a cross section is provided with cantilevers, each girder supporting the cantilever slab should be designed for the total dead load on the cantilever plus the reactions produced upon it by the cantilever bending moments. The cantilever loads affect also the reactions upon all other girders of the bridge. The reactions upon the girders of a bridge by cantilever slabs are given in following formulas. In these formulas it is assumed that the slabs are continuous, and that they are not monolithic with the supporting girders.

Let P = concentrated loads on cantilever.
w = uniformly distributed load on cantilever.
l_1 = length of cantilever.
M_c = maximum bending moment due to cantilever loads.
s = spacing of girders.
c_1, c_2, c_3 = constants from the table on p. 59.

Reaction on Girder Carrying Cantilever Slab:

$$R_1 = (P + wl_1) + c_1 \frac{M_c}{s} \tag{2}$$

Reaction on Second Girder:

$$R_2 = -c_2 \frac{M_c}{s} \tag{3}$$

Reaction on Third Girder:

$$R_3 = c_3 \frac{M_c}{s} \tag{4}$$

The constants c_1, c_2, and c_3 should be taken from the table below where values are given for slabs of one to five equal spans, and for symmetrical and unsymmetrical loadings of cantilevers.

If the slabs are connected with the girders, the actual reactions upon the girders due to cantilever bending moments will be smaller than those given by formulas (2) to (4), because only a part of the cantilever bending moment is transferred to the interior spans. (See also discussion on p. 54.) For positive reactions full values are recommended; but not more than 30 to 50 per cent of the reduction of the girder load permitted by formula (3) should be used for dead load. No negative reactions should be used for live loads.

TABLE III

CONSTANTS FOR REACTIONS DUE TO CANTILEVER LOADS

To be used in formulas (2) to (4), pp. 58 and 59.

Number of Spans	Both Cantilevers Symmetrically Loaded			Left Cantilever Loaded		
	1st Girder c_1	2nd Girder c_2	3rd Girder c_3	1st Girder c_1	2nd Girder c_2	3rd Girder c_3
1	0	0	....	1.0	−1.0	....
2	1.5	−3.0	....	1.25	−1.5	0.25
3	1.2	−1.2	....	1.27	−1.6	0.4
4	1.29	−1.71	0.86	1.27	−1.61	0.43
5	1.26	−1.6	0.32	1.27	−1.61	0.43

Live Loads for Main Girders. — Concentrated wheel loads should be used in designing simply supported girders.

The proportion of the truck loads carried by one girder may be determined as explained on p. 17. The proportion of the truck loads, W, having been found, the maximum bending moments and shears should be computed as explained in Chapter III, pp. 19 to 22. Bending-moment diagram should be drawn as explained on p. 21. Shear diagram for live loads should be prepared as given on p. 22.

For a bridge with cantilevered slabs, the effect of the loads on the cantilevers upon the loading of the girders should be considered only when they increase the loads upon the girder. Formulas (1) to (4), p. 59, should be used, considering either one or both cantilevers as loaded, whichever gives the most unfavorable results.

Numerical Example of Design of Deck Girder Bridge. Girder and Slab Type

The use of the formulas for the design of slabs and girders given in the preceding pages is illustrated by the following example.

Example. — Design a deck girder bridge for the following assumptions:

Span of main girders	50 ft. between centers of bearings.
Width of roadway	30 ft.[2] between curbs (three traffic lanes)
Sidewalks, each	5 ft. 6 in. effective width
Live loads, 20-ton truck arranged in trains as in Fig. 23, p. 60	
Impact, see formula (1), p. 13	
Live loads for sidewalks	100 lb. per sq. ft.
Weight of paving	40 lb. per sq. ft.
Concrete railing	300 lb. per lin. ft.

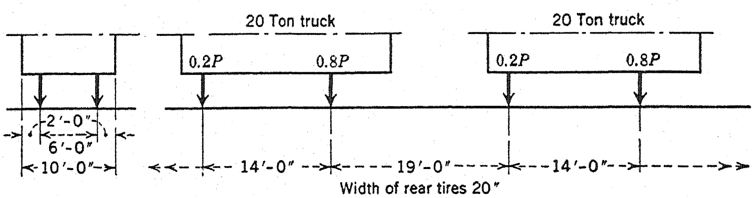


Fig. 23.—Truck Train Used in Numerical Example. *(See p. 60.)*

Specified working stresses, in pounds per square inch: $f_c = 800$; $f_s = 16\,000$; $n = 15$; $v = 40$ for plain concrete; $v = 120$ with web reinforcement; $u = 100$.

Solution. — Adopt the cross section of bridge shown in Fig. 24, p. 61. In actual design the spacing of girders should be determined by comparative estimates of the costs of materials and of formwork for several arrangements of girders.

In computations the accuracy obtained by the use of a 10 in. slide rule is sufficient.

Cantilever Slab

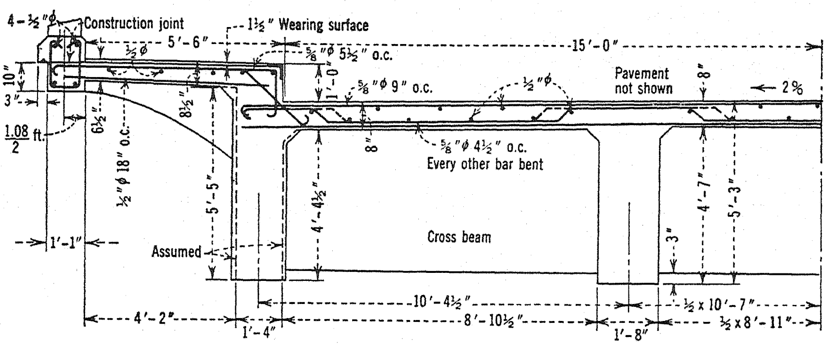
The cantilever slab carrying the sidewalk is designed first. Find the bending moments about the edge of the girder for dead and live load and for the horizontal pressure acting on the railing and amounting to 150 lb. per lin. ft. of railing. This horizontal pressure, assumed to be applied 3 ft. above the top of the sidewalk, produces negative bending moments throughout the sidewalk slab.

[2] A width of at least 32 ft. is preferable in practice. See p. 5.

Assume structural slab as varying in thickness from 5 to 7 in. Add $1\frac{1}{2}$ in. for wearing surface. Then weight of slab equals $\dfrac{6.5 + 8.5}{2 \times 12} \times 150 \times 4.17 = 391$ lb. per lin. ft. Moment arm is $\dfrac{1}{3}\,\dfrac{8.5 + 2 \times 6.5}{8.5 + 6.5} \times 50 = 24$ in.

Cantilever bending moment in slab at edge of girder:

Dead load of slab	$391 \times 24 =$	9 380
Railing	$300 \times \left(50 + \dfrac{13}{2}\right) =$	16 950
Edge beam	$\dfrac{10 \times 13}{144} \times 150 \times \left(50 + \dfrac{13}{2}\right) =$	7 650
Total dead load	$M_{c(d)} =$	33 980



All bars in slab are placed in one top and one bottom layers.
Two layers are used above to show clearly the bent bars.

FIG. 24.—Cross Section of Bridge. *(See p. 60.)*

Horizontal pressure on railing	$150 \times 36 =$	5 400
Live load	$\frac{1}{2} \times 100 \times 4.17^2 \times 12 =$	10 430
Total for live load	$M_{c(l)} =$	15 830
Grand total	$M_c =$	49 810 in-lb.

For accepted unit stresses, the constants are $R = 146.9$, $j = 0.857$.
The thickness of cantilever slab

$$d = \sqrt{\frac{49\,810}{12 \times 146.9}} = 5.3 \text{ in.}$$

Accept $h = 7$ in.; using 1-in. protective covering, $d = 7 - 1.3 = 5.7$ in.

$$A_s = \frac{49\,810}{0.857 \times 5.7 \times 16\,000} = 0.64 \text{ sq. in.}$$

Use $\frac{5}{8}$-in. round bars $5\frac{1}{2}$ in. on center, which gives $A_s = \dfrac{0.307}{5.5} \times 12 = 0.67$ sq. in.

In the bottom of the cantilever slab use arbitrarily $\frac{1}{2}$-in. bars spaced 18 in. on centers.

For longitudinal reinforcement, use 0.2 per cent of cross section of slab, or $0.002 \times 12 \times 5.7 = 0.14$ sq. in. per lin. ft.

Longitudinally use $\frac{1}{2}$-in. bars 18 in. on centers.

Roadway Slab

Spans. — As explained on p. 50, assume the theoretical span of slab as equal in each panel to the clear distance between the faces of the supporting girders.

$$l_1 = 8 \text{ ft. } 10\tfrac{1}{2} \text{ in.} = 8.87 \text{ ft.}; \quad l_2 = 8 \text{ ft. } 11 \text{ in.} = 8.91 \text{ ft.}$$

Dead load: Slab (assumed) 105

Paving 40

Total w_d = 145 lb. per sq. ft.

$$w_d l_1^2 = 145 \times 8.87^2 = 11\,400 \text{ ft-lb.} = 136\,800 \text{ in-lb.}$$

$$w_d l_2^2 = 145 \times 8.91^2 = 11\,550 \text{ ft-lb.} = 138\,600 \text{ in-lb.}$$

Assume that the conditions of restraint are equal to the average between those for a continuous slab of three spans and those for a slab fixed at both ends. The coefficients are found using Tables I and II, p. 53.

Coefficients — Bending moments

End span

$$-\frac{0 + 0.083}{2} = -0.042 \qquad M_1 = -0.042 \times 136\,800 = -\ 5\,700.0 \text{ in-lb.}$$

$$-\frac{0.1 + 0.083}{2} = -0.092 \qquad M_2 = -0.092 \times 136\,800 = -12\,600.0 \text{ in-lb.}$$

$$\frac{0.08 + 0.042}{2} = 0.061 \qquad M_{1\text{max.}} = 0.061 \times 136\,800 = 8\,400.0 \text{ in-lb.}$$

Second span

$$\frac{0.025 + 0.042}{2} = 0.034 \qquad M_{2\text{max.}} = 0.034 \times 136\,800 = 4\,700.0 \text{ in-lb.}$$

Effective Width of Slab to Carry Concentrated Wheel Loads. — Since the slab in all panels is continuous, the effective width is based upon 0.7 of the clear span. (See p. 16.) Using formula (2) on p. 15, the effective width is

$$l_e = (0.7 \times 0.72 \times 8.875) + 0.82 = 5.29 \text{ ft.}$$

This value is used for all slab panels because the difference between their span lengths is small.

Live Loads. — Concentrated wheel loads are used in designing the slab. The wheel load is assumed to be distributed at right angles to the span, but not along the span. One rear wheel of the design truck weighing 16 000 lb., placed in the center of slab span, gives the largest results. Since the impact ratio is $I = \dfrac{50}{8.9 + 200} = 0.24$, multiply the live load by 1.24 to take care of the impact. Divide the load by the effective width to get the concentrated load per lineal foot of width of slab.

Live load per foot of width of slab, including impact, $P = \dfrac{1.24 \times 16\,000}{5.29} = 3\,750$ lb.; $Pl_1 = 3\,750 \times 106.5 = 399\,300$ in-lb.; $Pl_2 = 3\,750 \times 107 = 401\,200$ in-lb.

The coefficients of bending moments are found from Tables I and II in the same manner as for dead load.

Coefficients	Bending moments
End span	
$-\dfrac{0 + 0.125}{2} = -0.063$	$M_1 = -0.063 \times 399\,300 = -25\,000$ in-lb.
$-\dfrac{0.175 + 0.125}{2} = -0.150$	$M_2 = -0.150 \times 399\,300 = -59\,900$ in-lb.
$\dfrac{0.213 + 0.125}{2} = 0.169$	$M_{1\text{max.}} = 0.169 \times 399\,300 = 67\,500$ in-lb.
Second span	
$\dfrac{0.175 + 0.125}{2} = 0.150$	$M_{2\text{max.}} = 0.150 \times 401\,200 = 60\,200$ in-lb.

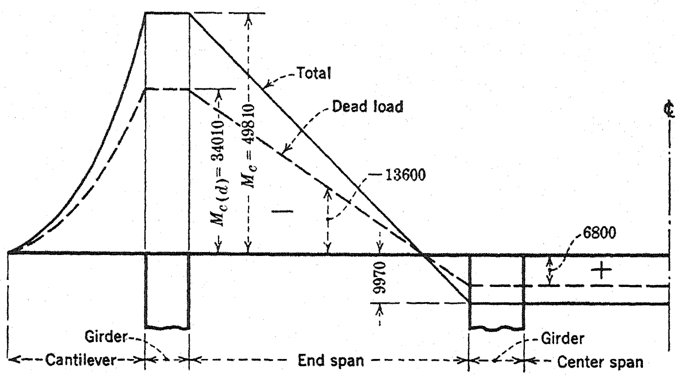


FIG. 25.—Bending Moments in Slab for Cantilever Loads. (*See p.* 63.)

Bending Moments in Main Span Due to Cantilever Loads. (See Fig. 25, p. 63.) — Bending moments of cantilever loads at edge of the first girder

Total $M_c = -49\,810$ in-lb. For dead load, $M_{c(d)} = -34\,010$ in-lb.

The bending moments at the edge of the second girder equals $-0.2M_c$ as shown in Fig. 25, p. 63, or $-0.2M_c = 9\,960$ in-lb., and $-0.2M_{c(d)} = 6\,800$ in-lb. See p. 182.

In this case, the effect of the cantilever loads upon the main spans of the slab is found as follows.

At the first girder, the total negative cantilever bending moment at the edge of the girder is assumed to be transmitted from the cantilever slab to the slab in the main span. The interior span of the slab is assumed to be subjected to a positive bending moment equal to $0.2M_c$.

The positive bending moments in the central portion of the first span and the negative bending moments in the slab at the second girder are reduced by the cantilever bending moments for dead load. To be on the safe side, in this case it is assumed

that owing to the monolithic connection between the slab and the girders only one-third of the theoretical values is effective. Therefore use as reduction in the center of the first span $-\frac{1}{3} \times 13\,600 = -4\,530$ in-lb., and at the second support $\frac{1}{3} \times 6\,800 = 2\,300$ in-lb. The value 13 600 is obtained by scaling.

Summary of Bending Moments:

Negative bending moments:

$$
\begin{array}{lll}
M_1 & = & -\ \ 5\,700 \text{ dead load} \\
 & & -25\,000 \text{ live load} \\
 & & \underline{-49\,810} \text{ cantilever} \\
\text{Total} & = & -80\,510 \text{ in-lb.}
\end{array}
\qquad\qquad
\begin{array}{lll}
M_2 & = & -12\,600 \text{ dead load} \\
 & & \underline{-59\,900} \text{ live load} \\
 & & -72\,500 \\
 & & \underline{\ \ 2\,300} \text{ cantilever} \\
 & & -70\,200 \text{ in-lb.}
\end{array}
$$

Positive bending moments:

$$
\begin{array}{lll}
M_{1\text{max.}} & = & \ \ 8\,400 \text{ dead load} \\
 & & \underline{67\,500} \text{ live load} \\
 & & 75\,900 \\
 & & \underline{-\ \ 4\,530} \text{ cantilever} \\
\text{Total} & & 71\,370 \text{ in-lb.}
\end{array}
\qquad\qquad
\begin{array}{lll}
M_{2\text{max.}} & = & \ \ 4\,700 \text{ dead load} \\
 & & 60\,200 \text{ live load} \\
 & & \underline{\ \ 9\,970} \text{ cantilever} \\
 & & 74\,870 \text{ in-lb.}
\end{array}
$$

Thickness of Slab and Reinforcement. — The thickness of slab is made the same in all slab panels, and it is based upon the maximum positive bending moment in the center span.

$$d = \sqrt{\frac{74\,870}{12 \times 146.9}} = 6.52 \text{ in.}$$

Use $h = 8$ in.; $d = 8.0 - 1.3 = 6.7$ in., for 1-in. protection for bars.

Area of reinforcement:

$$A_s = \frac{74\,870}{0.857 \times 6.7 \times 16\,000} = 0.82 \text{ sq. in.}$$

Use $\frac{5}{8}$-in. round bars $4\frac{1}{2}$ in. on centers, for which $A_s = \dfrac{0.307}{4.5} \times 12 = 0.82$ sq. in.

To simplify the arrangement of bars, the same spacing of the positive reinforcement is used in all spans. Every other bar is bent up and arranged as shown in Fig. 26, p. 65. Additional $\frac{5}{8}$-in. round bars, spaced 9 in. on centers, are used at the top. They extend the whole width of the roadway, and are anchored by hooks in the outside girders. The sum of the bent-up bars and of the straight top bars, amounting to 0.82 sq. in., is sufficient to take care of the negative bending moments at all sections with the exception of the points along the exterior girders. Therefore additional short top bars are used there. These are not shown in Fig. 26.

The continuous top bars take care of negative bending moments in the central portions of the span produced in the first panel by the cantilever loads, and in the second panel by a condition when the end spans are loaded and the center span is not loaded.

Bending of Slab Reinforcement. — The bending sketches for the slab reinforcement are shown in Fig. 26, p. 65. In the roadway two types of bent bars are used, marked as bars a and b, respectively. The two bars, placed end to end, extend the

whole width of the roadway. Bars a are bent up at different points from bars b; therefore by alternating them the slab is reinforced by bars bent at two points at each end of each span. This increases their effectiveness in resisting diagonal tension.

Diagonal Tension in Slab. — To determine diagonal tension, end shears are computed for dead load and live load.

Dead load end shear. $V_d = \frac{1}{2} \times 145 \times 8.875 = 643$ lb. per lin. ft.

Live load. In determining the end shear in the slab for concentrated wheel loads, it is permissible to assume a distribution with the span, in addition to the

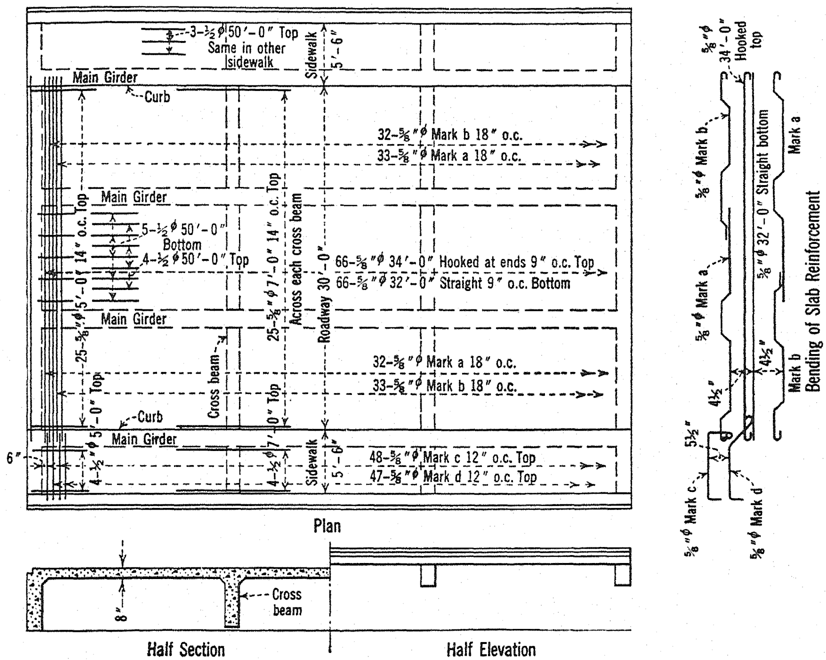


FIG. 26.—Arrangement of Reinforcement. (*See p.* 64.)

distribution of the load at right angles to the span. It is assumed that each wheel load is distributed over the width of the tire, 1 ft. 8 in., and on each side of the tire at 45° by the aggregate thickness of the pavement and of the slab. This makes the total distribution $1.67 + 2 \times (0.33 + 0.67) = 3.67$. For maximum end shear, the wheel loads should be placed as shown in Fig. 27, p. 66. This gives, for a rear wheel load of 16 000 lb. each, multiplied by 1.24 to allow for impact, and for the previously determined effective width of slab $l_e = 5.29$ ft.,

$$\frac{P}{5.29} = \frac{16\,000 \times 1.24}{5.29} = 3\,750 \text{ lb. per wheel and per lin. ft. of slab}$$

$$V_{L+I} = 3\,750\left(\frac{7.08}{8.92} + \frac{2.92}{3.67} \times \frac{2.92}{2 \times 8.92}\right) = 3\,480 \text{ lb. per lin. ft. of slab}$$

Finally the total end shear for live and dead load

$$V = 643 + 3\,480 = 4\,123 \text{ lb. per lin. ft.}$$

For this end shear, unit shearing stress to be used as a measure of diagonal tension is

$$v = \frac{4\,123}{12 \times 0.857 \times 6.7} = 60 \text{ lb. per sq. in.}$$

This unit stress is satisfactory, because the slab is provided with bent bars, and also because each negative reinforcing bar is anchored by being extended beyond the negative points of inflection. Either of these two facts is sufficient to permit the use of a unit stress $v = 60$ lb. per sq. in.

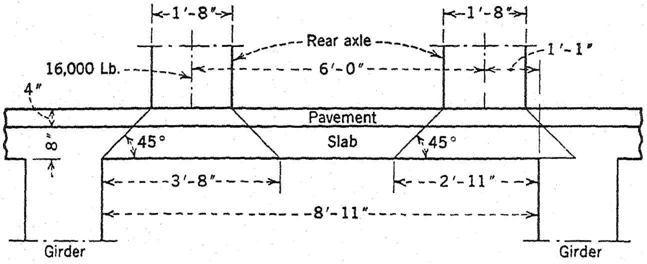


FIG. 27.—Position of Wheel Loads for Maximum Shear in Slab. (*See p.* 65.)

Bond Stresses in Slab Reinforcement. — Use same end shears as for diagonal tension. The tensile reinforcement at the points of maximum end shear consists of $\frac{5}{8}$-in. round bars $4\frac{1}{2}$ in. on centers. Their perimeter per foot of width of slab is

$$\Sigma o = \frac{12}{4.5} \times 1.964 = 5.23 \text{ sq. in.}$$

and the bond stresses are

$$u = \frac{4\,123}{5.23 \times 0.857 \times 6.7} = 138 \text{ lb. per sq. in.}$$

These stresses are satisfactory because the tensile reinforcement is anchored at the free ends, as explained in connection with diagonal tension. For anchored bars, the American Joint Committee recommendations allow 50 per cent larger unit stresses than for bars without anchorages.

Cross Reinforcement of Slabs. — Cross reinforcement is provided of an area equal to 0.002 times the effective area of the slab. (See p. 54.)

$$A = 0.002 \times 6.7 \times 107 = 1.43 \text{ sq. in. per panel}$$

Use in each panel nine $\frac{1}{2}$-in. round bars, four bars near the top and five bars near the bottom. This gives $A_s = 9 \times 0.196 = 1.76$ sq. in.

Reinforcement across Cross Beams. — Required short top bars across cross beams are found as explained on p. 55.

$$A_s = 0.002 \times 6.7 \times 12 = 0.16 \text{ sq. in. per lin. ft.}$$

Use $\frac{3}{8}$-in. round bars 8 in. on centers, for which $A_s = 0.17$ sq. in.

Main Girders

Dead Load. — For the interior girders, the dead load from the slab is computed as if the slabs were freely supported. The uplift due to the cantilever loads reduces the dead load on the interior girder. Also, since this girder is the first interior girder, the slab reactions should be increased as recommended on p. 57. In this case it is assumed that both these values cancel.

Slab	$\frac{8.0}{12} \times 150 \times \frac{8.875 + 8.917}{2} =$	889
Paving	$40 \times \frac{10.375 + 10.583}{2} =$	419
Girder (assumed)	$1.67 \times 5.0 \times 150 =$	1 252
Allowance for cross beams		70
Total		2 630 lb. per lin. ft.

Maximum static bending moment for dead load

$$M_D = \tfrac{1}{8} \times 2\,630 \times 50^2 \times 12 = 9\,862\,000 \text{ in-lb.}$$

Live Load and Impact. — The proportion of the truck loads carried by one girder are found from formula (1), p. 18

$$W = \frac{10.375 + 10.583}{2 \times 10} P = 1.05P$$

The impact ratio is $I = \dfrac{50}{50 + 200} = 0.2$; therefore, to take care of the impact multiply the truck load by 1.2, so that the truck load is $P = 1.2 \times 40\,000 = 48\,000$ lb., including impact, and

$$W = 1.05 \times 48\,000 = 50\,400 \text{ lb.}$$

Bending Moment for Live Load and Impact. — Maximum bending moment in a longitudinal girder may be found from the formula in the table on p. 20, item 3, by substituting proper values for W and l.

$$M_{L+I} = \left[\left(0.548 - \frac{0.455}{50}\right)^2 - \frac{2.8}{50}\right] \times 50 \times 50\,400 \times 12 = 7\,080\,000 \text{ in-lb.}$$

Sum of Maximum Bending Moments. —

$$M_{\text{max.}} = 9\,862\,000 + 7\,080\,000 = 16\,942\,000 \text{ in-lb.}$$

External Shears. — Dead load $V_D = \tfrac{1}{2} \times 2\,630 \times 50 = 65\,750$ lb.

The shears vary according to a straight line.

Live load and impact. From the table on p. 24, item 4, for load $W = 50\,400$ lb. and span length $l = 50$ ft., the external shears are:

$$V_{1(L+I)} = \left(2 - \frac{38.6}{50}\right) \times 50\,400 = 61\,900 \text{ lb.}$$

$$V_{x1} = \frac{55.4}{50} \times 50\,400 = 55\,800 \text{ lb. for } x_1 = 50 - 47 = 3 \text{ ft.}$$

$$V_{x2} = \frac{30.2}{50} \times 50\,400 \qquad = 30\,400 \text{ lb. for } x_2 = 50 - 33 = 17 \text{ ft.}$$

$$V_{x3} = \frac{11.2}{50} \times 50\,400 \qquad = 11\,300 \text{ lb. for } x_3 = 50 - 14 = 36 \text{ ft.}$$

These shears are plotted in the shear diagram in Fig. 28, p. 68.
Maximum external shear for dead load, live load, and impact:

$$V_1 = 65\,750 + 61\,900 = 127\,650 \text{ lb.}$$

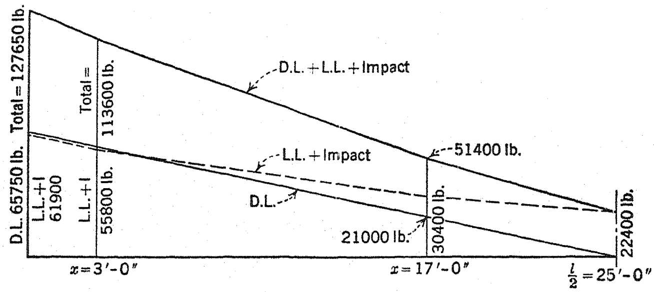


FIG. 28.—Diagram of External Shears in Girder. *(See p. 68.)*

Dimensions of Girders

Before selecting the dimensions for the girder, the depth required by external shear and the economical depth will be computed. The depth required by bending moments is always appreciably smaller than either of these two values.

Depth required by external shear. Assume that $b' = 20$ in., $j = 0.93$ for T-beam

$$d = \frac{127\,650}{20 \times 0.93 \times 120} = 57.3 \text{ in.}$$

Economical Depth of T-beam.[3] — Assume that the ratio of cost of steel in place per cubic foot to cost of concrete in place, including cost of formwork reduced to a basis per cubic foot of concrete is, $r = 55$, then the economical depth is

$$d = \sqrt{\frac{55 \times 16\,942\,000}{16\,000 \times 20}} + \frac{8.0}{2} = 54.0 + 4.0 = 58.0$$

Accepted Dimensions. —

Use $h = 64$ in., $b' = 20$ in., $t = 8$ in.

$$d = 64 - 5.2 = 58.8 \text{ in.}$$

Required area of steel at center, using $j = 0.93$ for $\frac{t}{d} = \frac{8.0}{58.8} = 0.14$:

$$A_s = \frac{16\,942\,000}{0.93 \times 58.8 \times 16\,000} = 19.4 \text{ sq. in.}$$

Use two $1\frac{1}{4}$-in. plus thirteen $1\frac{1}{8}$-in. square bars. $A_s = 2 \times 1.56 + 13 \times 1.266 = 19.58$ sq. in.

[3] See "Concrete, Plain and Reinforced," Vol. I, p. 220.

Place the bars in three layers, five bars per layer. The two $1\frac{1}{4}$-in. square bars are placed in the top layer. To accommodate in one layer two $1\frac{1}{4}$-in. square bars and three $1\frac{1}{8}$-in. square bars with a protective cover of 2 in. and the clear space between two square bars equal to the sum of their sides, the minimum width of stem is: min. $b' = 2 \times 2 + 9 \times 1\frac{1}{8} + 4 \times 1\frac{1}{4} = 19\frac{1}{8}$ in. This is smaller than the adopted width of 20 in., hence this dimension is satisfactory. (See Fig. 29, p. 69.)

To keep the bars in place, use, between the horizontal layers of bars, separators consisting of short pieces of $1\frac{1}{2}$-in. square bar placed crosswise about 4 ft. on centers. The bars should be properly tied to avoid misplacement. If possible, mechanical spacers and separators should be used. Support the lowest layer of the bars on concrete blocks and not on steel supports so as to avoid the danger of rust. Keep the outside bars from the forms the required distance. Stirrups must be kept the proper distance from the face of formwork. Misplaced stirrups are a frequent cause of disintegration of concrete.

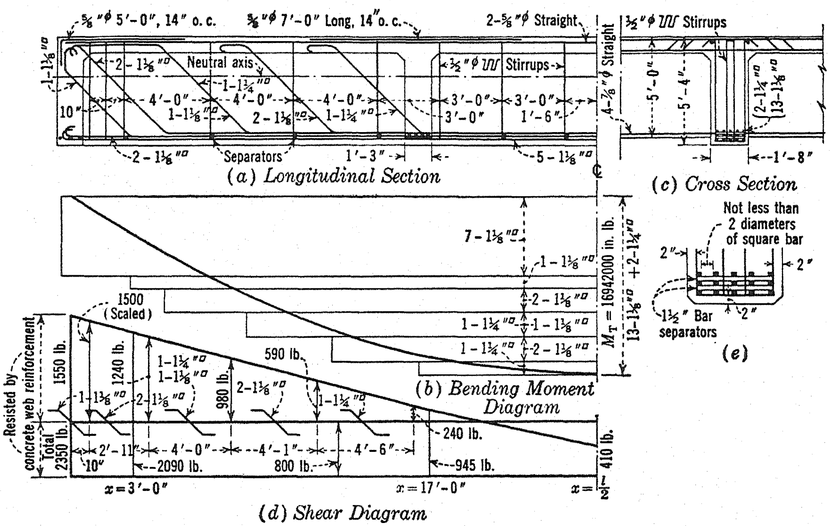


FIG. 29.—Longitudinal Girder. (*See p.* 69.)

Points of Bending of Reinforcement. — A bending-moment diagram is prepared for live load as explained on p. 21 and for dead load; and the two diagrams are combined as shown in Fig. 29 (*b*), p. 69. This diagram is used to determine points of bending of reinforcement. In this example, the point of bending has been selected so as to make the bent bars effective in resisting diagonal tension. (See also p. 181.)

Diagonal Tension Reinforcement. — Shearing stresses are accepted as a measure of diagonal tension. Using external shears in the diagram in Fig. 28, p. 68, find the shearing stresses in the girder per inch of length from formula $vb' = \dfrac{V_x}{jd}$. These are: at support $vb' = 2\,350.0$; at $x_1 = 3$ ft., $2\,090.0$; at $x_2 = 17$ ft., 945; and at center, 410 — all values being in pounds per lineal inch of girder.

These values are plotted in Fig. 29 (*d*). Since, in accordance with American practice, concrete is assumed to resist a unit shear of 40 lb. per sq. in., the total shear

resisted by the 20-in.-wide girder is $20 \times 40 = 800$ lb. per lin. in. Plot this on the shear diagram, and draw a horizontal line. The shear below this line is resisted by concrete, and the shear above the line is provided for by web reinforcement.

In the diagram, indicate the points of intersection of the bent bars with the neutral axis. Since the spacing of the adjoining sets of bent bars is smaller than the effective depth of the beam, each set of bent bars is fully effective in taking care of the shear developed on each side of that set in a distance equal to one-half of the longitudinal spacing. The total shear to be resisted by any one set of bent bars is equal to the area in the shear diagram tributary to that set, i.e., the average shear per lineal inch multiplied by the tributary distance in inches. The value of bent bars in resisting diagonal tension is equal to their cross-sectional area multiplied by the unit tensile stress in steel and by 1.4, or $1.4 \times A_s f_s$.

The stresses to be resisted and the resistance value of bent bars are given in the following table.

Diagonal Tension Reinforcement (See Fig. 29, p. 69)

Set	Bent Bars			Average Shear, lb. per lin. in.	Tributary Distance, in.	Shear to be Resisted, lb.
	Number and Size Square Bars	Area A_s, sq. in.	Resistance $1.4 \times A_s \times 16\,000$			
1st	$1 - 1\frac{1}{8}''$	1.27	28 450 lb.	1 550	10	15 500
2nd	$2 - 1\frac{1}{8}''$	2.54	56 900 lb.	$\frac{1\,500 + 1\,240}{2}$	35	48 000
3rd	$1 - 1\frac{1}{4}'' + 1 - 1\frac{1}{8}''$	2.83	63 350 lb.	$\frac{1\,240 + 980}{2}$	48	53 400
4th	$2 - 1\frac{1}{8}''$	2.54	56 900 lb.	$\frac{980 + 590}{2}$	49	38 400
5th	$1 - 1\frac{1}{4}''$	1.56	34 900 lb.	$\frac{590 + 240}{2}$	54	22 400

From a comparison of the resistance values of the bent bars with the shears to be resisted, it is evident that bent bars are sufficient to take care of all diagonal tension in the regions in which they are assumed to be effective. However, it is always advisable to use some stirrups and to distribute them over the whole length of the girder. Therefore, $\frac{1}{2}$-in. round stirrups with four prongs are placed as shown in the figure.

Bond Stresses in Main Reinforcement. — Maximum bond stresses act near the support where the end shear is $V_1 = 127\,650$ lb., and the tension reinforcement there consists of seven $1\frac{1}{8}$-in. square bars. The perimeter of these bars is

$$\Sigma o = 7 \times 4 \times 1\tfrac{1}{8} = 31.5 \text{ sq. in.}$$

Since $j = 0.93$ and $d = 58.8 + 2 = 60.8$ in., the bond stresses are

$$u = \frac{127\,650}{0.93 \times 60.8 \times 31.5} = 71.7 \text{ lb. per sq. in.}$$

The bond stresses are lower than the allowable stresses, hence the design is satisfactory.

Girder Carrying Cantilever Sidewalk

Dead Load. — The dead load on the girder carrying the cantilever sidewalk consists on one side of all the loads on the cantilever plus $1.2 \frac{M_c}{s}$, where M_c is the bending moment of all dead loads on the cantilevers about the center line of the girder and s is the girder spacing. (See the table, p. 59.) On the other side, the dead load consists of one-half of the reaction of the slab multiplied by 1.1 to take care of the possibility of a larger portion of the dead load being transmitted to the girder on account of its greater rigidity in comparison with the interior girders.

Live Load. — The live load on the girder consists of the live load on the roadway plus the uniformly distributed live load on the sidewalk, plus the effect of the cantilever bending moment equal to $1.27 \frac{M_{c(l)}}{s}$, where $M_{c(l)}$ is the cantilever bending moment of the live load on the sidewalk plus the bending moment due to the horizontal forces acting on the railing. (See design of sidewalk slab.)

The loading of the roadway consists of trucks; and the proportion carried by the girder is found from formula (1), p. 18, making s_1 equal to zero. Longitudinally the trucks are placed in the same position as for the interior girder, and the same formula is used for maximum bending moment.

Design of Girder. — After bending moments and shears are found, the dimensions and the amount of reinforcement are computed in the same manner as for the interior girder. The work is not here shown.

Cross Beams

Two intermediate cross beams are used as shown in Fig. 26, p. 65. The depth of cross beams is such that its bottom reinforcement can be inserted between the second and the third layer of the girder reinforcement.

The area of the bottom reinforcement of the cross beam is taken as 0.003 times the effective area of the cross section of the beam:

$$A_s = 0.003 \times 15 \times 57.5 = 2.6 \text{ sq. in.}$$

Use four $\frac{7}{8}$-in. round bars. $A_s = 4 \times 0.6 = 2.4$ sq. in.

No additional top reinforcement is needed because the slab reinforcement above the cross beam is sufficient to take care of any tensile stresses there developed.

Cantilever Ribs under Sidewalk. — The cantilever ribs under the sidewalk are intended to stiffen the construction, but not to carry any loads. Their strength, however, is sufficient to carry any loads transmitted to them by the slab.

To take care of secondary stresses, short cross bars are placed in the top of the slab across the ribs.

2. TYPE OF FLOOR CONSISTING OF GIRDERS, FLOOR BEAMS, AND SLAB

General Description. — In a bridge with wide spacings of longitudinal girders, the girder and slab design described in the preceding pages of this chapter would require a thick slab. Such design would not be economical, not only on account of the large quantities of material in the slab, but also because the large dead load would require heavy

longitudinal girders. The design may be improved by the introduction of floor beams spanning between the longitudinal girders, and by supporting the slab on these floor beams.

The span of floor beams may be reduced by placing the exterior girders some distance from the edges of the bridge; and then the slab may be either cantilevered out, or it may be supported by cantilever extensions of the floor beams. Bridge floors with floor beams are shown in Fig. 30, p. 72.

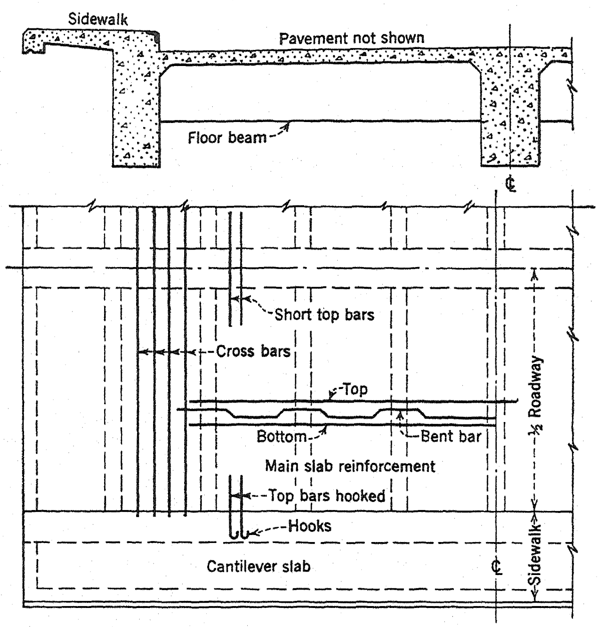


FIG. 30.—Bridge Floor Design: Girders, Floor Beams, and Slabs. (*See p.* 72.)

Spacing of Floor Beams. — The spacing of floor beams should be close enough to permit the use of thin slabs, so as to make the dead load as small as possible.

Design of Slab. — The slab in this type of bridge floor is always continuous over a number of supports. The floor beams are monolithic with the slabs, which affects the bending moments in the slabs in a manner similar to that explained on p. 51 in connection with the girder and slab floor type. But since the stiffness of the floor beams is usually much smaller than that of the girders, their effect upon the slab is appreciably smaller than that of the longitudinal girders discussed in the previous type. With narrow floor beams, their effect upon bending

moments in the slabs may be entirely disregarded, and the slab designed for bending moments given in Table I, p. 53.

For live loads consisting of trucks, maximum bending moments for live loads are produced when the rear axles are placed side by side, and in the center of the slab span. Laterally the concentrated loads should be considered as distributed over an effective width found as explained on p. 15 in connection with two concentrated loads. Usually it is permissible to consider the axle load as distributed uniformly over the width of a traffic lane. No longitudinal distribution of the concentrated load should be assumed in computing bending moments.

Reinforcement of Slabs. — The main reinforcement of the slab consists of bars placed near the bottom of the slab at right angles to the floor beams; and its amount is determined by the maximum positive bending moments. From one-half to two-thirds of the bottom bars are usually bent up and extended as top reinforcement into the adjacent panels, as shown in Fig. 30, p. 72. Negative bending moments in the slab are resisted partly by the top portions of the bent-up bars; and the difference between the required amount of the negative reinforcement and the total area of the effective bent bars is usually supplied by straight bars placed near the top of slab and extending over several panels. These straight top bars take care also of the negative bending moments in the central portions of each panel produced by conditions of loading in which the adjoining panels are loaded, but the panel under consideration is not loaded. At the end floor beams all top bars must be anchored, because they receive their maximum stress at the edge of the floor beam.

Reinforcement parallel to the floor beams should be provided to serve as distributing bars, and also as temperature and shrinkage reinforcement. (See p. 54.) Short top bars should be placed across the top of each girder to prevent cracks due to secondary bending moments, as explained on p. 55.

Design of Floor Beams. — Floor beams are designed for a combination of dead load, live load, and impact. For dead load, the reactions of the slabs may be computed by the rules of static.

When live load consists of lines of trucks, its most unfavorable position longitudinally so far as the floor beam is concerned is when the rear axles of the trucks are placed directly over the floor beam. Transversely, as many trucks should be placed as can be accommodated on the floor beam. If possible, the trucks should be placed as explained on p. 26 for maximum bending moments, and on p. 27 for maximum end shears. For continuous floor beams, each rear axle load should be considered as uniformly distributed laterally over the width of the traffic

lane, and the uniformly distributed unit load thus obtained should be used in design.

For spacings of floor beams of more than 6 ft., the total load should be considered as carried by one floor beam. For smaller spacings, some of the load may be considered as transferred by the slab to the adjoining floor beams, so that the loaded floor beam may be assumed to carry the loading multiplied by the ratio $\frac{s}{6.0}$, where s is the spacing of floor beams.

One-Span Floor Beam. — One-span floor beams, such as in through bridges, may be considered as restrained at the ends by the girders. The degree of restraint depends upon the relation between the rigidity of the floor beam and the resistance to torsion of the girder. This resistance is increased by the other floor beams acting as struts, because usually only one floor beam at a time is subjected to maximum bending moments for live loads.

The restraint at the ends of the floor beam reduces all positive bending moments in the floor beam; and this reduction may be assumed to be equal to 20 per cent of the maximum static bending moment. This restraint also produces negative bending moments at the ends; and to be on the safe side the amount of this bending moment should be taken as not less than 50 per cent of the maximum static bending moment. Larger negative bending moment should be used if it appears advisable, but, of course, it should not exceed that required for fixed beams. The negative bending-moment reinforcement should be anchored in the girder. It should cover at each end a distance from the edge of the girder not less than 0.15 of the clear span.

Continuous Floor Beams. — Floor beams of two or more spans should be treated as continuous beams; and bending moments and shears should be computed as explained in Chapters IX and X for continuous girders. The effect of the monolithic connection between the girders and the floor beams should be taken into consideration somewhat in the same manner as explained on p. 51 in connection with slabs.

Floor Beams with Cantilevers. — In floor beams with cantilevers, the effect of the cantilever loads should be taken care of in the manner explained on p. 53 in connection with slabs with cantilevers.

3. TYPE OF FLOOR CONSISTING OF GIRDERS, FLOOR BEAMS, AND TWO-WAY SLABS

General Description. — In this arrangement, consisting of parallel longitudinal girders, floor beams, and slabs, the spacing of the floor beams is made appreciably larger than in the previously described example. Where possible, the spacing of floor beams is made equal to the spacing

of the longitudinal girders. In square crossings, the floor beams are placed at right angles to the girders, and the slab is divided into a number of square or rectangular panels supported on four sides and reinforced by two sets of bars placed at right angles to each other.

In skew crossings, floor beams may be placed parallel to the skew piers, and the slab panels become rhombic or rhomboidal. The reinforcement is then placed parallel to the supporting beams and girders, respectively. Often, to simplify formwork, it is preferable in skew crossings to place the floor beams at right angles to the girders in the same manner as in square crossings. In such an arrangement, at each end of each span there are odd panels, which to some extent complicate the placing of reinforcement.

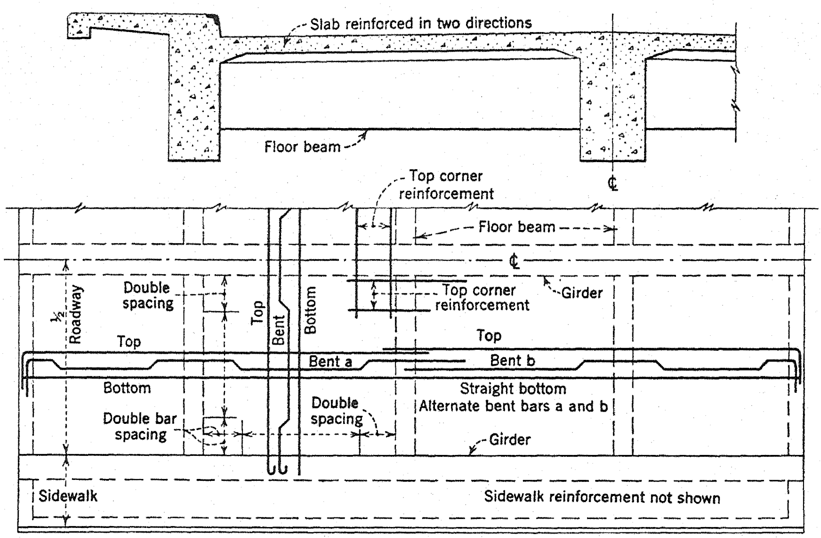


FIG. 31.—Bridge Floor Design: Slab Reinforced in Two Directions. *(See p. 75.)*

For the sake of brevity, slabs reinforced in two directions and supported on four sides are here often called "two-way slabs." A typical illustration of such arrangement is shown in Fig. 31, p. 75. For numerical example see p. 83.

Advantages of Slab Designs Reinforced in Two Directions. — Bridge floor designs consisting of slab panels reinforced in two directions are more economical than either the girder and slab arrangement described on p. 44, or the girder, floor beam, and slab arrangement discussed on p. 71. In Europe in recent years two-way-slab designs have largely supplanted the arrangements formerly in general use, consisting of closely spaced girders with slabs reinforced in one direction. In America, slabs

reinforced in two directions have been used for years in building construction, but in bridge design they have been used but rarely. Yet it is in bridge design that this type is particularly advantageous.

In comparison with the girder and slab type, the arrangement with two-way slabs has the following advantages: (*a*) For the same spacing of main girders, the thickness of slab in the two-way system is decidedly smaller, because the loads are carried in two directions, which reduces appreciably the bending moments upon which the slab thickness is based. (*b*) In a two-way design, the concentrated loads are distributed in two directions, and over the whole panel as explained on p. 79. (*c*) In a two-way slab, all reinforcement is effective in resisting bending moments, whereas in one-way slabs the cross bars and all secondary reinforcement are not directly effective in carrying loads. (*d*) Owing to the decrease in slab thickness, the dead load upon the girder is appreciably reduced, which in turn reduces the dimensions of the girders.

The number of floor beams needed in designs with two-way slabs is not much larger than the number of cross beams in a properly designed bridge with one-way slab. For instance, in Fig. 34, p. 84, showing a bridge 60 ft. long three intermediate floor beams are used in the two-way design, while in the one-way design two intermediate cross beams would have been needed. Therefore, with the same spacing of longitudinal girders, the cost of formwork is not much greater than in the one-way design. However, the two-way-slab design permits a much wider spacing of main girders, thus appreciably reducing the cost of formwork.

In comparison with the designs consisting of floor beams and one-way slabs the two-way designs have the advantage that they require a much smaller number of floor beams, and at the same time a smaller thickness of slab. All slab reinforcement is effective in the two-way slab, while in the one-way design only the main bars carry the loads. The distribution of live loads is also superior. The dimensions of floor beams for the two-way arrangement, even with the appreciably wider spacing, do not need to be made larger for the following reasons. First, because the dead load carried by them is smaller than for the one-way arrangement; second, because the live load resisted by one floor beam for spacings up to 14 ft., i.e., the spacing of axles in a design truck, is the same in both cases, and consists of the rear axles of the trucks placed directly over the floor beam. For larger spacings of floor beams the increase in live load is small.

Bending Moments in Slab Reinforced in Two Directions. — Accurate determination of bending moments in a slab reinforced in two directions is very complicated and will not be here discussed.

The approximate method here given is simple, easy to understand,

and easy to apply. It not only gives structures of ample strength to carry the loads for which they are designed; but, actually, two-way slabs designed as here recommended have an appreciably greater factor of safety than the one-way slabs, because this approximate method does not utilize to the full extent the advantages of this type of construction.

In the approximate method here recommended, it is assumed that a two-way slab is replaced by two separate one-way slabs, one of which extends between the girders and is reinforced crosswise, while the other spans between the floor beams and is reinforced by longitudinal bars. Part of the total load on the slab is assumed as carried by one of these two slabs, and the balance by the other slab. The procedure in designing is as follows: (1) Find the loadings carried by each of the two one-way slabs. (2) Treat each imaginary one-way slab, with its loading, separately; and compute for each of them bending moments in the same manner as for an actual one-way slab of the same number of spans as there are panels in that direction, and with the same conditions of restraint as actually exist at the supports in that direction.

In this manner, two sets of bending moments at critical sections are obtained, one set in each direction; and for these bending moments the thickness of the slab and the amounts of reinforcement are computed in the usual way. When the two imaginary slabs are combined, the tensile stresses in each direction are resisted by the set of bars running in that direction; and the compression stresses are resisted by the common concrete slab, each particle of concrete being subjected simultaneously to stresses acting at right angles to each other. It should be noted that being subjected to compression in one direction does not reduce, but actually increases, the resistance of concrete to stresses acting simultaneously in the direction at right angles.

Division of Slab Loading in Two Directions. — The division of the slab loading between the two imaginary one-way slabs depends primarily upon the relation between the length and the width of the slab panel. Everything else being equal, the shorter span of the panel carries the larger proportion of the slab load.

The division of the slab loading also depends upon whether the degree of restraint along the edges of the slab in one direction is equal or unequal to the degree of restraint in the other direction. An increase in the degree of restraint along two opposite edges over those along the other two edges is equivalent to a reduction in the span length at right angles to the edges with increased restraint. This means that in a square panel in which the slab is continuous in one direction and simply supported in the other direction the continuous slab will carry the larger proportion of the loading.

In a two-way slab, the uniformly distributed loading on the slab, w, may be considered as divided into two parts: w_s carried by the short span, and w_l carried by the long span. The sum of the two values is equal to the loading w; or $w = w_s + w_l$.

Notation:

l_s = span in short direction.
l_l = span in long direction.
$m = \dfrac{l_s}{l_l}$ = ratio of span in short direction to span in long direction.
w_s = unit load carried in short direction.
w_l = unit load carried in long direction.
w = total unit load on slab.
α_s = deflection coefficient for short span.
α_l = deflection coefficient in long span.

Then[4]

Unit Loading Carried by Short Span:

$$w_s = \frac{\alpha_l}{\alpha_l + m^4\alpha_s w} \tag{5}$$

Unit Loading Carried by Long Span:

$$w_l = w - w_s \tag{6}$$

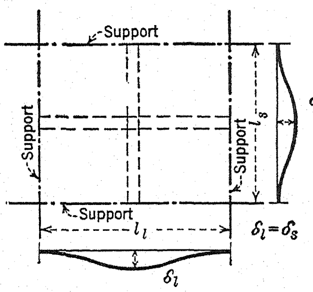


Fig. 32.—Deflection of Slab Reinforced in Two Directions.

α_s and α_l in the formulas depend upon the restraint of the edges of the respective slabs. When the slab in the short direction is fixed at both edges, the constant α_s equals 1.0; when freely supported on both sides, α_s = 5.0; and for intermediate conditions of restraint, intermediate values should be used. The same applies to the values α_l. When conditions on all sides are the same, values of α_s and α_l in the formulas cancel and are replaced by 1.0.

[4] This formula was developed by the authors.

The deflection formulas for a slab in the short and long directions, respectively, are

$$\delta_s = \frac{\alpha_s}{384EI} w_s l_s^4, \quad \text{and} \quad \delta_l = \frac{\alpha_l}{384EI} w_l l_l^4$$

Since the deflection in the center of the slab must be the same, see Fig. 32, p. 78, $\delta_s = \delta_l$. Formula (5) is obtained from this requirement, and from the requirement that the sum of the unit loads carried in each of the two directions must be equal to the total load unit w.

Use of Formulas for Unit Loads w_s and w_l — The formulas for unit loads w_s and w_l may be used in each case where the loading of the slab is uniformly distributed. After these values are found, bending moments and shears are computed as for a slab reinforced in one direction.

Dead Loads. — The dead load is usually composed of the weight of the slab and the weight of the pavement. Both are either uniformly distributed, or may be replaced by a uniformly distributed loading. Therefore, for dead load, formulas (5) and (6) may be used directly.

Live Loads. — The loading on sidewalks and on foot bridges may be considered as uniformly distributed; and it may be divided between the two floor systems using formulas (5) and (6) directly.

Concentrated wheel loads are carried by the whole panel upon which they rest because it is impossible to conceive how any part of the panel could fail to be effective. The distribution of the concentrated loads in the lateral and the longitudinal directions may be found with sufficient accuracy for practical purposes in the following manner.

Lateral Distribution. — Laterally, each concentrated axle load may be considered as distributed uniformly over the whole width of the traffic lane. Since the traffic lanes adjoin, the whole slab may be considered as loaded uniformly by a loading equal to the load on a rear axle divided by the width of the traffic lane.

Longitudinal Distribution. — So far as the slab is concerned, the truck loads are in the most unfavorable position when as many trucks are placed side by side as can be accommodated on the slab panel and when the rear axles of all trucks are placed along the center line of the longitudinal span of the panel. As stated in the preceding paragraph, the concentrated loads may be considered as uniformly distributed laterally over the whole lateral span. Now each of these loads is also distributed longitudinally; and it may be assumed that each load is distributed uniformly over the effective width computed in the same manner as for a one-way slab, using the rule given on p. 16 for restrained slabs. In this case the effective width would be $0.7 \times 0.72l_2 = 0.5l_2$. Over the balance of the panel the live load may be assumed as distributed with intensities varying according to a straight line from the maximum value to zero at the supports. This distribution is shown in Fig. 33, p. 79.

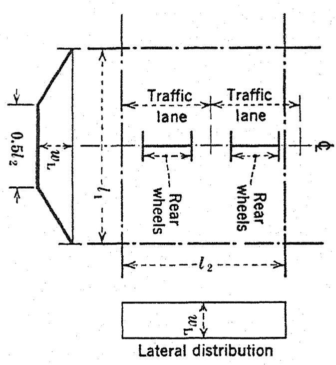


FIG. 33.—Distribution of Wheel Loads. Slab Reinforced in Two Directions. (*See p.* 79.)

Intensity of Live Load Due to Rear Axles. — Using the distribution of the rear axle load as given in the preceding paragraphs, the unit live load acting upon the slab may be found from the following formula.

Let P = truck load.
$0.8P$ = load on rear axle of truck.
l_1 = longitudinal span of panel.
l_2 = transverse span of panel.
c = width of traffic lane.
w_L = unit live load on slab.

Unit Live Load on Slab Due to Rear Axle of Truck:

$$w_L = \frac{0.8P}{(0.5l_1 + 0.25l_2)c} \tag{7}$$

This unit load acts as shown in Fig. 33, p. 79; but to simplify computations it may be assumed that the slab is subjected to uniformly distributed loading extending over the whole panel, which assumption gives somewhat more unfavorable results.

Division of Unit Live Load w_L in Two Directions. — It is now necessary to divide the unit live load w_L between the two imaginary one-way slabs; i.e., the unit live load, w_L, is assumed to be divided into the unit load carried by the short span and that carried by the long span, and their values are computed using formulas (5) and (6), p. 78.

The unit loads acting in each direction having been determined, the bending moment in each one-way slab is computed as for slabs reinforced in one direction.

Dimensions of Slab and Amount of Reinforcement. — The maximum positive and negative bending moments for dead load should be added to the corresponding maximum bending moments for live load, including impact. As a result, two sets of combined positive and combined negative bending moments are obtained.

In each panel, the larger of the two maximum positive bending moments governs the thickness of the slab in the center; and the reinforcement corresponding to this bending moment is placed as the first layer of the bottom bars. The amount of reinforcement in the other direction is then based upon the smaller depth for the upper layer of the bottom bars.

When the negative bending moments at any support are larger than the maximum positive bending moments, the depth of the slab there may be increased by a shallow haunch of proper depth. To be effective, the length of the haunch should not be less than three times its depth. (See p. 83 for numerical example.) In many cases, even with larger

negative bending moment than the maximum positive bending moment, it is possible to get along without haunches because at the supports there is always some effective compression reinforcement, and because the allowable unit stresses at support are larger than in the center of the span.[6]

Arrangement of Slab Reinforcement for Two-Way Slabs. — The required cross sections of reinforcement at the supports and in the center of the span having been found, the reinforcement in each direction is arranged in the same manner as for slabs reinforced in one direction. (See p. 52.) It is important to make the arrangement of bars as simple as possible. Multiplicity of bars of different lengths should be avoided, and, where possible, continuous bars extending over several spans should be used. However, the bars should not be too long for handling.

The spacing of bars found for the maximum bending moments should be maintained in the central six-tenths of the panel width and the panel length, respectively. In the end portions of each panel, the spacing of bars may be doubled, and all these bars made continuous at the bottom; also, one or two spaces of bars next to the beam or girder may be omitted. These reductions in reinforcement are permissible because in the end sections the maximum positive bending moments are appreciably smaller than in the central portion of the panel; and next to the support no tensile stresses parallel to that support can be developed in the bottom of the slab. Corner top reinforcement should be used as explained under separate heading.

A simple arrangement of reinforcement in a two-way slab is shown in Figs. 31, p. 75, and 34, p. 84.

Corner Reinforcement. — In addition to the main reinforcement arranged as explained in the previous paragraph, it is necessary to provide reinforcement in each corner of the slab. This should consist of two sets of bars placed near the top of the slab at right angles to the supporting beams or girders. The size and spacing of these bars should be the same as those for the negative bending moments in the central portion of the panel under consideration. Each bar should extend on each side of the beam a distance equal to one-quarter of the clear span in that direction. At the end beam or girder, the top bars should be anchored in the beam or girder.

The object of the corner reinforcement is to prevent cracks in the corners of the slab produced by the peculiar action of the two-way slabs.

[6] In American practice, when the allowable unit stress in the center is $f_c = 800$ lb. per sq. in., the corresponding stress at the supports of continuous slabs and girders is $f_c = 900$ lb. per sq. in.

These cracks have been found both in tests and in actual structures where no corner reinforcement has been used.

Design of Floor Beams for Two-Way Slabs. — Floor beams should be assumed to carry their proportion of the dead load.

When the spacing of floor beams is equal to, or smaller than, the longitudinal spacing of the axles of the design truck, the most unfavorable loading for a floor beam consists of the rear axles of as many trucks as can be there accommodated, placed side by side directly above the floor beam. This loading should be considered as carried by one floor beam.

For spacings of floor beams larger than the axle spacing, the reactions of the front axles located on the slab panels should be added to the loading of the floor beam. Since these front axles are much nearer the adjoining floor beam, and also since a large part of these loads is transferred to the girders by the two-way slab, only a small part of this loading reaches the floor beam under consideration.

Design of Main Girders Carrying Two-Way Slabs. — In a bridge with slabs reinforced in two directions, part of the loads is transferred directly from the slab to the girders, and the balance is transferred to them by the floor beams as concentrated reactions.

For dead load, when the number of the slab panels in the girder span is even, the maximum bending moments in the girder, for the assumption that the slab loads are transmitted partly directly to the girder and partly as concentrated loads at the panel points, are the same as for an assumption that all slab loads are uniformly distributed along the girder. When the number of panels is odd, the results obtained by the first assumption is smaller; and the difference reaches a maximum of 5.5 per cent of the maximum bending moment due to the slab load for a design with three panels. This difference decreases with the increase in the number of the slab panels.

As far as external shears are concerned, there is always a difference in the results obtained by the two methods, because in the first method the reaction of the end floor beam does not produce any shear in the girder.

For live loads, maximum bending moments in the girder may be computed as for a girder carrying one-way slabs. If desired, however, the result may be multiplied by the ratio of maximum bending moments for the slab dead load computed by the two methods discussed in the preceding paragraph.

It is a difficult task to find exact external shears for concentrated wheel loads because concentrated loads placed upon a slab panel are transferred in four directions: directly to the girders and to the floor

beams which then carry the loads to the girders. For practical purposes it is accurate enough to use for the girder the same external shear diagram as for a girder with a slab reinforced in one direction, except that in each end panel the shear in the girder may be taken as equal to the shear from the diagram at the first interior floor beam.

Numerical Example of Design of Two-Way Slab

The following numerical example illustrates the procedure to be used in the design of a floor system consisting of slabs reinforced in two directions.

To make possible a direct comparison between the design with slabs reinforced in one direction, and the design with two-way slabs, in this example the same general dimensions of the bridge are accepted as were used in the example on p. 60. The resulting designs are compared and discussed at the end of this example.

Example. — Design a floor system for a bridge the general dimensions of which are given on p. 60, using slabs reinforced in two directions.

Length of girders, 52 ft. overall.

Width of roadway, 30 ft.; two sidewalks, 5 ft. 6 in. each.

Loading of roadway, 20-ton truck trains as shown in Fig. 23, p. 60.

Allowable unit stresses same as used in the example on p. 60.

Solution. — In this example, three main girders are used in place of the four girders in the example on p. 60. Each sidewalk is supported on one side by the exterior main girder, and on the other side by the facia girder which is carried by the cantilever extensions of the floor beams. Three intermediate floor beams are used, spaced as shown in Fig. 34, p. 84. In the longitudinal direction, the end panels are made somewhat shorter than the interior panels to equalize bending moments in that direction.

Design of Slab

Interior Panels. — The clear spans of the interior slab panels are used in determining bending moments in the slab. The spans are

$$l_l = 13 \text{ ft. } 2 \text{ in.}; \quad l_s = 12 \text{ ft. } 3 \text{ in.}; \quad \text{ratio } m = \frac{12.25}{13.17} = 0.93$$

Longitudinally, the slab is continuous over four spans. To take into account the restraining effect upon the slab of the floor beams, bending-moment coefficients are assumed to be intermediate between those for a continuous slab with free ends and those for a continuous slab with fixed ends. For both conditions, coefficients are taken from Table I, p. 53, for a slab of four equal spans. Since the end spans in this case are somewhat shorter than the interior spans, this assumption is not accurate; but it is exact enough for practical purposes. The coefficients for continuous slab with free ends is multiplied by 2 and added to the coefficients for continuous slab with fixed ends, and the sums are divided by 3.

Laterally, the slab consists of two short end spans carrying the sidewalks, and of

two interior panels under the roadway. In computing bending-moment coefficients for the roadway panels, the two sidewalk panels are disregarded, and considered only as affecting the degree of restraint at the exterior girders. The roadway panels are restrained at the supports by the girders which are not only heavy but also stiffened laterally by the floor beams. Under these circumstances, the coefficients may be

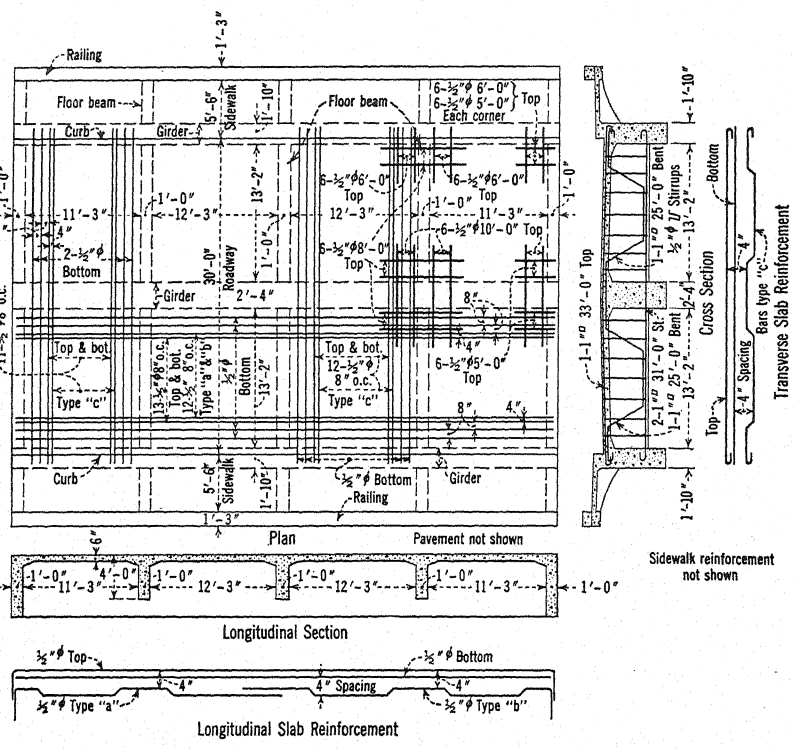


FIG. 34.—Example of Two-Way Slab Design. (*See p.* 83.)

assumed to be intermediate between those for a continuous slab of two spans with free ends and those for a one-span slab fixed at both ends. Tables I and II, p. 53, are used in finding the coefficients. The coefficients for the fixed span condition are multiplied by 2 and added to the coefficients for the continuous two-span slab, and the sums are divided by 3.

The live load in both the lateral and the longitudinal direction is assumed as uniformly distributed.

Division of Loads in Two Directions. — Interior Panels. By comparing the restraint of the interior panels at the four supports, the following relation is accepted between the deflection constants to be used in formula (5), p. 78, $\alpha_s = 1.2\alpha_l$. Use $m = 0.93$. α_l cancel in the formula, and the loads carried in the long and short directions are

$$w_s = \frac{1.0}{1.0 + 1.2 \times 0.93^4} w = 0.53w; \quad w_l = (1 - 0.53)w = 0.47w$$

Coefficients for Bending Moments. Interior Panels. — As explained in the previous paragraphs, values from Tables I and II are used to find coefficients in the two directions.

	Maximum Positive Bending Moments		Negative Bending Moments at Support	
	Dead load	Live load	Dead load	Live load
4-span continuous slab, free ends.......	2 × 0.036 0.072	2 × 0.08 0.160	2 × 0.107 0.214	2 × 0.121 0.242
fixed ends........	0.042	0.074	0.083	0.098
	3)0.114	3)0.234	3)0.297	3)0.340
Coefficients for short span...........	**0.038**	**0.078**	**0.099**	**0.113**
2-span continuous slab, free ends.......	0.07	0.096	0.125	
one-span, fixed ends	2 × 0.042 0.082	2 × 0.042 0.084	0.083 × 2 0.166	
	3)0.152	3)0.180	3)0.291	
Coefficients for long span...........	**0.051**	**0.060**	**0.097**	

Unit Loads for Slab Panel. —

Dead Load. — Slab plus pavement: $w_d = 75 + 40 = 115$ lb. per sq. ft. Short way, $w_s = 0.53 \times 115 = 61$ lb.; long way, $w_l = 115 - 61 = 54$ lb. per sq. ft.

Live Load. — The load on the rear axle, allowing 24 per cent for impact, is $1.24 \times 32\,000 = 39\,700$ lb. Using formula (7), p. 80, the unit live load is

$$w_{L+I} = \frac{39\,700}{(0.5 \times 12.25 + 0.25 \times 13.17) \times 10} = 422 \text{ lb. per sq. ft.}$$

Divide this unit load in two directions in the same manner as for dead load.

Live load plus impact, short way $w_s = 0.53 \times 422 = 224$ lb. per sq. ft.

long way $w_l = 422 - 224 = 198$ lb. per sq. ft.

Bending Moments. — Using coefficients and unit loads just computed, bending moments are:

Short Span. — $l_s = 12.25$ ft. Dead load $12 w_s l_s^2 = 12 \times 61 \times 12.25^2 = 110\,000$ in-lb.

Live load $= 12 \times 224 \times 12.25^2 = 403\,000$ in-lb.

		Maximum positive		Negative at support
Dead load	0.038	4 200	0.099	−11 000
Live load	0.078	31 400	0.113	−45 500
Total		35 600 in-lb.		−56 500 in-lb.

Long Span. — $l_l = 13.17$ ft. Dead load $12 w_l l_l^2 = 12 \times 54 \times 13.17^2 = 112\,100$ in-lb.
Live load $= 12 \times 198 \times 13.17^2 = 412\,000$ in-lb.

		Maximum positive		Negative at support
Dead load	0.051	5 700	0.097	−10 900
Live load	0.060	24 700	0.097	−40 000
Total		30 400 in-lb.		−50 900 in-lb.

Dimensions and Amount of Reinforcement. Interior Panels. — The thickness of slab is governed by the largest positive bending moment in the center of the panel. For the specified stresses and the corresponding constants $R = 146.9$ and $j = 0.857$.

Thickness of slab:

$$d = \sqrt{\frac{35\,600}{12 \times 146.9}} = 4.5 \text{ in.} \quad \text{Use } h = 6 \text{ in.}$$

Reinforcement, short span, using 1 in. of concrete cover:

$$d = 6.0 - (1.0 + \tfrac{1}{4}) = 4.75 \text{ in.} \quad A_s = \frac{35\,600}{0.857 \times 4.75 \times 16\,000} = 0.55 \text{ sq. in.}$$

Use $\frac{1}{2}$-in. round bars 4 in. on centers, in bottom layer. $A_s = 0.196 \times \dfrac{12}{4} = 0.59$ sq. in.

Reinforcement in long span, second layer:

$$d = 6.0 - (1.0 + \tfrac{1}{2} + \tfrac{1}{4}) = 4.25 \text{ in.} \quad A_s = \frac{30\,400}{0.857 \times 4.25 \times 16\,000} = 0.53 \text{ sq. in.}$$

Use $\frac{1}{2}$-in. round bars 4 in. on centers, second layer.

Thickness of Slab at Supports. — The negative bending moments at supports are larger than in the center of the span. To use the same areas of steel at the supports as in the center, it is necessary to increase the depth of the slab at the supports by introducing a haunch.

Since it is desired to limit the negative bending-moment reinforcement to the available area of steel $A_s = 0.59$ sq. in., to take care of the negative bending moment of $M = -56\,500$ in-lb., increase the depth of the slab at the supports to:

$$d_1 = \frac{56\,500}{0.857 \times 0.59 \times 16\,000} = 7.0; \; d_1 - d = 7.0 - 4.75 = 2.25 \text{ in.}$$

Use a haunch $2\frac{1}{2}$ in. deep and 8 in. long.

The same haunch is used at all four supports.

End Panels. — The end panels are made somewhat shorter than the interior panels so as to get the same bending moments in the longitudinal direction in the end panels as in the interior panels. This permits there the use of the same thickness of slab and the same reinforcement as in the interior panels. The computations are not repeated here.

The relation between the degrees of restraint in the two directions is different in the end panels from that in the interior panels. The accepted relation between the deflection constants to be used in formula (5), p. 78, is $\alpha_s = 1.8\alpha_l$.

Arrangement of Main Slab Reinforcement. — The arrangement of main slab reinforcement is shown in Fig. 34, p. 84. Bending sketches of the bars are also there given.

Corner Reinforcement. — As shown in Fig. 34, the corner reinforcement consists of straight bars of the same size and spacing as the main negative bending-moment reinforcement.

Floor Beams

It is assumed that each floor beam carries the proportion of the slab dead load for which the slab was designed in the longitudinal direction. Design span l = 13.17 ft.

Dead Load. —

Slab and pavement	$61.0 \times 12.25 =$	747.0
Beam	$4 \times 150.0 =$	600.0
Pavement above beam	$=$	40.0
Total w_d	$=$	1 387.0 lb. per sq. ft.

$$12 \times w_d l^2 = 2\,880\,000 \text{ in-lb.}$$

Live Load. — The most unfavorable position of the trucks in longitudinal direction is when the rear axles are placed directly over the floor beam. The load is assumed as distributed transversly over the width of the traffic lane, and this uniformly distributed loading is used in designing the floor beam. Therefore $w_l = \dfrac{1.24 \times 32\,000}{10.0} = 3\,970$ lb., including impact, and

$$12 w_l l^2 = 8\,260\,000 \text{ in-lb.}$$

Assuming the same conditions of restraint as were used in designing the slab in the same direction, the bending moments are:

		Positive		Negative
Dead load	$0.051 \times 2\,880\,000$	147 000	0.097	280 000
Live load	$0.06 \times 8\,260\,000$	496 000	0.097	801 000
Total		643 000 in-lb.		1 081 000 in-lb.

Depth of Beam and Amount of Reinforcement. — A larger depth of floor beam is accepted than required by bending moments and shears, because the beams serve also to stiffen the bridge laterally. Accept h = 48 in. and b = 12 in. For one layer of steel, $d = 48 - 2.5 = 45.5$ in. Assuming $j = 0.88$ at support, and $j = 0.9$ for T-beam, the required areas of steel are:

$$\text{Positive } A_s = \frac{643\,000}{0.9 \times 45.5 \times 16\,000} = 1.0 \text{ sq. in.}$$

$$\text{Negative } A_s = \frac{1\,081\,000}{0.88 \times 45.5 \times 16\,000} = 1.70 \text{ sq. in.}$$

To these areas is added the reinforcement usually provided in cross beams to stiffen the bridge. Therefore, use bottom reinforcement consisting of three 1-in. square bars. Bend one bar at each end. Extend one end into the adjoining span across the interior girder and at the other end provide a hook placed in the end girder. Also add one 1-in. square bar at the top, provided with hooks at both ends.

Because of this arbitrary increase in reinforcement, the effect of the cantilever loads will not be computed.

Web Reinforcement of Floor Beam. — The shearing stresses in the floor beam are low. However, several $\frac{1}{2}$-in. round stirrups of two prongs, spaced as shown in the figure, will be used.

Main Girders

In this case, in computing bending moments in girders, it is accurate enough to assume that the dead load is uniformly distributed along the girder. Advantage should be taken, however, of the reduced end shears.

For live load bending moments, concentrated wheel loads should be used in the same manner as for the main girder in the example on p. 67. To get maximum loading of the girder, all three lanes must be fully loaded by trucks running in the same direction. When all three lines of trucks are simultaneously in the most unfavorable position as far as the girder is concerned, the girder would carry 1.5 truck loads, or $W = 1.5P$. Such condition of loading is not likely to occur; therefore, W may be reduced as explained on p. 11. If, however, the girder is designed for the full load, the bridge would be able to carry safely occasional trucks of much larger weight than the design truck.

The details of design are the same as for the girder designed on p. 67 and will not be repeated here.

Comparison of Design Using One-Way Slab with Design Using Two-Way Slab. — The designs worked out in the example on p. 60 and in the example on p. 83 make possible a direct comparison of the merits of the two types of floor designs.

In the design using the two-way slab, one more cross beam is used than in the design with the one-way slab. However, one main girder has been omitted, and the thickness of the slab is reduced from 8 in. to 6 in. The amounts of material in the three main girders of the two-way design are appreciably smaller than in the four girders in the one-way design for the following reason: the dead load of the slab is much smaller in the two-way design; the live load acting on the girder may be reduced; these reductions reduce the dimensions and the dead load of the girder.

The advantages of the two-way arrangement may be summarized as follows: (1) reduced cost of slab; (2) appreciable reduction in cost of formwork because the number of main girders has been reduced and only one floor beam added; (3) appreciable reduction in quantities of materials in the girders.

EXAMPLES FROM PRACTICE

Crossing over Savannah River Delta in Georgia. — In. Fig. 35, p. 89, is shown the cross section through a 25-ft. girder span used in the pile trestle over the Savannah River Delta. In this structure, the girders crossing the marsh are 25 ft. long and are supported on pile bents. Where the trestle crosses deeper water, the span lengths are increased to

35 ft. and the deck is supported by pier bents on piles (see p. 429). In the channel, through girder spans 48 ft. long are used, and these are described on p. 117.

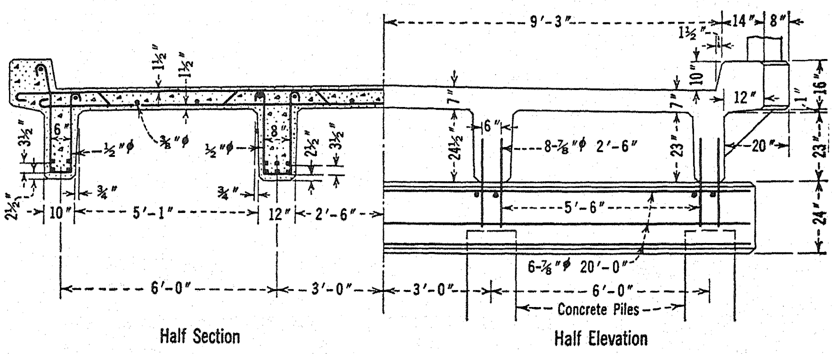


Fig. 35.—Pile Trestle Across Savannah River Delta. (*See p.* 88.)

Bridge across Truckee River near Hilton. (See Fig. 36, p. 90.) In this structure, the end spans are 34 ft. long, and the interior spans 60 ft. long. The abutment is replaced by two columns, one under each girder, each resting upon an isolated footing. The wing walls of the abutment are replaced by shallow wings attached to the girders.

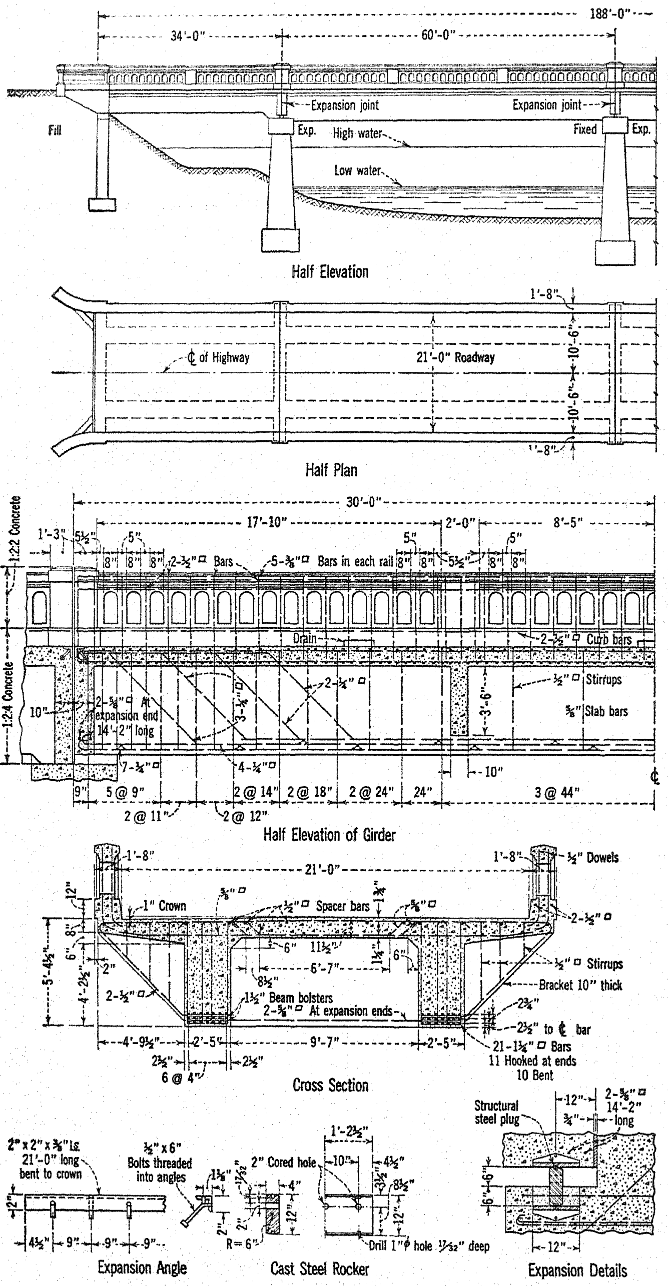
The design of the 60-ft. girders is clearly shown in Fig. 36. The ratio of depth to span is 1 : 10.8. Expansion bearings for each interior span consist of a segmental steel rocker 12 in. in diameter, placed between two steel bearing plates.

At each expansion joint, the edges of the slabs are protected by $2 \times 2 \times \frac{3}{8}$ in. angles properly anchored to the slabs. The $\frac{3}{4}$-in. expansion joint is filled with a premolded elastic strip.

Bridge across San Dieguito River, California. (See Fig. 37, p. 91.) This structure is 595 ft. 10 in. long, and consists of eleven 54-ft. spans. The useful width is 48 ft., of which 40 ft. form the roadway and the balance two sidewalks, each 4 ft. wide.

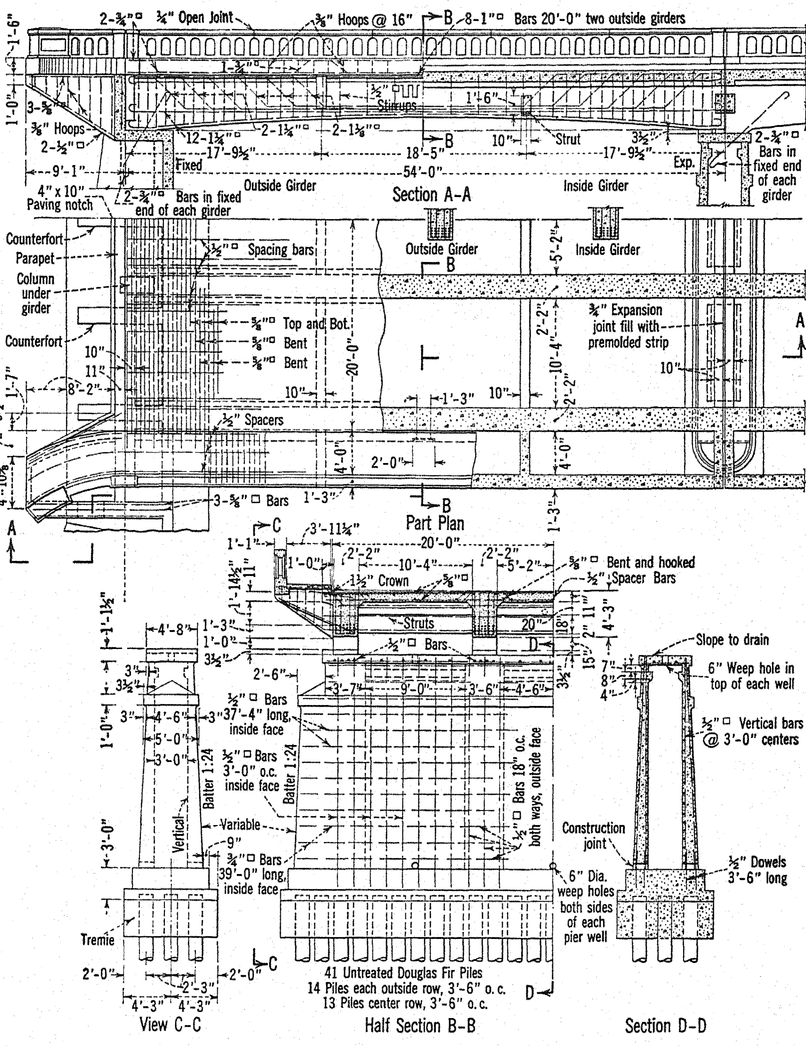
In cross section, the bridge consists of four main girders and of two facia girders supported by the cantilevered cross struts. The sidewalk slab is built separately from the roadway slab. In each girder span there are two end struts and two intermediate struts. Openings are provided between the tops of the struts and the bottom of the slab for the accommodation of pipes and ducts.

The arrangement of the girders and their details are shown in Fig. 37, p. 91. The ratio of the depth of girder to span is 1 : 12.3. Each girder has a fixed bearing at one end and an expansion bearing at the



Harlan D. Miller, Bridge Engineer.

FIG. 36.—Bridge Across Truckee River, California. (*See p.* 89.)



Chas. E. Andrews, Bridge Engineer.

FIG. 37.—Bridge Across San Dieguito River, California. (*See p.* 89.)

other. The expansion bearing consists of a steel rocker placed between two bearing plates. Each rocker is $5\frac{1}{2}$ in. deep, and when in place is buried to a depth of 2 in. in an asphalt seal.

The girders are supported by hollow piers, each consisting of four columns resting on a common foundation. The effect of a solid pier is obtained by connecting the columns by thin reinforced-concrete vertical slabs. On the top of the columns a continuous slab and a continuous coping are provided.

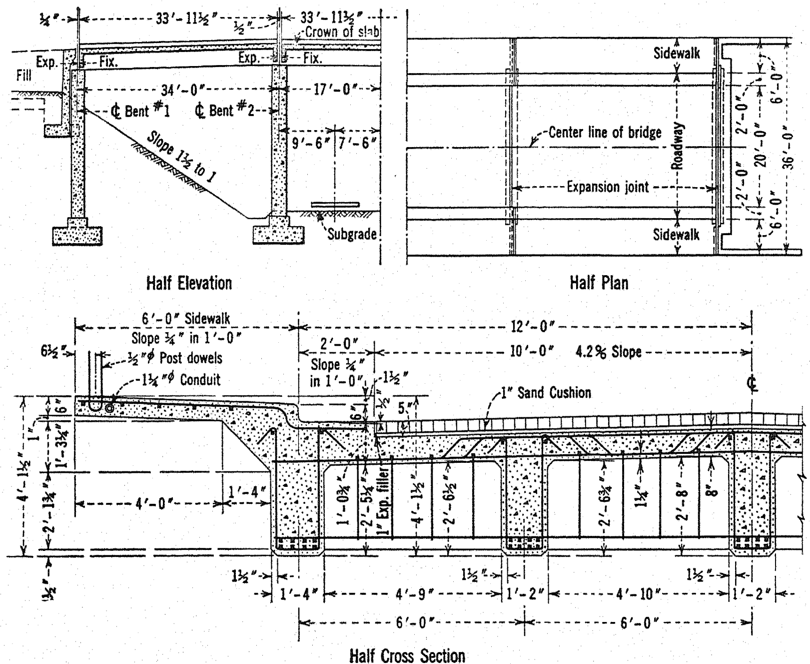


FIG. 38.—Pells Street Crossing, Illinois Central Railroad, Paxton, Illinois. (*See p.* 92.)

Paxton Grade Separation. — The Pells Street crossing in Paxton, Illinois, designed by the engineering department of the Illinois Central Railroad, consists of three spans. In cross section, the superstructure consists of five girders spaced 6 ft. on centers. The sidewalks are cantilevered.

The arrangement of the wood pavement is of interest.

CHAPTER VI

THROUGH GIRDER BRIDGES

Simply supported through girder bridges are treated in this chapter with complete description and analysis of designs with solid and open webs respectively. A numerical example illustrates the use of the formulas in design. Details of design are shown from the examples from practice.

General Description. — Through girder bridges are structures in which the main longitudinal girders extend above the roadway; and the depth of the construction, so far as the headroom below is concerned, is governed by the depth required for the floor construction. The floor construction may consist of: (1) a solid slab spanning between the main girders; (2) closely spaced floor beams spanning between the girders with slab supported by the floor beams; (3) floor beams spaced farther apart than in the previous case, one or more lines of longitudinal beams spanning between floor beams, and a slab divided into panels, which are supported on four sides and reinforced in two directions. The third arrangement is likely to be the most economical because it gives the smallest dead load of the floor.

As a rule, through girder bridges are less economical than the deck girder designs described in the previous chapter; and they are used mainly where the headroom is too small for deck girder designs. Their use is naturally restricted to structures in which the width is appreciably smaller than the span.

In highway construction, intermediate girders in through bridges are seldom used in modern practice although there is no objection to their use to separate traffic in four- or more lane roads having reservations in the middle of the highway. In railroad construction, on the other hand, multi-track through girder bridges are usually built with intermediate girders placed between two adjacent tracks.

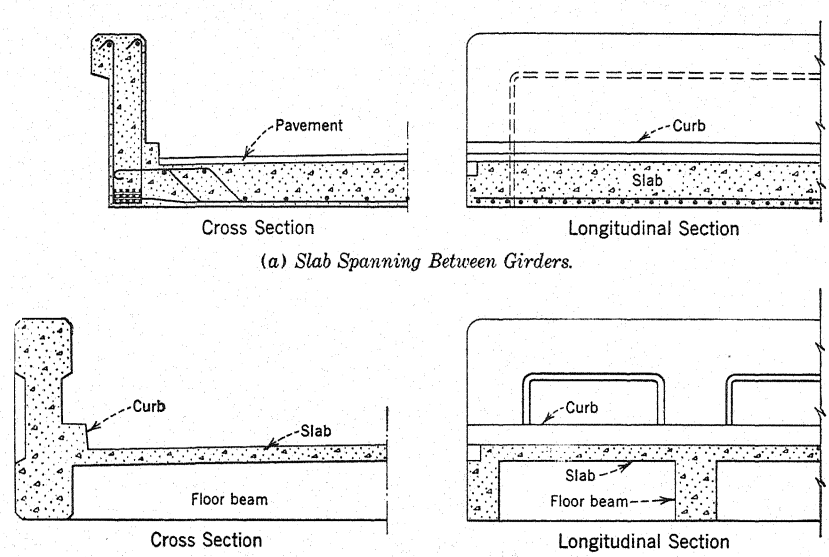
Through girders should not be used on roads which may require future widening, unless a four-lane road is projected with a separation in the center. It must be borne in mind that, with the normal increase in traffic, all through roads are likely to require three- or four-lane highways in the future.

Although this chapter deals mainly with simply supported girder bridges, the details of floor design and construction here described and discussed may be used for cantilever and continuous through bridges as well as for rigid through frames.

A cantilever through bridge is shown in Fig. 63, p. 147; a continuous through bridge is described on p. 163; finally a one-span rigid through frame is shown in Fig. 147, p. 324.

THROUGH BRIDGE WITH SOLID SLAB SPANNING BETWEEN GIRDERS

The simplest design of floor construction for a through bridge consists of a slab spanning between the main girders, as shown in Fig. 39 (*a*), p. 94. Such arrangement is economical only for narrow bridges. For wider crossings, the dead load of the slab would be excessive.



(*a*) *Slab Spanning Between Girders.*

(*b*) *Floor Beam and Slab Design.*

FIG. 39.—Typical Floor Designs for Through Girder Bridges. (*See p.* 94.)

Bending Moments in Slab for Dead Load. — In computing bending moments for the slab, the only difficulty is to estimate correctly the effect of the restraint of the slab due to the heavy main girders with which the slab is monolithic. This restraint depends upon the relation between the torsional resistance of the girder and the stiffness of the slab. Under ordinary conditions, the bending moments for dead load may be found as follows: Compute the maximum static bending moment in the

center of the span assuming it as simply supported, and assume that owing to the restraint all positive bending moments in the slab are reduced by 20 per cent of the maximum static bending moment. At the supports, provide for negative bending moments of a magnitude equal to at least 50 per cent of the previously determined maximum static bending moment.

When sidewalks are supported by cantilever slabs placed outside of the girders, the effect of the cantilever bending moments upon the slab should be determined in the same manner as explained on p. 54.

Bending Moments in Slab for Live Load. — Where live load consists of concentrated wheel loads of trucks, rear wheels of two lines of trucks placed on the slab side by side produce the largest bending moments in the slab. To determine bending moments for live load, proceed as follows: Find the maximum positive bending moment considering the slab as simply supported. Reduce the positive bending moments in the same manner as explained in connection with the dead load. Also, use the same rule for negative bending moments as for dead load. Find the effective width of the slab (see p. 15), and compute bending moments per foot of width of slab. Take care of impact by multiplying the truck loads by the proper ratio. At each end of the bridge the effect of the live load upon the slab is larger than inside of the bridge because for concentrated loads placed near the edge of the slab the effective width is smaller. The consequent necessity of thickening the slab there may be avoided by strengthening it by means of an end cross beam.

Depth of Slab. — The depth of slab is found for the combined maximum positive bending moment for dead and live loads, including impact, using the ordinary slab formulas.

Reinforcement. — Main reinforcement of the slab consists of bars placed near the bottom at right angles to the girders. Its cross section is found for the bending moments used in computing the depth of slab. About one-half of the bottom bars are bent up and carried near the top of the slab into the girder, where they should be anchored. These serve as negative bending-moment reinforcement. Often, additional top bars are needed at each girder, and these are anchored in the girder and extended into the span for a distance not less than 0.15 of the clear span of the slab.

Cantilever slabs carrying sidewalks should be reinforced with top bars placed at right angles to the girders, and properly anchored in the girders or in the roadway slab.

In addition to the main reinforcement, longitudinal bars should be provided for the reasons, and in the amounts, suggested on p. 54.

Hangers for Slab Loads. — The loads from the slab are transferred to the girders partly by shear and partly by direct tension in the web of the girders. This tension may be taken as equal to one-half of the total slab reaction, and it should be provided for by means of steel hangers of sufficient strength, preferably in the form of stirrups spaced uniformly along the whole length of the girder. These should not be considered as a part of web reinforcement of the girder.

FLOOR OF THROUGH BRIDGE CONSISTING OF FLOOR BEAMS AND SLABS

For spacings of girders larger than for two-lane traffic, it is usually more economical to use, instead of a solid slab, a floor design consisting of floor beams and slabs spanning between the floor beams, as shown in Fig. 39 (*b*), p. 94. Floor beams should be spaced closely to permit the use of thin slabs, thus making the dead load on the girder as small as possible.

Design of Slab. — The design of slab supported by floor beams is fully treated on p. 72.

Design of Floor Beams. — For treatment of the design of floor beams, see general discussion on p. 73, and the discussion of the design of one-span floor beams given on p. 74.

Hangers for Floor Beams. — The loads from the floor beams are transferred to the girder in the same manner as explained on p. 96 in connection with the slab floor. The hangers may be concentrated at each floor beam, or they may be distributed over the whole girder.

In the example from practice shown in Fig. 46, p. 115, the function of hangers is performed by bent-up beam bars which extend into the compression zone of the girder, and by stirrups which are added to those required for diagonal tension reinforcement.

FLOOR OF THROUGH BRIDGE CONSISTING OF FLOOR BEAMS AND TWO-WAY SLABS

The cost of the structure may be appreciably reduced by using floor beams with wide spacings and a continuous longitudinal beam, placed midway between the main girders and supported by the floor beams. In this manner the slab is divided into square or rectangular panels which are supported on four sides and reinforced in two directions. For wide bridges it may be necessary to use two or three lines of longitudinal beams.

The advantage of this arrangement is as follows: The number of floor beams is reduced, and at the same time a thinner slab than would be required with the close spacing of floor beams in the one-way-

slab design may be used. The reduction in the slab thickness not only lowers the cost of the floor system, but also, by reducing the dead load carried by the girder, permits smaller dimensions for the main girders and their reinforcement.

The design of a floor system with slabs reinforced in two directions is fully explained on p. 74 and illustrated by a numerical example on p. 83. The relative economy of a floor system for a through bridge using floor beams with one-way slab, and one using two-way slab, may be determined by comparing the designs shown in Fig. 42, p. 103, and in Fig. 44, p. 110. Both floor systems are designed for the same loads using the same unit stresses. The computations for the one-way-slab design are given in the example on p. 99; but the computations for the two-way design are not given because they are similar to those in the example on p. 83.

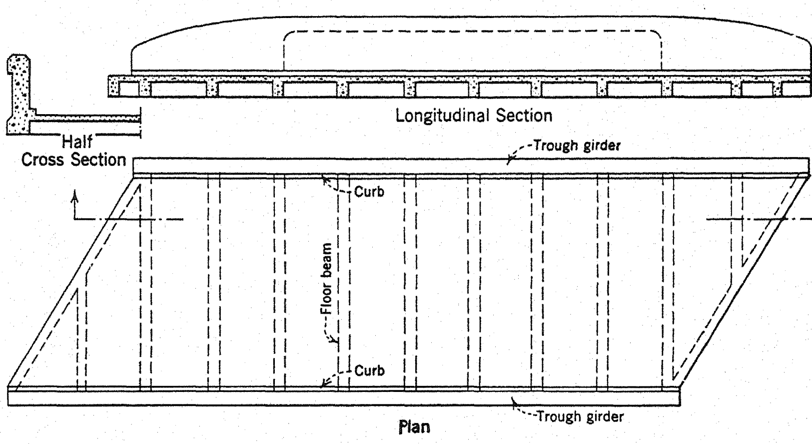


FIG. 40.—Skew Through Bridge. *(See p. 97.)*

FLOOR DESIGN IN SKEW THROUGH GIRDER BRIDGES

In a skew through girder bridge with a floor consisting of a solid slab, the main reinforcement is placed at right angles to the girders, and the span of the slab is measured at right angles to the girders. The triangular section of the slab at each end of the span is supported along one edge by the girder and along the other edge by the end cross beam spanning between the girders.

When the floor construction of a skew bridge consists of floor beams and slabs, the floor beams are placed at right angles to the girders, as shown in Fig. 40, p. 97. At each end of each span, the short floor beams are supported by the girder and by the end cross beam.

MAIN GIRDERS OF THROUGH BRIDGE

After the design of the floor system is completed, the dimensions of the main girder and their reinforcement are determined as follows.

Dead Loads. — When the floor consists of a solid slab, the dead load from the floor is distributed uniformly over the whole length of the girder.

When the floor consists of floor beams and slabs, the dead load from the floor is concentrated at the panel points. With close spacing of floor beams, the computations may be simplified without any appreciable error by considering all dead load as uniformly distributed along the girder. (See also discussion on p. 82.)

Uniformly Distributed Live Loads. — Live loads consisting of uniformly distributed loading may be treated in the same manner as suggested for dead load. However, external shears should be found as explained on p. 28 for moving loads.

When cantilever sidewalks are used, the reactions of the cantilever loads upon the girder are equal to the cantilever load increased by $\frac{M_c}{s}$, where M_c is the maximum cantilever bending moment, and s is the spacing of the girders.

Truck Loads. — When the roadway of the bridge is equal in width to two traffic lanes, each girder should be considered as carrying one line of trucks. For roadways wider than two but less than three traffic lanes, larger reactions upon the girder may be found by placing the two lines of trucks as close as possible to the girder under consideration.

For bridges accommodating three lanes of traffic or more, the roadway may be considered as loaded uniformly over its entire width with a load per foot of width equal to the load on one traffic lane divided by the width of the traffic lane. Proper reduction in live load may then be made as explained on p. 11.

The proportion of the truck loads carried by one girder having been determined, bending moments and external shears are computed as explained on pp. 59 and 82 in connection with deck girder bridges.

Dimensions of Girder. — As far as computations are concerned, the main girder of a through bridge is rectangular in section because the slab, being in the tensile zone, is not considered effective in resisting stresses.

The ratio of the depth of the main girder in the center to the span length is larger than in deck girder bridges, varying from $\frac{1}{6}$ to $\frac{1}{8}$. The design is most economical when compression stresses are resisted entirely by the concrete. However, where for any reason the depth of the girder and the width of its compression zone are restricted, it may be necessary to use compression reinforcement, preferably consisting of longitudinal

bars. Occasionally the compression zone is reinforced by longitudinal and spiral reinforcement.

Simplest Design of Girders. — In the simplest design of through girders, the width and depth of the girders are made constant, throughout. In such designs all surfaces of the girders are plane, which simplifies formwork. In determining dimensions the width is usually assumed, and the corresponding depth of the girder found for the maximum bending moment for combined live and dead loads.

Curved Tops of Girders. — If the depth of girder required by end shears is smaller than that required by bending moments in the center of the span, the top of the girder may be curved downward at the ends, as shown in Fig. 40, p. 97. Such designs are more graceful in appearance, and also they obstruct the view less. The saving due to the reduction in the amount of concrete in this case may be offset by the greater cost of formwork and of additional web reinforcement.

Paneling of Sides of Girders. — The sides of through girders are often paneled, as shown in Fig. 46, p. 115, to reduce the amount of concrete in the girder, and also to improve the appearance of the bridge. Paneling of the girder sides is possible because the external shears governing diagonal tension are much smaller in the central portions than at the ends, thus requiring smaller breadths of girder there.

Girder with Open Web. — In long-span girders, the cost of the structure may be reduced by using solid webs only at the ends of the girders, where the end shears are large. In the central portions of the girder, actual openings in the web may then be provided, as shown in Fig. 45, p. 111. This reduces the amount of concrete in the girder, and also its dead load. The design of girders with open webs is treated on p. 110.

NUMERICAL EXAMPLE OF DESIGN OF THROUGH BRIDGE

The use of formulas for the design of through girder bridges is illustrated by the following example. The floor system used in the example consists of closely spaced floor beams with a slab spanning between the floor beams.

For the sake of comparison, in Fig. 44, p. 110, a floor system is shown consisting of floor beams and two-way slabs designed for the same conditions as used in this numerical example.

Example. — Design a girder bridge with a span of 50 ft., measured between centers of supports, and a clear roadway of 20 ft.[1] between curbs. Two sidewalks

[1] The clear roadway of 20 ft. here used was selected arbitrarily. For safety in traffic greater widths than this should be used. See p. 5 for discussion of proper width of bridges.

are used, each 5 ft. wide in the clear. The available depth for the construction as determined by the headroom is limited to 3 ft. 4 in.

Live loads: two lines of 20-ton trucks arranged in trains as in the example on p. 60. Sidewalk live load, 100 lb. per sq. ft. Impact: 25 per cent for floor system; 20 per cent for girders.

Paving: 50 lb. per sq. ft.

Allowable unit stresses, pounds per square inch: $f_c = 800$; $f_s = 16\,000$; $n = 15$, $v = 120$ (with web reinforcement); $v = 40$ for concrete only; $u = 100$.

Solution. — Since the available depth is too small for a deck girder design, it is necessary to use through girders.[2] A floor design consisting of floor beams and slabs will be used. For cross section of bridge, see Fig. 41, p. 101.

Spacing of Floor Beams. — The overall length of the girder is 52 ft. Cross beams are used at each end of the bridge. The floor is then divided into seven panels, of which the interior panels are 7 ft. 6 in. and the exterior 6 ft. 9 in. long, both spacings measured from center to center of beams.

Design of Slab

Assume breadth of end cross beams $b' = 12$ in., and of the interior floor beams $b' = 1$ ft. 6 in. This gives clear span for interior panels $l_2 = 6$ ft.; and for exterior panels $l_1 = 5$ ft. 6 in. Clear spans will be used in design.

Dead load:		
	Slab	90
	Paving	50
	Total w_d =	140 lb. per sq. ft.

$$12 w_d l_1^2 = 50\,800 \text{ in-lb.} \quad 12 w_d l_2^2 = 60\,500 \text{ in-lb.}$$

Bending-moment coefficients will be taken from Table I, p. 53, for a five-span slab. The coefficients for a slab with free ends are multiplied by 2 and added to those for a continuous slab with fixed ends, and the sum is divided by 3.

	Coefficients		Bending moments
End span	$\frac{2 \times 0.078 + 0.042}{3} =$	0.066	$M_{1\max.} = 3\,400$ in-lb.
Interior spans	$-\frac{2 \times 0.105 + 0.083}{3} =$	-0.098	$M_2 = -5\,900$ in-lb.
	$\frac{2 \times 0.046 + 0.042}{3} =$	0.045	$M_{2\max.} = 2\,700$ in-lb.

For interior spans the positive bending moment coefficients for the third span are used because they are larger than those for the second span.

Live Loads. — For maximum bending moments, one rear wheel is placed in the center of the slab span. Multiplied by 1.25 to allow for impact, the wheel load is $1.25 \times 16\,000 = 20\,000$ lb.

Divide the concentrated load by the effective width determined as explained on p. 15 for continuous slab. Width of tire assumed to be $g = 20$ in.

[2] The available depth may be sufficient for a rigid frame design, which would give a much more economical solution of the problem.

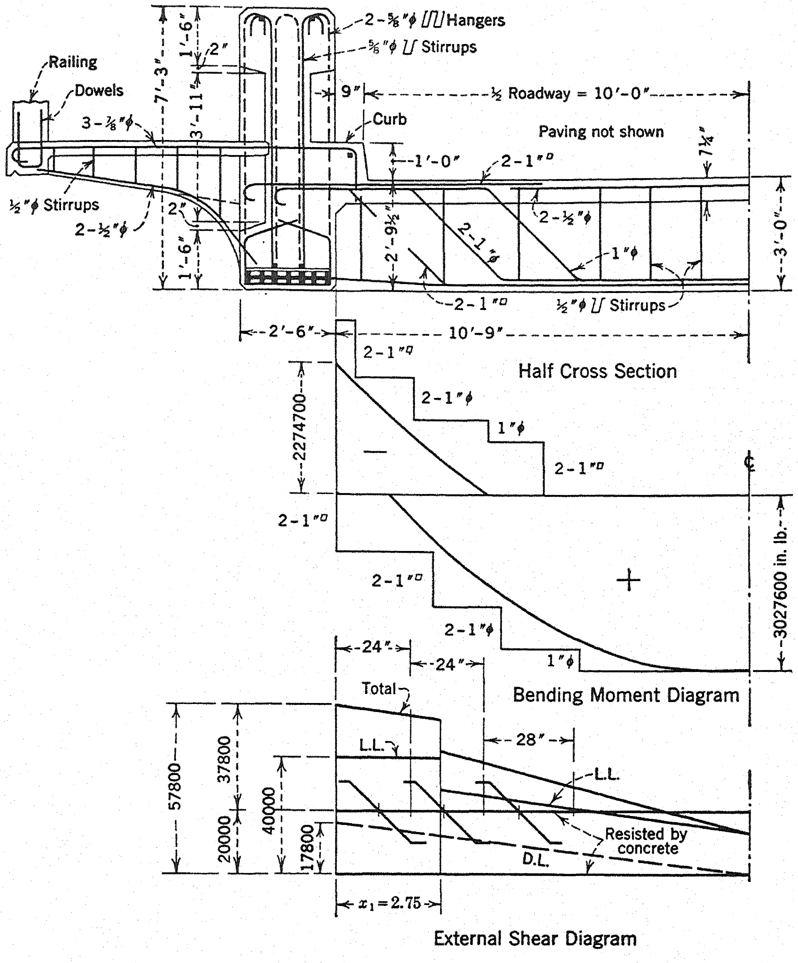


FIG. 41.—Cross Section of Through Bridge. Design of Floor Beam. (*See p.* 100.)

Effective width, end panels $l_e \quad = 0.72 \times 0.8 \times 5.5 + 1.67 = 4.84$

interior panels $l_e = 0.72 \times 0.7 \times 6.0 + 1.67 = 4.69$ ft.

Concentrated loads per foot of width of slab:

$$\text{End panel} \qquad P = \frac{20\,000}{4.84} = 4\,150 \text{ lb.}; \;\; 12Pl_1 = 274\,000 \text{ in-lb.}$$

$$\text{Interior panel } P = \frac{20\,000}{4.69} = 4\,280 \text{ lb.}; \;\; 12Pl_2 = 308\,000 \text{ in-lb.}$$

Bending-moment coefficients are found as for dead load.

	Coefficients		Bending moments
End span	$\dfrac{2 \times 0.21 + 0.148}{3} =$	0.189	$M_{1\max.} = 51\,800$ in-lb.
Interior span	$-\dfrac{2 \times 0.18 + 0.148}{3} =$	-0.169	$M_2 = 52\,000$ in-lb.
	$\dfrac{2 \times 0.190 + 0.183}{3} =$	0.188	$M_{2\max.} = 57\,900$ in-lb.

Summary of bending moments for dead load and live load, including:

$$\text{Negative } M_2 = -(5\,900 + 52\,000) = -57\,900 \text{ in-lb.}$$

$$\text{Positive } M_{1\max.} = 3\,400 + 51\,800 = 55\,200 \text{ in-lb.}$$

$$M_{2\max.} = 2\,700 + 57\,900 = 60\,600 \text{ in-lb.}$$

Depth of Slab and Reinforcement. — Depth of slab determined for the largest positive bending moment. Using constants $R = 146.9$ and $j = 0.857$:

$$d = \sqrt{\frac{60\,600}{12 \times 146.9}} = 5.85 \text{ in.; use } h = 7\tfrac{1}{4} \text{ in.}$$

$$d = 7.25 - (1.0 + 0.3) = 5.95 \text{ in. and } A_2 = \frac{60\,600}{0.857 \times 5.95 \times 16\,000} = 0.74 \text{ sq. in.}$$

Use $\frac{5}{8}$-in. round bars 5 in. on centers, $A_s = \dfrac{0.307}{5} \times 12 = 0.74$ sq. in.

The reinforcement is arranged as shown in the plan, Fig. 42, p. 103.

Design of Floor Beam Cantilever

Bending Moments and Shears in Cantilever. — Bending moments and shears in cantilever are found in the usual manner. The work is not shown here.

	End shear	Bending moment
Dead load	5 125	−234 250
Live load, including pressure on railing	4 370	−153 000
Total	9 495 lb.	−387 250 in-lb.

Dimensions of Cantilever. — Assume breadth of stem $b' = 10$ in. $R = 146.9$, $p = 0.0107$. The cantilever beam is considered in design as a rectangular beam.

Depth: $$d = \sqrt{\frac{387\,250}{10 \times 146.9}} = 16.2. \quad \text{Use } h = 19 \text{ in.}$$

Reinforcement: $A_s = 0.0107 \times 10 \times 16.2 = 1.74$ sq. in.

Use three $\frac{7}{8}$-in. round bars. $A_s = 3 \times 0.60 = 1.8$ sq. in.

Diagonal tension: $V = 9\,495$ lb.

$$v = \frac{9\,495}{10 \times 0.857 \times 16.2} = 68 \text{ lb. per sq. in.}$$

Web reinforcement consisting of stirrups is provided.

MAIN SPAN OF FLOOR BEAM

The theoretical span is assumed to be equal to the clear span, $l = 20.0 + 2 \times 0.75 = 21.5$ ft.

Dead Load. — Slab $\dfrac{7.25}{12} \times 150 = 91$

Paving $\underline{50}$

Total $141 \times 7.5 = 1\,058$

Beam, below slab $\underline{600}$

Total w_d $= 1\,658$ lb. per lin. ft.

Static bending moment:

$$M_s = \tfrac{1}{8} \times 1\,658 \times 21.5^2 \times 12 = 1\,150\,000 \text{ in-lb.}$$

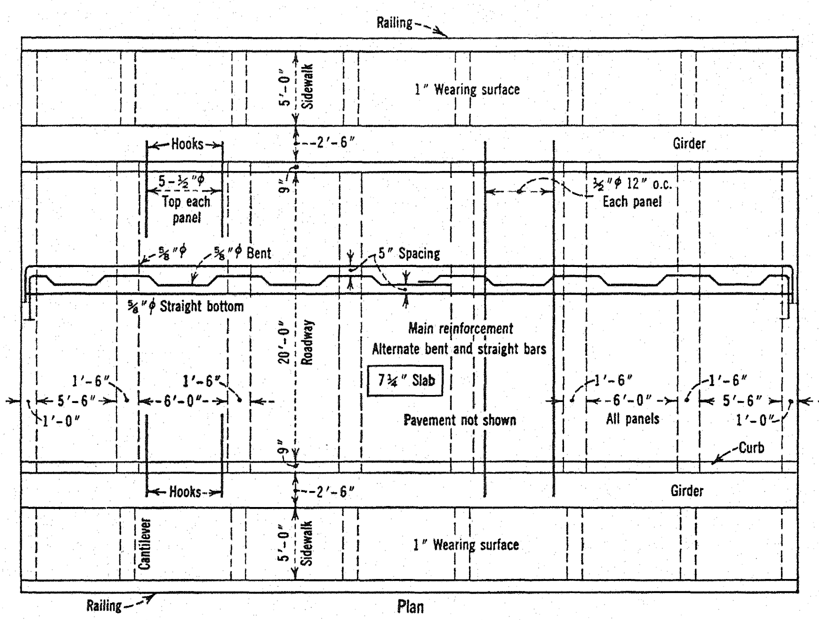


FIG. 42.—Plan of Through Bridge. (*See p.* 102.)

Since the floor beam is partly restrained by the girders, use for maximum positive bending moment eight-tenths of the static bending moment. In addition, reduce the positive bending moment by one-half of the negative cantilever bending moment for dead load. Therefore:

Positive bending moment for dead load:

$$M_D = 0.8 \times 1\,150\,000 - 0.5 \times 234\,250 = 803\,000 \text{ ft-lb.}$$

Negative bending moment for dead load:

$$M = -(0.5 \times 1\,150\,000 + 0.8 \times 234\,000) = -762\,200 \text{ ft-lb.}$$

Live Load and Impact. — Maximum bending moments are produced in a floor beam when the rear axles of the trucks are placed directly above it. Transversely, two trucks are used arranged as shown in Fig. 9, p. 26. The load on a rear axle, plus impact, is $1.25 \times 32\,000 = 40\,000$ lb. The maximum static bending moment for live load is

$$M_{s(L+I)} = 40\,000 \times 105 - 20\,000 \times 72 = 2\,760\,000 \text{ in-lb.}$$

Taking advantage of the restraint, similarly as for dead load, largest positive bending moment is

$$M_{L+I} = 0.8 \times 2\,760\,000 = 2\,208\,000 \text{ in-lb.}$$

Cantilever loads for live load have no effect upon positive bending moments.

Largest negative bending moment for live load, assuming eight-tenths of the cantilever bending moment as transmitted:

$$M_1 = -(0.5 \times 2\,760\,000 + 0.8 \times 153\,000) = -1\,502\,400 \text{ in-lb.}$$

Summary of Bending Moments:

Positive bending moments

$$M_{\max.} = M_D + M_{L+I} = 803\ 000 + 2\,208\,000 = 3\,011\,000 \text{ in-lb.}$$

Negative bending moments

$$M_1 = -762\,200 - 1\,502\,400 = -2\,264\,600 \text{ in-lb.}$$

Reinforcement for Floor Beams. — The depth of the construction, as far as the headroom is concerned, is limited to 3 ft. 4 in. Using 4 in. for paving, the useful depth of floor beams is $h = 36$ in. in the center. Assuming two layers of steel and a concrete cover of 2 in., the effective depth of floor beam is $d = 36.0 - 3.5 = 32.5$ in. Assuming $j = 0.9$ for T-beam, the area of positive reinforcement is

$$A_s = \frac{3\,011\,000}{0.9 \times 32.5 \times 16\,000} = 6.44 \text{ sq. in.}$$

Use four 1-in. square bars plus three 1-in. round bars. $A_s = 4 \times 1.0 + 3 \times 0.785 = 6.4$ sq. in.

At the support, the depth of the floor beam is 33.5 in., giving an effective depth $d = 30.0$ in. Using $j = 0.88$, the amount of negative bending moment reinforcement is

$$A_s = \frac{2\,264\,600}{0.88 \times 30.0 \times 16\,000} = 5.36 \text{ sq. in.}$$

Add two 1-in. square short bars at each end, and use bent bars as shown in Fig. 41.

Shears for Dead Load. — External end shear for dead load is

$$V_d = \tfrac{1}{2} \times 1\,658 \times 21.5 = 17\,800 \text{ lb.}$$

External Shears for Live Load plus Impact. — External shears for floor beams are computed using formulas (29) to (32), p. 27. The value of W in the formulas is equal to the load on the rear axle multiplied by 1.25, or $W = 40\,000$ lb. Also, substitute $b = 6.0$ ft.; $e = 4.0$ ft.; $f = 2.75$ ft. This assumes that the outside edge of the truck clearance coincides with the curb line. For these values, external shears are

End shear: $V_1 = 40\,000$ lb.

$V_{x1} = 40\,000$ lb. to left of x_1. $(x_1 = 2.75)$.

$V_{x1} = 29\,400$ lb. to right of x_1

Center of span: $V_{\frac{l}{2}} = 14\,400$ lb.

External Shears for Cantilever Loading. —

$$V = \frac{153\,000}{21.5 \times 12} = 590 \text{ lb. (may be disregarded because relatively small)}$$

This is constant throughout the span.

Shear Diagram. — The values just computed are sufficient for a shear diagram. This is shown in Fig. 41, p. 101.

Maximum Combined End Shears. — Add the maximum value for dead loads to that for live loads:

$$V = 17\,800 + 40\,000 + 590 = 58\,390 \text{ lb.}$$

Dimensions of Floor Beam as Determined by Shear. — The depth of the floor beam is fixed by the limitation of headroom. The breadth is determined by the shear requirement. Since at support $d = 31.5$ in., and $j = 0.88$,

$$b' = \frac{V}{vjd} = \frac{58\,390}{120 \times 0.88 \times 31.5} = 17.6 \text{ in.}$$

Use $b' = 1$ ft. 6 in.; $h = 2$ ft. 10 in. at ends.

Points of Bending of Reinforcement. — The bending-moment diagram shown in Fig. 41, p. 101, is used to determine the points of bending of the reinforcement. These are selected so as to make the bent bars effective in resisting diagonal tension as well as negative bending moments.

Web Reinforcement of Main Span of Floor Beam. — The shear diagram is used to determine spacing of web reinforcement, since shear is considered as a measure of diagonal tension.

The external shear resisted by concrete alone, based on $v = 40$ lb. per sq. in., is $40 \times 18 \times 0.88 \times 31.5 = 20\,000$ lb. per lin. in. This shear is marked on the diagram.

Locate the bent bars on the diagram, showing the points where they intersect the neutral axis. Determine the tributary areas of shear for each set of bent bars. Compare the shear to be resisted with the resistance of bent bars.

For the first set of bars, the average external shear scaled from the diagram is $V_x = 36\,000$ lb. The tributary distance is $s = 24$ in. The shear to be resisted (considering shear as a measure of diagonal tension) is

$$s_x \frac{V_x}{jd} = 24 \times \frac{36\,000}{0.88 \times 31.5} = 31\,200 \text{ lb.}$$

The value of two 1-in. bars bent at about 45° in resisting diagonal tension is $1.4 \times 2 \times 16\,000 = 44\,800$ lb. This value is larger than the shear to be resisted.

The other sets of bent bars do not need to be investigated, because it is evident from inspection of the diagram that the conditions there are more favorable than for the first set of bars. Stirrups with wide spacings are added, although they are not required by computations.

Design of Main Girder

Dead Load. — Dead load carried by the girder consists of the floor-beam reactions concentrated at the panel points, and of uniformly distributed dead load of the girder.

Floor-beam reactions, main span	17 800
cantilever	5 125
Total	22 925 lb.

For the sake of simplicity, this reaction is used at all panel points, instead of taking somewhat smaller values at the first and the last panel point.

Maximum bending moment in girder for concentrated dead load

$$M_{d1} = 22\,925 \times (3 \times 21.25 - 3 \times 7.5) \times 12 = 11\,300\,000 \text{ in-lb.}$$

Uniformly distributed dead load of girder is assumed at 2 300 lb. per sq. in.

Maximum bending moment, uniformly distributed dead load

$$M_{d2} = \tfrac{1}{8} \times 2\,300 \times 50^2 \times 12 = 8\,620\,000 \text{ in-lb.}$$

Sum of maximum bending moments for dead load

$$M_d = M_{d1} + M_{d2} = 19\,920\,000 \text{ in-lb.}$$

Live Load. — Each girder is assumed to carry one line of trucks. Adding 20 per cent for impact, the value of the truck load per girder is

$$W = 1.2 \times 40\,000 = 48\,000 \text{ lb.}$$

Maximum bending moment for live load on roadway is found using the table on p. 20, item 3

$$M_{L+I} = \left\{\left(0.548 - \frac{0.455}{50}\right)^2 - \frac{2.8}{50}\right\} 48\,000 \times 50 \times 12 = 6\,740\,000 \text{ in-lb.}$$

Live loads on sidewalks are considered as concentrated at the panel points. For one-sided loading of sidewalk, panel load is $3\,750 + \dfrac{112\,000}{12 \times 21.5} = 4\,190$ lb. Maximum bending moment for live load on sidewalks

$$M = 4\,190 \times (3 \times 21.25 - 3 \times 7.5) \times 12 = 2\,070\,000 \text{ in-lb.}$$

Sum of maximum bending moments for live load

$$M_L = 6\,740\,000 + 2\,070\,000 = 8\,810\,000 \text{ in-lb.}$$

Sum of bending moments for live and dead loads. (See diagram, Fig. 43, p. 108.)

$$M_{\text{max.}} = 19\,920\,000 + 8\,810\,000 = 28\,730\,000 \text{ in-lb.}$$

Dimensions of Girder. — The dimensions of the girder are governed by the maximum bending moment. Using constants $R = 146.9$ and $j = 0.857$,

$$d = \sqrt{\frac{28\,730\,000}{30 \times 146.9}} = 80.6 \text{ in.; for } b = 30 \text{ in.}$$

The combined bending-moment diagram in Fig. 43, p. 108 is used to determine the points of bending of reinforcement. (See also p. 181.)

Add 2.4 in. to the depth of the girder to compensate for the paneling of the web. Accepted dimensions are: $b = 30$ in.; $h = 87$ in.; $d = 87.0 - 4.0 = 83.0$ in.

$$A_s = \frac{28\,730\,000}{0.857 \times 83 \times 16\,000} = 25.2 \text{ sq. in.}$$

Use sixteen $1\frac{1}{4}$-in. square bars, for which $A_s = 25.0$ sq. in. (See Fig. 43, p. 108.)

External Shears. —

Dead load, floor beam reactions	$V_a = 3 \times 22\,925 = 68\,775$
weight of girder	$V_b = \frac{1}{2} \times 2\,300 \times 50 = 57\,500$
Total dead load end shear	$V = 126\,275$ lb.

Live load. For truck loads the shears are computed from the formula in the table on p. 24, item 4. Substitute $l = 50.0$ ft.; and $W = 48\,000$ lb., including impact.

End shear

$$V_1 = \left(2 - \frac{38.6}{50.0}\right) 48\,000 = 59\,000 \text{ lb.}$$

Shears at intermediate points

$$V_{x1} = \frac{55.4}{50} \times 48\,000 = 53\,300 \text{ lb.}; \; x_1 = 50 - 47 = 3.0 \text{ ft.}$$

$$V_{x2} = 28\,800 \text{ lb.}; \; x_2 = 50 - 33 = 17.0 \text{ ft.}$$

$$V_{x3} = 10\,750 \text{ lb.}; \; x_3 = 50 - 14 = 36.0 \text{ ft.}$$

Shears for Sidewalk Live Load. This loading is considered as a moving load. End shear

$$V_1 = 3 \times 4\,190 = 12\,570 \text{ lb.}$$

Second panel $V_2 = 10\,800$ lb.

Third panel $V_3 = 7\,500$ lb.

Fourth panel $V_4 = 4\,200$ lb.

Total end shear

$$V_1 = 126\,275 + 59\,000 + 12\,570 = 197\,845 \text{ lb.}$$

Shear Diagram. — To get shears at intermediate points, the shear diagram is drawn as shown in Fig. 43, p. 108. Separate lines are drawn for the dead load of the girder and the dead-load reactions of the floor beams. Separate lines are drawn for live load on roadway and live load on sidewalk. The values are added to give total shear in girder.

Thickness of Web as Determined by Shear. — The required thickness of web in different panels of the girder is found from the formula $b = \dfrac{V_x}{vjd}$, in which V_x is scaled from the shear diagram. Values are given in the table on next page.

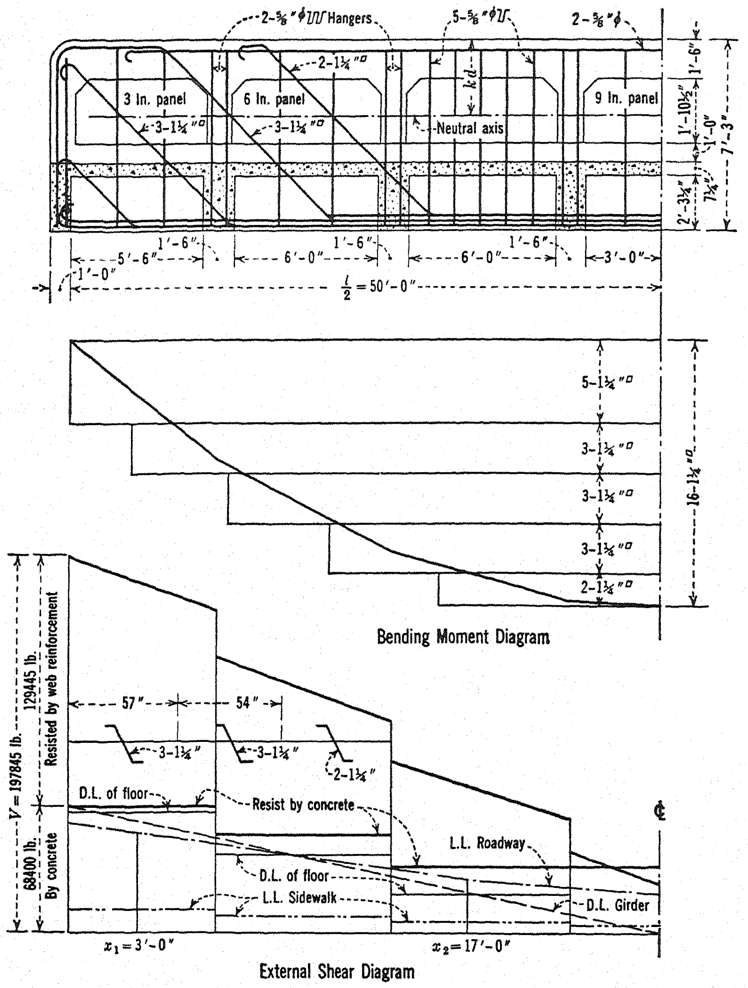


Fig. 43.—Girder Details for Through Bridge. (See p. 106.)

Thickness of Web

Panel Point	External Shear V_x	Computed	Accepted	Paneling
1	197 845 lb.	23.1	24.0	3.0
2	140 000 lb.	16.3	18.0	6.0
3	88 000 lb.	10.4	12.0	9.0
4	42 000 lb.	4.9	8.0	11.0

Shear Resisted by Concrete. — External shear resisted by concrete is obtained from the formula $V = vbjd$, in which $v = 40$ lb. per sq. in.

First panel, $b = 24$ in.; $V = 68\,400$ lb.

Second panel, $b = 18.0$ in.; $V = 51\,300$ lb.

Third panel, $b = 12.0$ in.; $V = 34\,200$ lb.

These values are marked off in the shear diagram. Shear below the line thus obtained is resisted by concrete; that above the line, by web reinforcement.

Web Reinforcement. — Longitudinal bars are bent up so that the bent parts can be utilized as diagonal tension reinforcement. Mark on the external shear diagrams the points of intersection of the bent bars with the neutral axis. Measure the distance s tributary to each set of bent bars. Scale the average external shear for each set of bars. Compute the average shear per foot of length of girder from the formula $vb = \dfrac{V_x}{jd}$. Find the total shear vbs to be resisted. Compare the shear to be resisted with the resistance of the bars.

For the first set of bars, $vb = 1\,700.0$ lb.; $vbs = 97\,000$ lb. The value of the bent bars in the first set for resisting diagonal tension $= 3 \times 1.56 \times 16\,000 \times 1.4 = 105\,000$ lb. The bent bars are sufficient to take care of their share of diagonal tension.

In the same manner, all other sets of bent bars are investigated.

Stirrups. — In the third and fourth panels of the girder, it is necessary to use stirrups. In the third panel, the total external shear to be resisted by stirrups is: at second panel point, $V_x = 54\,000$ lb.; at third panel point, $V_x = 24\,000$ lb. Average shear $V_{\text{av.}} = \dfrac{54\,000 + 24\,000}{2} = 39\,000$ lb., and $vb = \dfrac{39\,000}{0.857 \times 83} = 547$ lb. Total shear in the panel to be resisted by stirrups $= 547 \times 7.5 \times 12 = 49\,200$ lb.

Required area of stirrups $A_s = \dfrac{49\,200}{16\,000} = 3.1$ sq. in.

Use, in third panel, five $\frac{5}{8}$-in. round U stirrups. $A_s = 5 \times 2 \times 0.307 = 3.1$ sq. in.

In the center panel, wide spacing of stirrups is used, as shown in Fig. 43, p. **108**.

In this example, the external shear diagram is used directly in determining web reinforcement. If desired, a separate diagram may be drawn for unit shears, or for shears per inch of length of girder, $v_x b$.

Hangers. — One-half of the floor-beam reactions is assumed as transferred to the girder by shear, and the other half is taken care of by tension in hangers.

Reaction of floor beam	58 390
Reaction of cantilever	9 495
Total	67 885 lb.

Tension to be resisted by hangers at each floor beam $= \dfrac{67\,885}{2} = 33\,840$ lb.

Area of hangers $\dfrac{33\,840}{16\,000} = 2.1$ sq. in.

Use at each end of floor beam in the girder two $\frac{5}{8}$-in. round four-legged stirrups. Their area is $A_s = 2 \times 4 \times 0.307 = 2.5$ sq. in.

Example of Floor Design with Two-Way Slab

In Fig. 44, p. 110, is shown the floor plan, with slabs reinforced in two directions, for a bridge of the same dimensions and carrying the same loads as in the example on p. 99 in which slabs reinforced in one direction are used. Computations are not given here because they are the same as those in the example on p. 83.

Comparing the relative economy of the design using two-way slab and the design with one-way slab, it is evident that in the two-way-slab arrangement two floor beams

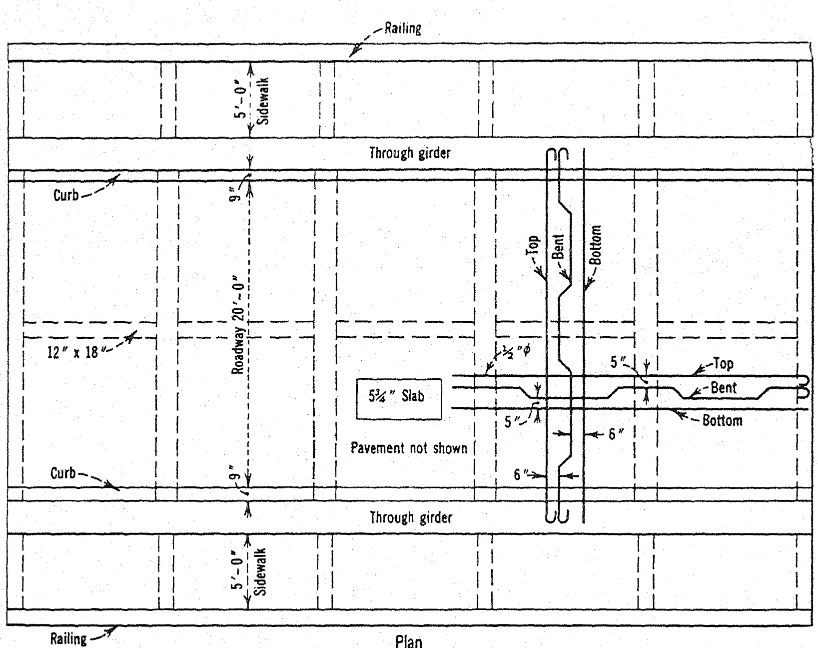


Fig. 44.—Example of Two-Way Slabs in Through Bridge. *(See p. 110.)*

of the aggregate length of 43 ft., are omitted, while a longitudinal beam of the aggregate length of 44 ft. is added. Since the quantities in the longitudinal beam per foot are less than one-third of the quantities in the floor beam, there is a direct saving of the cost of more than 29 ft. of the floor beam, including formwork.

The slab in the two-way design shows a direct saving of $1\frac{1}{2}$ in. of slab thickness and also more than $\frac{3}{4}$ lb. of steel per square foot of slab.

The dimensions of the floor beams are practically the same in both cases.

The dimensions of the main girders may be somewhat reduced because the dead load is reduced by 180 lb. per lin. ft. of girder.

TRUSSES WITHOUT DIAGONALS. APPROXIMATE METHOD

The approximate method of design of open web trusses here given may be used safely for girders with open web, and for trusses without diagonals of moderate spans. For long-span trusses, such as the

Vierendeel trusses, the use of more exact methods, not given in this volume, is recommended.

Component Parts of Trusses without Diagonals. — As shown in Fig. 45, p. 111, a truss without diagonals and a girder with open web consists of the following members: top chords, bottom chords, and posts.

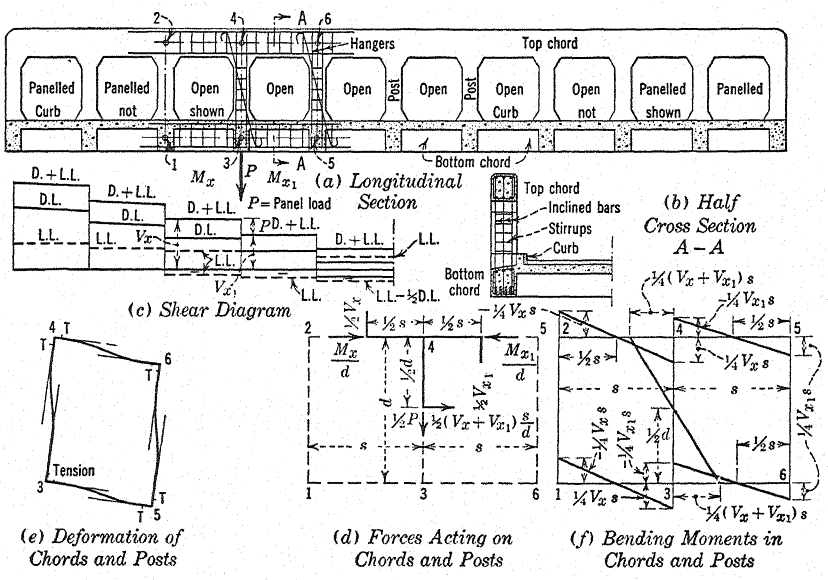


FIG. 45.—Through Girder with Open Web. (See p. 111.)

Bending Moments and Shears in Trusses and Open-Web Girders. — A truss and an open-web girder are subjected to bending moments produced by the dead and live loads, and computed in the same manner as for a girder with a solid web. These bending moments are here called "principal bending moments," to distinguish them from the "secondary bending moments" produced in the truss members by unbalanced external shears.

External shears acting upon a truss are computed in the same manner as for a girder with a solid web.

Notation:

d = moment arm for principal bending moment.
s = panel length, center to center of posts.
$M_{\text{max.}}$ = principal maximum bending moment in truss.
M_x = principal bending moment at any point x.
V_x = average external shear in panel to the left of post under consideration.

V_{x1} = average external shear in panel to the right of post.
V_P = external shear to be resisted by post.
M_L, M_R = maximum secondary bending moments in lower and upper chord.
M_P = maximum secondary bending moment in post.
P = panel load.

Direct Stresses in Chords. — In a truss, principal bending moments produce direct stresses in chords, namely compression in the top chords and tension in the bottom chords. They may be found from the following formulas:

Maximum Total Tension or Compression in Chords:

$$F = \pm \frac{M_{\text{max.}}}{d} \tag{1}$$

Total Tension or Compression in Chords at any Point x:

$$F_x = \pm \frac{M_x}{d} \tag{2}$$

External shears produce vertical shearing stresses in the chords. Each chord is assumed to resist one-half of the external vertical shear acting in the panel under consideration.

Direct Stresses in Posts. — In through bridges, the direct stress to which a vertical post is subjected is tension, and it is equal to one-half of the panel load. The other half of the panel load is assumed as transferred by shear to the bottom chord.

The posts are subjected to horizontal external shears of a magnitude given by the following formula.

External Horizontal Shear in Post:

$$V_P = \frac{1}{2}(V_x + V_{x1})\frac{s}{d} \tag{3}$$

This is the same type of formula as used in computing shearing stresses resisted by the web of a solid web girder.

Effect upon Truss Members of Unbalanced External Shears. — Since the truss has no diagonals, external shears in each panel are not resisted directly by a diagonal member, but must be resisted by all panel members. These shears produce distortion of the panel; deflection of the panel members; and consequent bending moments in the truss members, which here are called "secondary bending moments." The distortion of a panel due to external shear and the deflection of members

are shown in Fig. 45 (*e*), p. 111. Letters T indicate the points where cracks are most likely to occur.

The magnitude of the secondary bending moments produced by external shears may be found from the following formulas.

Maximum Bending Moment in Post at Top:

$$M_P = -\frac{1}{2}(V_x + V_{x1})\frac{s}{d} \times \frac{1}{2}d = -\frac{1}{4}(V_x + V_{x1})s \qquad (4)$$

Maximum Bending Moment in Chord to Left of Post:

$$M_L = \tfrac{1}{2}V_x \times \tfrac{1}{2}s = \tfrac{1}{4}V_x s \qquad (5)$$

Maximum Bending Moment in Chord to Right of Post:

$$M_R = -\tfrac{1}{4}V_{x1}s \qquad (6)$$

These formulas are based on the assumption that the points of zero bending moments in each member of the truss are located in the center of that member. This is not always so; and in this respect the approximate method differs from exact methods, in which the positions of the points of zero bending moments are obtained from computations.

In Fig. 45 (*d*) is shown by solid lines a part of a truss separated from the rest of the structure by sections through the assumed points of zero bending moments. To keep this portion of the truss in equilibrium, it is necessary to replace at the points of contraflexure the forces acting there before the separation from the truss. In the top chords, these forces are the direct compression $\frac{M_x}{d}$ and $\frac{M_{x1}}{d}$ acting centrally, and vertical shears $\frac{1}{2}V_x$ and $\frac{1}{2}V_{x1}$. In the vertical post, act the direct tension equal to $\frac{1}{2}P$ and the horizontal shear $\frac{1}{2}(V_x + V_{x1})\frac{s}{d}$.

The respective shears act at right angles to the truss members and produce in them bending moments varying, according to a straight line, from zero in the center of member to a maximum at its end. For positive external shears in a girder, bending moments in the chords turn clockwise, and in the post counterclockwise. The sum of bending moments at a joint turning in one direction must be equal to the bending moments turning in the opposite direction, because the joint is in equilibrium. The effect of secondary bending moments upon the posts is much more severe than upon the adjoining chords, because the maximum bending moment in the post is equal to the sum of the bending moments in the two chords on both sides of it.

In Fig. 45 (*f*), diagrams are drawn for secondary bending moments.

It is evident that one-half of each member is subjected to positive bending moments, and the other half to negative bending moments. The bending moments in the figure are for positive external shears. For negative external shears the signs of the bending moments are reversed.

Design of Top Chords. — Top chords are subjected to direct compression due to principal bending moments and to secondary bending moments; they should be designed for direct stress and bending. In effect, they should be considered as eccentrically loaded columns and reinforced with at least 0.5 per cent of longitudinal reinforcement, even when no tensile stresses are developed for the combined stresses. Bars should be placed near the top and the bottom surfaces of the chord.

To resist shearing stresses, stirrups or hoops should be used, spaced uniformly throughout the chord.

Design of Bottom Chord. — Bottom chords are subjected to direct tension and to secondary bending moments, and also to external vertical shears.

Bottom chords may form an integral part of the floor system, in which case their depth is governed by the depth of the floor beams. Often bottom chords are not connected with the floor system; and then their depth may be selected according to the stress requirements. The floor beams may then run directly into the posts.

The main element of a bottom chord is the tension reinforcement required to resist direct tension. This reinforcement may be distributed uniformly throughout the whole cross section of the chord; or it may be concentrated near its bottom. The concrete cross section of the chord may be governed either by external shears or by secondary bending moments. It is necessary to supply additional reinforcement to resist the secondary bending moments.

Stirrups or hoops should be provided to resist vertical shear.

Posts. — In through bridges, vertical posts are subjected to direct tension which should be resisted in full by steel hangers properly anchored at the top to the top reinforcement of the top chord, and at the bottom to the bottom reinforcement of the floor beam. The smallest concrete dimension of the post is governed by the horizontal external shears. At points of maximum secondary bending moments, i.e., the top and bottom of the post, a larger cross section may be required; and for this purpose fillets may be used.

In a truss, the external shears to which the posts are subjected vary from a minimum in the center panel of the truss to a maximum at the end. Therefore concrete dimensions of the posts may vary with the stresses. Often, however, for the sake of appearance the same concrete

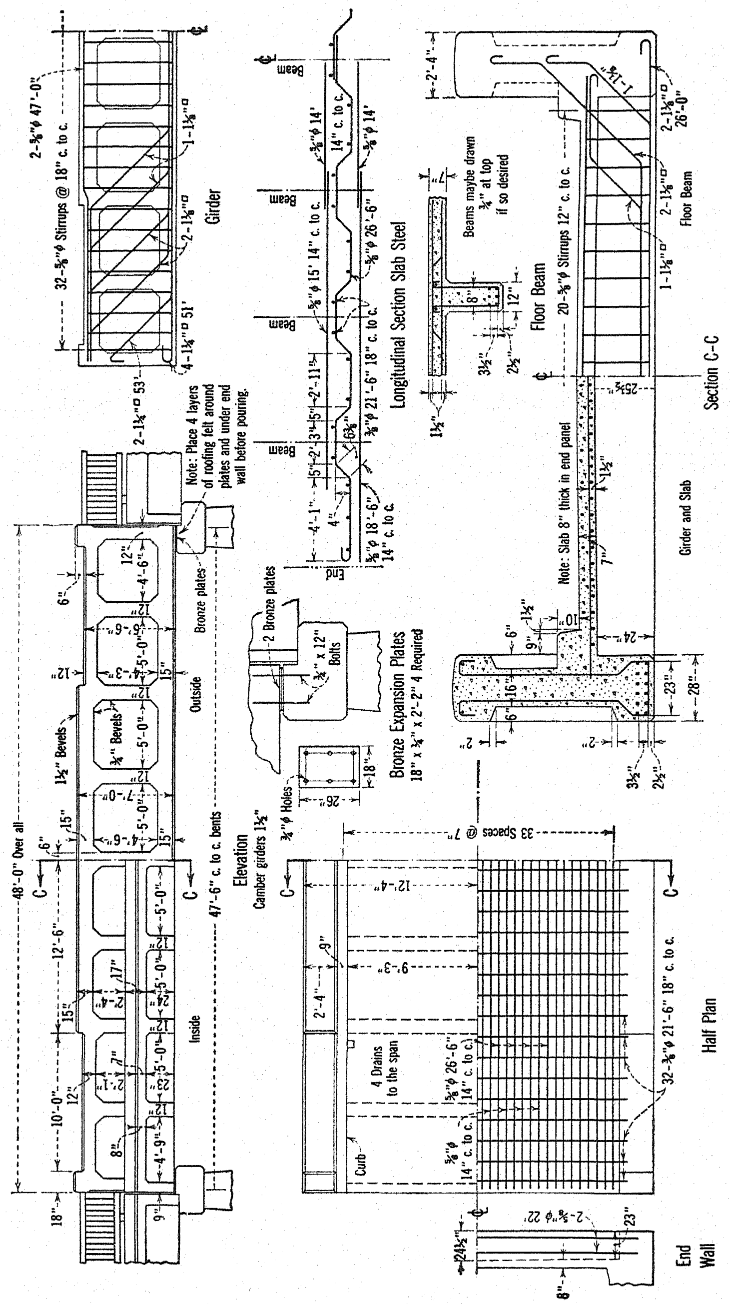


FIG. 46.—Through Girder Span. Savannah Delta Trestle. (*See p. 117.*)

cross section is used for all posts, but the amount of reinforcement may be varied.

To resist secondary bending moments in the posts, inclined bars may be used. In the left half of the truss near the ends, where only positive shears are possible, secondary bending moments in the posts are always positive in their lower half and negative in their upper half. One set of

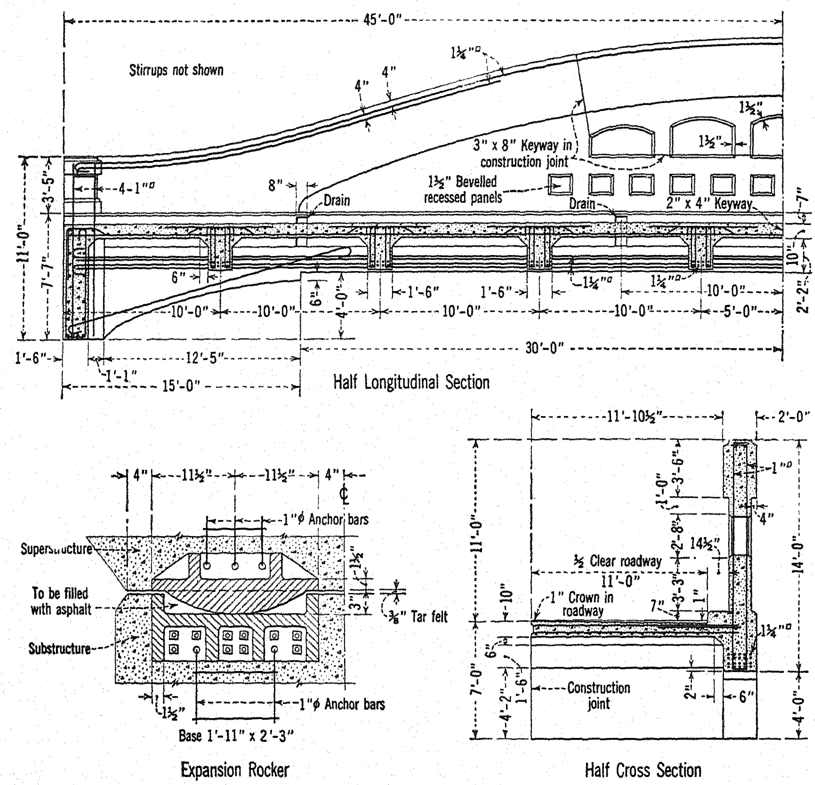


FIG. 47.—Through Girder Bridge. Kalamazoo, Michigan. (*See p.* 117.)

inclined bars arranged as shown in Fig. 45, p. 111, is sufficient to resist bending moments. Where reversal of external shears is possible, i.e., where negative shears for live load are more than one-half[4] of the positive shear for dead load, two sets of bars are necessary to resist bending moments.

[4] To get a factor of safety of 2, only one-half of the external shear for dead load is used to balance the negative shear for live load.

EXAMPLES OF THROUGH GIRDERS FROM PRACTICE

Two examples from practice of through bridges will be described and illustrated.

Through Girder Span, Savannah Delta Trestle. — In the through span used in the Savannah Delta Trestle and shown in Fig. 46, p. 115, the girders are 48 ft. long, overall, and the theoretical span is 46 ft. With a depth of girder of 7 ft., the ratio of depth to span is 1 : 6.5. The structure was designed for two 15-ton trucks, abreast. The floor beams are 2 ft. $8\frac{1}{2}$ in. deep and are spaced 6 ft. on centers. The thickness of slab is 7 in. for inside and 8 in. for outside panels.

The girders are supported by piers resting on piles. Expansion bearings consist of two $\frac{1}{2}$-in.-thick bronze plates. The bearing area outside of the plates is covered by four plies of roofing felt.

Bridge across Kalamazoo River at Charleston, Michigan. — In Fig. 47 is shown a 90-ft. through span which forms the center of the bridge with three openings. Each of the end spans, also of through design, is 60 ft. long. The depth of the center span is 14 ft. in the center, giving a ratio of depth to span of 1 : 6.4.

The top of the girder is curved; the central portion of the girder is provided with five openings; also paneling is used in the web. At the ends of the span the girder extends below the slab.

Of interest is the steel rocker used at one end of each 90-ft. girder.

CHAPTER VII

CANTILEVER BRIDGES

Cantilever bridges described in this chapter are structures in which the longitudinal girders are one span long, and some or all of them are provided with cantilevers either at one or at both ends. In some designs, the ends of the cantilevers are free; in others they carry short simply supported spans. Since all girders simply rest on the supporting piers, they are statically determinate; and all bending moments and shears can be determined by the basic rules of statics. Such structures are not affected to any appreciable degree by unequal settlement of foundations.

Cantilever designs of girders are usually cheaper than simply supported girders, but they are more expensive than continuous girders and rigid frames. They are useful where it is desired to reduce the depth of the girders without resorting to statically indeterminate designs, such as continuous girders, frames, and arches. Cantilever girders may be used advantageously in locations where unyielding foundations, suitable for statically indeterminate structures, are not easily obtainable. They have been used for spans up to 220 ft. and with ratios of depth in the center to span length as small as 1 : 35.

For purposes of discussion, cantilever bridges are divided into two groups:

1. Bridges with free cantilevers.
2. Bridges in which cantilevers support at their ends short simple spans.

For each of these two groups, design formulas are here given, arrangements of spans are discussed, and the subject is illustrated by examples from practice.

1. BRIDGES WITH FREE CANTILEVERS

The following arrangements of girder spans with free cantilevers will be discussed.

(*a*) One-span girders with two cantilevers, forming a large center opening and two small side openings.

(*b*) Bridges with counterweighed cantilevers.

(*c*) Combinations of any number of one-span girders, each of which is provided with one or two cantilevers.

First, formulas for bending moments and shears are given for one-span girders with cantilevers. These are followed by the description of the various uses of the girders with free cantilevers. Finally, examples from practice are given.

Bending Moments and Shears in One-Span Girders with Two Cantilevers

One-span girders with cantilevers are statically determinate. The loads placed on the cantilevers produce bending moments and shears not only in the cantilevers, but also in the main span of the girders. On the other hand, loads placed on the main span have no effect upon bending moments and shears in the cantilevers.

Notation:

l_1 = length of cantilever.
l = length of main span of girder.
P_1, P_2, P_3 = concentrated loads on cantilever.
a_1, a_2, a_3 = distance from support of loads P_1, P_2, P_3.
M_c = bending moment at support due to loads on cantilever.
V_c = external shear at support due to loads on cantilever.
M_{cx} = bending moment in cantilever at any point x from support.
V_{cx} = shear in cantilever at any point x from support.

Bending moments and shears due to the loads on the cantilever, produced in the cantilever and in the main span, may be found from the following formulas.

Cantilevers. — For cantilevers loaded by uniformly distributed loading or by concentrated loading, the formulas for shears and for bending moments in the cantilevers are as follows:

		Uniformly Distributed Loading on Cantilever	Concentrated Loads on Cantilever
Shears:	V_c at support	wl_1	$P_1 + P_2 + P_3$
	V_{cx} at point x	$w(l_1 - x)$	$P_1 + P_2$
Bending moments	M_c at support	$-\frac{1}{2}wl_1^2$	$-(P_1a_1 + P_2a_2 + P_3a_3)$
	M_{cx} at point x	$-\frac{1}{2}w(l_1 - x)^2$	$-[P_1(a_1 - x) + P_2(a_2 - x)]$

Main Span. — Bending moments and shears in the main span due to loads on the cantilevers are as follows:

Shears (constant throughout the span. — See Fig. 48 (*a*), p. 121).

$$V_x = -\frac{M_{c1}}{l} \qquad \text{Left cantilever loaded} \qquad (1)$$

$$V_x = \frac{M_{c2}}{l} \qquad \text{Right cantilever loaded} \qquad (2)$$

$$V_x = -\frac{M_{c1} - M_{c2}}{l} \qquad \text{Both cantilevers loaded} \qquad (3)$$

$$V_x = 0 \qquad \text{when } M_{c1} = M_{c2} \qquad (4)$$

Bending Moments. — For one-sided loading, negative bending moments in the span vary according to a straight line from a maximum at the support next to the loaded cantilever to zero at the other support. These are indicated in Fig. 48 (*a*) by dash lines.

When both cantilevers are loaded, bending moments in the span vary according a straight line from the cantilever bending moment M_{c1} at one support, to the cantilever bending moment M_{c2} at the other support. When $M_{c1} = M_{c2}$, bending moments in the main span are constant and are equal to M_{c1}. See Fig. 48 (*a*), solid line.

Most Unfavorable Position of Live Loads on Cantilevers. — The most unfavorable position of uniformly distributed live load is when the cantilever is fully loaded.

The most unfavorable position of concentrated wheel loads is when the heaviest load is placed at the end of the cantilever, and the balance of the cantilever is loaded with as many loads as can be accommodated there.

Bending Moments and Shears for Dead Load in Girder with Cantilevers. — For dead load, both cantilevers and the main span should be considered as loaded simultaneously. The bending-moment diagram for dead load is shown in Fig. 48 (*b*), p. 121. At the supports the negative bending moments are equal to the maximum negative cantilever bending moments M_{c1} and M_{c2}, respectively. Considering line *ab* as a closing line of a bending-moment diagram, a parabola is drawn for the bending moments due to the loads on the span, which completes the bending-moment diagram.

The shears in the main span are equal to the shears due to the loads on the main span, and to the shears due to the cantilever loads computed from formula (1), p. 120. For symmetrically loaded cantilevers, this last item is equal to zero.

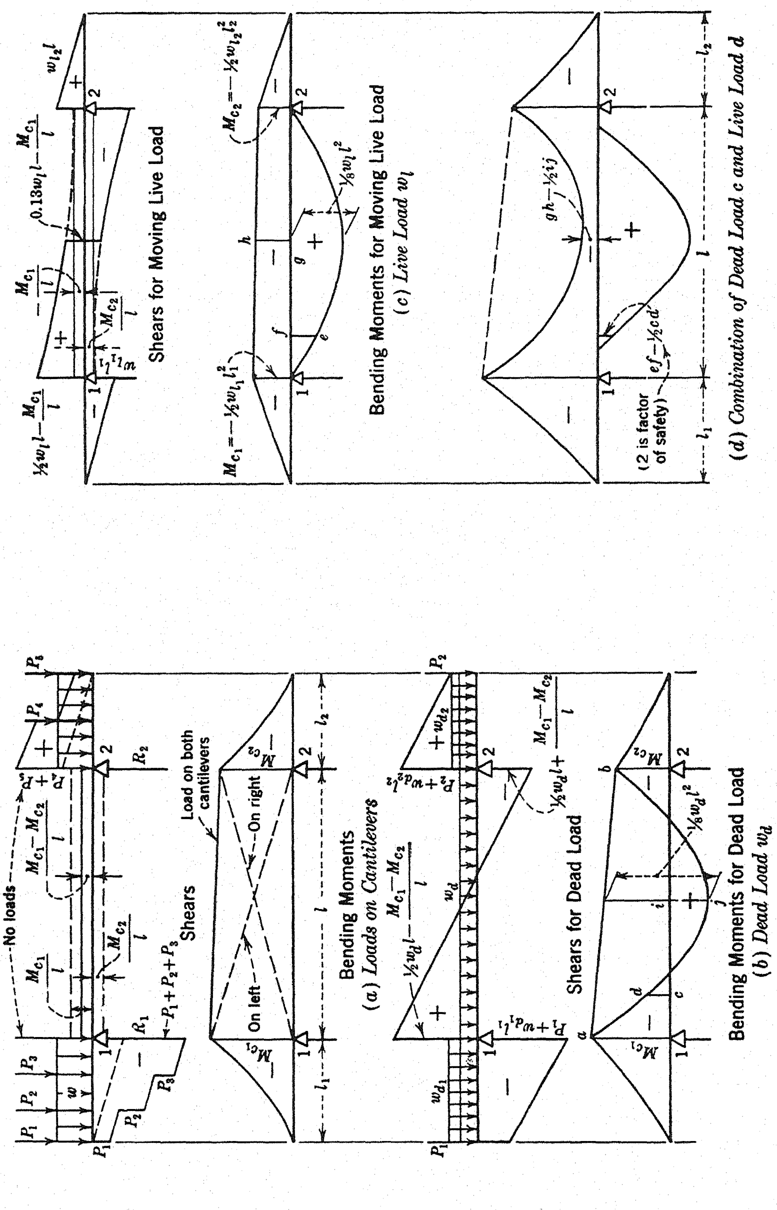


FIG. 48.—Single Span with Two Cantilevers. (See p. 120.)

Bending Moments and Shears for Live Loads. — For moving live loads, the positive bending moments in the main span are the same as for a simply supported span. The negative bending moments due to the cantilever loads may not be considered as reducing the positive bending moments in the main span, because the main span may be fully loaded when there is no live load on the cantilevers.

The largest negative bending moments for live load in the main span are produced when both cantilevers are fully loaded, and there is no live load on the main span.

Maximum shears in the main span are produced when, in addition to the most unfavorable position of the loading on the main span, one of the cantilevers is loaded. For positive shears, the left cantilever should be loaded. (See Fig. 48, p. 121.)

Combined Bending-Moment Diagram for Dead and Live Loads. — The combined diagram for dead load and for live load, including impact, shown in Fig. 48 (*d*), p. 121, is obtained by combining bending moments for dead loads with bending moments for live loads. Where both values are of the same sign, they are combined by a simple addition. Where, however, the bending moment for dead load is of opposite sign to the bending moment for live load, it must be divided by a factor of safety, usually 2, before being subtracted from the bending moment for live load. (For explanation, see p. 171.)

Combined bending-moment diagrams should be used to determine the dimensions of the girder at different points, the number and the length of reinforcing bars, and the points of bending of reinforcement.

Bending Moments and Shears in One-Span Girder with One Cantilever

For a girder with one cantilever, bending moments and shears may be found using formulas (1) to (4), p. 120. The bending-moment and shear diagrams are drawn in the same manner as for girders with two cantilevers, except that the cantilever bending moments at the end without a cantilever are made equal to zero. The bending-moment and shear diagrams for dead load, live load, and the combined bending moments are shown in Fig. 49, p. 123.

Particular attention should be paid to the possibility of uplift at the free end of the girder. Unless the end without a cantilever is anchored to a heavy support, the dead load on the main span must be large enough to balance the dead loads of the cantilever plus the cantilever live load multiplied by a desired factor of safety. In computing uplift for a bridge sufficiently stiffened laterally by cross beams, it is permis-

sible to treat the structure as a unit, instead of computing uplift separately for each girder for its most unfavorable loading.

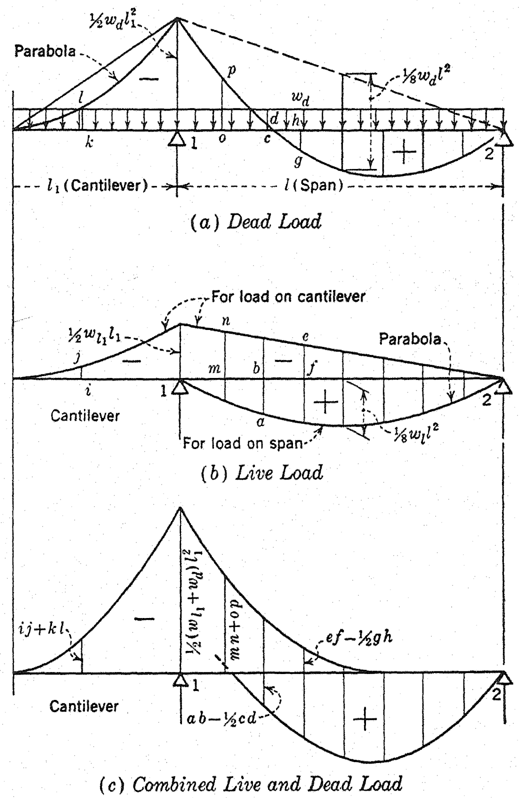


FIG. 49.—Girder with One Cantilever. (*See p.* 122.)

ONE-SPAN GIRDER WITH TWO CANTILEVERS

A one-span girder with two cantilevers may be used for the superstructure of a bridge with a large center opening and two small side openings. An example of such use is shown in Fig. 50, p. 124. In the figure are shown the combined bending-moment diagram for which the bridge was designed and the arrangement of the girder reinforcement. At the supports the arrangement of reinforcing bars is similar to that for a continuous girder. In this design, abutments are provided at each end of the bridge to retain the embankment, but the ends of the cantilevers do not rest upon them, a clearance having been provided between each wall and the end of the cantilever. The bending moments and

shears in this design may be found by means of formulas and suggestions on p. 119.

In some cases the cost of the construction may be reduced by omitting the abutments, and replacing them by apron walls suspended from the ends of the cantilevers. The apron walls and the side walls should be designed to resist the earth pressures. The total weight of the apron

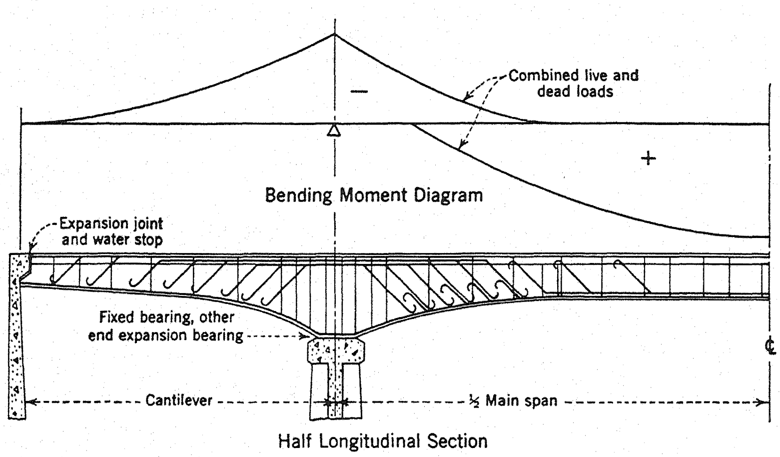


FIG. 50.—Girder with Two Cantilevers. Details of Reinforcement. (*See p.* 123.)

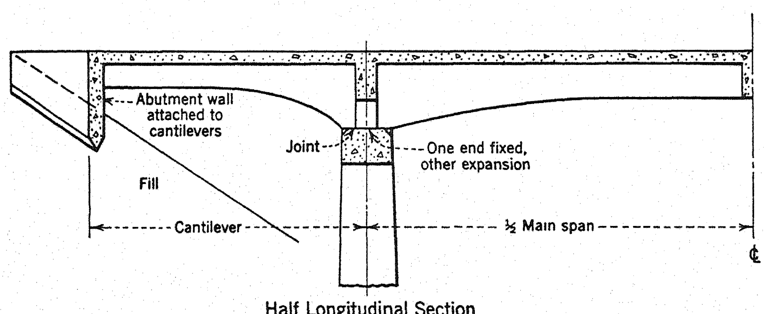


FIG. 51.—One-Span Bridge with Cantilevers. Aprons at Ends. (*See p.* 124.)

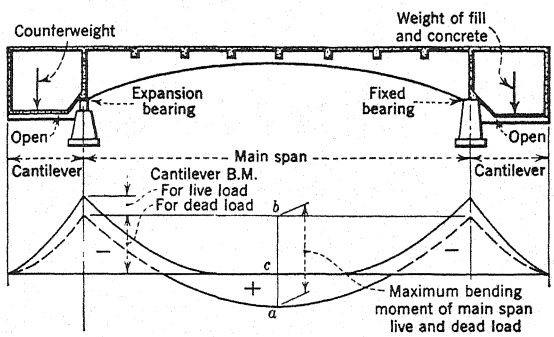
and of the side walls should be considered as a dead load carried by the cantilever. In addition, earth pressures produce negative bending moments in the cantilever, and these should be taken into account in design. A design with apron walls is shown in Fig. 51, p. 124.

The arrangements here described should be compared with right-angle rigid frames with cantilevers described on p. 270. Where foundations permit, a rigid-frame design should be used in preference to a cantilever-girder design.

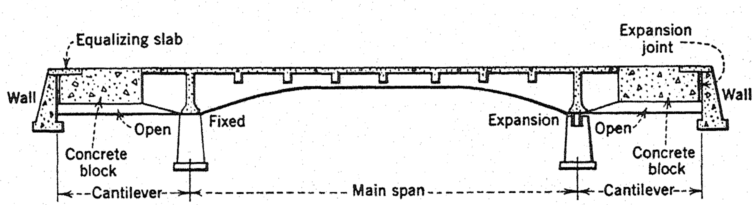
Bridges of One Opening with Concealed Cantilevers

Designs consisting of one-span girders provided with concealed cantilevers are useful for bridges of one opening when, on account of small available headroom, it is desired to reduce the depth of the girders, but at the same time it is not advisable to use rigid-frame designs.

Where an appreciable reduction of the positive bending moments is necessary, the cantilevers may be provided with counterweights. In America, designs with concealed counterweighed cantilevers are comparatively little known. In Europe, and particularly in Germany where this type appears to have originated, such designs have been used successfully for spans up to 150 ft., with ratios of depth in the center to span as small as 1 : 35.



(*a*) *Bridge with Short and Heavy Cantilever*



(*b*) *Bridge with Long Light Cantilever*

Fig. 52.—Bridge with Counterweighed Cantilevers. General Arrangement. (*See p.* 125.)

The general scheme of this design is evident from Fig. 52, p. 125. The structure consists of a number of longitudinal girders, each provided at both ends with cantilevers carrying heavy counterweights. Usually the cantilevers are concealed in the fill, but sometimes they form short side spans.

Since such structures are statically determinate, all reactions at the

supports are vertical and central, and there are no horizontal thrusts. This permits the use of comparatively light piers. At one pier the superstructure is provided with fixed bearings, and at the other pier with expansion bearings.

Design of Girders with Counterweighed Cantilevers. — The purpose of the counterweighed cantilevers is to produce in the main span large negative bending moments for dead load, and thereby to reduce the positive bending moments in the center of the span. In such designs much smaller depths for the main girders may be used than would be possible for a simply supported girder design.

As a first step in designing the structure, the floor construction in the main span should be designed. It is of advantage to adopt as light a floor design as possible, because by reducing the dead load of the main span the required weight of the counterweight is also reduced. Concrete of light-weight aggregates may be found economical here. An arrangement consisting of slab panels supported on four sides and reinforced in two directions may be found economical as giving smaller dead load than one-way slabs.

Next, from local requirements as to clearances, adopt tentatively the depths of the girders at the center and at the supports. The underside of the girder may be parabolic or segmental; or it may be straight in the central portion and provided at the ends with haunches. In existing structures the ratios of the depth at the center to the span length varies from 1 : 15 to 1 : 35; and the ratio of the depth at the center to the depth at the supports from 1 : 2 to 1 : 4.

After the dimensions of girders are tentatively chosen, find for the main span the maximum positive bending moments for dead load and for live load, considering the girder as simply supported. This is indicated in the bending-moment diagram, Fig. 52, p. 125, by *ab*. Using the assumed depths of the girders as a guide, decide upon the desired distribution of the total maximum bending moment in the girder between the positive and the negative bending moment. In the bending-moment diagram in Fig. 52, p. 125, the positive portion of the bending moments is represented by *ac* and the negative portion by *cb*. The maximum cantilever bending moment for dead load must be made equal to the negative bending moment just accepted. This determines the length and the weight of the cantilever. To complete the bending-moment diagram for the main span, the bending moment due to live loads on the cantilever should be added as shown in Fig. 52, p. 125.

Counterweight. — To get the most economical length and weight of the cantilever for the desired maximum cantilever bending moment for dead load, several combinations should be tried. It is possible to solve

the problem by using short cantilevers with a large unit weight, as in Fig. 52 (*a*), p. 125; or long cantilevers may be used with correspondingly smaller unit weights, as in Fig. 52 (*b*). In existing structure the ratio of the length of the cantilevers to the span length varies from 0.2 to 0.35.

The counterweight may consist of a concrete box supported by the cantilever ends of the girders, with bottom slab and side walls, filled with well-tamped earth fill. Or, to increase the weight of the counterweight without increasing its volume, it may consist of a solid concrete block of very lean proportions carried by the cantilevers. A heavy slab extending the full width and length of the cantilever may also serve as a counterweight.

To get a statically determinate structure, it is necessary to provide the girder with an expansion bearing at one end; and also, to permit free expansion and contraction of the superstructure, it is desirable to provide vertical expansion joints at both ends. In Fig. 52 (*b*), separate retaining walls are used at the ends to retain the fill, and an expansion joint is provided between the structure and these walls. When the bridge is subjected to moving loads, the cantilevers move up and down; and to make these downward movements possible, open space should be provided below the cantilever. A break in the alignment in the roadway slab due to the vertical movements of the cantilever ends is avoided by the use, at each end, of a special loose reinforced-concrete slab several feet long, placed across the joint, as shown in Fig. 52 (*b*), p. 125. With such arrangement, any vertical movement of the cantilever end causes only a slight change in the inclination of this equalizing slab.

In some designs, the cantilever box serves also as a wing wall for the abutment; and in such case the freedom of movement of the cantilever is impaired to some extent by the frictional resistance of the fill pressing against its sides.

Examples from Practice of Bridges with Counterweighed Cantilevers

Two examples from practice of one-span bridges with concealed counterweighed cantilevers are here described. One of them is the bridge at Freiburg, Germany, and the other a bridge in Breslau across a navigable canal.

Bridge at Freiburg. — The bridge at Freiburg, shown in Fig. 53, p. 128, has a span of 131.2 ft. (40.0 meters), and the length of each of the cantilevers is 27.9 ft. (8.5 meters), giving a ratio of length of cantilever to length of span of 1 : 4.7. The dimensions of the members are evident from the figure.

The structure is composed of eleven girders spaced 4.3 ft. on centers. Their ratio of depth at center to span is 1 : 23; and the ratio of minimum to maximum depth is 1 : 1.8. To resist compression stresses at the supports due to negative bending moments, the structure is provided with a bottom slab extending the full length of the main span, but with

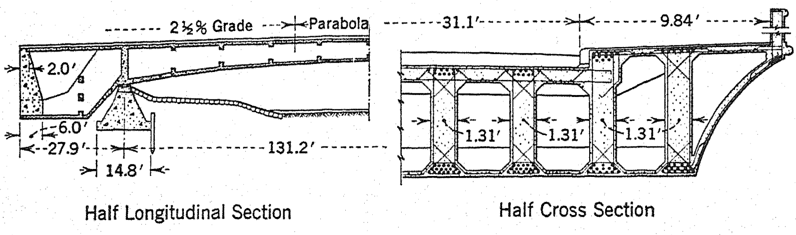


FIG. 53.—Bridge at Freiburg, Germany. (*See p.* 127.)

gradually reduced thickness. In addition to supplying compression areas, the bottom slab serves to stiffen the structure and to protect the bottoms of the girders from injury from drift and ice floes.

Each counterweight consists of a box filled with earth. It is 19.7 ft. deep and is formed by a reinforced-concrete bottom slab supported by the cantilevers, and by plain concrete walls on the sides and at the end. To permit a downward movement of the cantilevers, clear space is provided under the bottom slab of the box. Separate wing walls are used to retain the embankment.

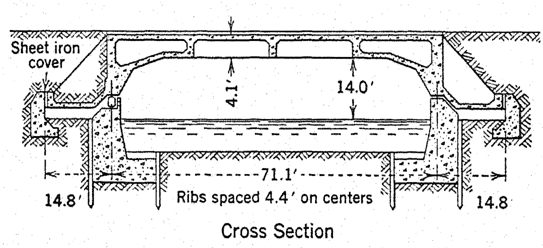


FIG. 54.—Bridge with Concealed Cantilevers at Breslau. (*See p.* 128.)

Bridge at Breslau. — The main span of the bridge across a navigable canal in Breslau is 71.1 ft. long, and each of the concealed cantilevers is 14.8 ft. long, which gives a ratio of cantilever length to span length of about 1 : 5. The depth of the girder in the center is 4.1 ft. (1.25 meters) giving a ratio of depth to span of 1 : 17.3. The bridge was designed for a truck weighing 20 metric tons, followed by a 12.5-ton trailer. For details of girder reinforcement and other details see *Beton und Eisen*, 1928, p. 340. Longitudinal section of the bridge is shown in Fig. 54, p. 128.

The counterweight box consists of a bottom slab spanning between girders and of side walls formed by the outside girders. The fill of the cantilever is allowed to merge with the fill of the embankment. However, free space is provided under the bottom slab, which is protected from the intrusion of the surrounding fill by a small retaining wall.

Combination of Several One-Span Girders, Each Provided with Free Cantilever

A number of multi-span bridges have been built in America in which the superstructure consists of a combination of one-span longitudinal girders, each of which is provided either with one or with two cantilevers. A general scheme of such designs is shown in Fig. 55, p. 129; their object is to reduce the depths of the girders in the center of the spans, as well as to reduce the cost of the girders, without resorting to statically indeterminate designs.

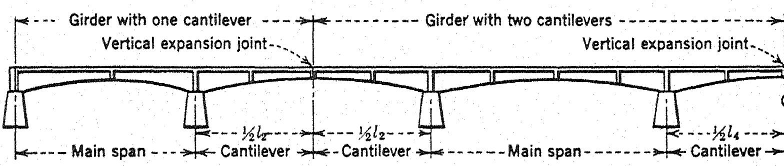


FIG. 55.—Bridge Design. Girders with Free Cantilevers. (*See p.* 129.)

In comparison with simply supported girders of one span, these cantilever designs have the advantage that they require a smaller depth of the main girders in the center of the span, and smaller quantities of materials for the girders. Also fewer bearings are required, and narrower piers may be used. The disadvantages of these arrangements are the deflection of the long cantilever arms and the constant vertical movements of the cantilever ends due to passing vehicles. These may be objectionable because they may make it difficult to keep the roadway in alignment, and this difficulty may be aggravated by unequal settlement of adjoining piers.

Several expedients have been tried to keep the ends of the adjoining cantilevers in alignment, but the weakness of all of them is that they either do not work, or, if they do work, loads are transmitted from one cantilever to the adjoining cantilever, entirely upsetting the anticipated stress conditions in both girders. The statically determinate design is then changed into an indefinite statically indeterminate construction.

The authors do not favor designs of this type. Where it is possible to secure good foundation at reasonable cost, it is preferable to use continuous girder designs, instead. Where, however, conditions are such as to

exclude definitely continuous girders, it is preferable to use designs consisting of cantilevered girders and of short spans supported by the cantilever described in following paragraphs.

2. BRIDGES IN WHICH CANTILEVERS SUPPORT SHORT SIMPLE SPANS

To the second group of cantilever bridges belong structures consisting of lines of one-span girders, each with one or two cantilevers, which carry short simply supported spans suspended from the ends. This arrangement is illustrated in Fig. 56, p. 130. Structures of this type are

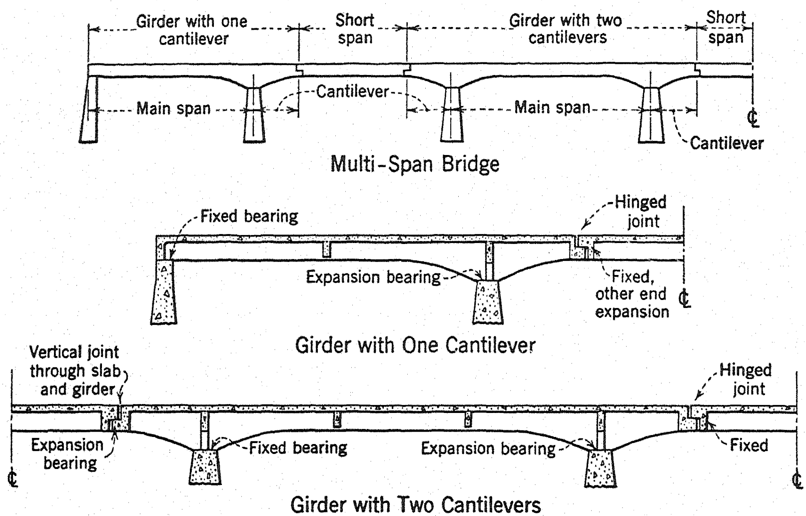


FIG. 56.—Combination of Girders with Cantilevers and Short Suspended Spans. (*See p.* 130.)

very useful for crossings of three or more openings in locations where it is not advisable to use statically indeterminate designs.

Principle of Design. — The object of the designs here described is to get most of the advantages of continuous girder designs without the disadvantages connected with statically indeterminate designs.

It is a well-known fact that, for any particular loading condition, a continuous girder may be changed into a number of statically determined members by providing in alternate spans hinges at the points of contraflexure, i.e., at the points of zero bending moments. The resulting structure then consists of a number of girders with cantilevers, and of short girders spanning between the cantilevers. The bending moments in such a structure for the particular loading are identical with those in a continuous girder.

In a girder carrying moving loads, the location of the points of contraflexure varies with the conditions of loading. For this reason it is impossible to reproduce in the substitute structure identically the same stress conditions at all times as are found in a continuous girder. It is possible, however, by a judicious selection of the lengths of cantilevers to reduce the maximum positive bending moments in the girders to the desired degree, and in this manner to reduce appreciably the required depths of girders in the center of the span.

Advantages and Disadvantages of Cantilever Designs. — The advantages of the cantilever design of the type here described in comparison with the simply supported girder designs are: (1) Cantilever designs require less concrete, steel, and formwork for the main girders. (2) At each pier the cantilever design uses one bearing per line of girders and, therefore, requires a smaller width of piers than the simply supported girders in which two bearings in a line are needed at each pier. (3) The reactions of the piers are always central. (4) There are fewer expansion bearings, which reduces the first cost and the cost of maintenance.

The only disadvantage in comparison with simple spans is that the cantilever designs require somewhat more skill on the part of the designer, and the arrangement of reinforcement is somewhat more complicated.

In comparison with continuous girder designs, the cantilever design has the only advantage that they are statically determinate, and that, therefore, the possibility of bad effects of unequal settlement of foundations is less. It has the disadvantages that the cost of the girders is somewhat larger; that there is an extra cost of the hinges and expansion joints between the short spans and the cantilever ends; and finally that the cantilever design is less rigid than the continuous design.

Cantilever designs should be used only where good foundations are not easily obtainable. When piers rest on gravel, hard clay, hardpan, rock, or piles driven to solid bearing, continuous girders or multi-span frames should be used. The computations of bending moments and shears for such continuous designs as given in this volume are simple and reliable; therefore, the fact that cantilever designs are statically determined should be of comparatively small importance.

Arrangements of Cantilever Structures. — Numerous combinations of short simple spans and of one-span girders with cantilevers are possible. The subject will be discussed under two headings:

(*a*) Multi-span bridges, consisting of cantilevered girders and of simply supported short girders spanning between cantilevers (p. 132).

(*b*) Bridges of one opening the superstructure of which consists of two concealed short girder spans, one on each side of the opening, each

girder provided with one cantilever extending into the opening, and a short center girder spanning between the ends of the cantilevers (p. 140).

(a) Multi-Span Bridges of Cantilever Design

Multi-span bridges may consist of a number of main girders placed in a line, each of them with one or two cantilevers, and of simply supported short spans suspended from the ends of the cantilevers. For this arrangement see Fig. 56, p. 130.

Cross Section of Bridge. — The cross section of a bridge of cantilevered design does not differ materially from those of simply supported or continuous slab or girder bridges. Usually the bridge consists of a number of longitudinal girder lines, which support a slab reinforced in one or two directions. The floor system is designed in the same manner as explained on pp. 49 and 74. The most economical spacing of girders should be found by comparative estimates, taking into consideration the cost of materials as well as that of formwork. When slabs reinforced in one direction are used, the bridge should be stiffened by cross beams, as explained on p. 48.

Main Girders. — The depths of the main girders in the center of the span and at the ends of the cantilevers usually are made smaller than at the supports. The underside of each girder may be parabolic or segmental, or it may be straight in the center and provided with straight or curved haunches at the ends. The main span of the girder is reinforced for positive bending moments in the center of the span, and for negative bending moments at the supports. In some cases some negative bending-moment reinforcement is needed along the whole length of the girder. The cantilevers are subjected only to negative bending moments, and require only negative bending-moment reinforcement placed near the top of the girder, which is usually a continuation of the negative bending-moment reinforcement of the main span. Often, compression reinforcement is required at the bottom of the cantilevers, some of which should extend the whole length of the cantilever. Bottom slab or increased width of stem may also be used to take care of compression at and near the supports.

The length of bars should be determined from the combined bending-moment diagram. The amount of web reinforcement should be determined from the shear diagram.

Ratio of Cantilever Lengths. — The ratio of the length of the cantilever to the length of the span should be selected so as to get the most economical design. Several arrangements should be studied before the final selection is made. In known examples from practice, the ratio

of the cantilever length to the span length varies from 0.19 to 0.28. In unusual cases, this ratio is even larger. (See p. 143.) In girders with one cantilever, the cantilever length must be small enough to prevent uplift at the free end of the girder. If uplift cannot be prevented, the end should be anchored to the end supports.

Supporting Simple Spans on Cantilever Ends. — Short girders are supported on brackets formed at the ends of the cantilevers in the manner shown in Fig. 57, p. 133. The aggregate depth of the bracket at the cantilever end and of the superimposed bracket at the end of the girder, including the bearing placed between them, is about the same as the depth of the cantilever on one side and the depth of the short girder on the other side, so that there is no sudden break in the outline of the structure at the hinges.

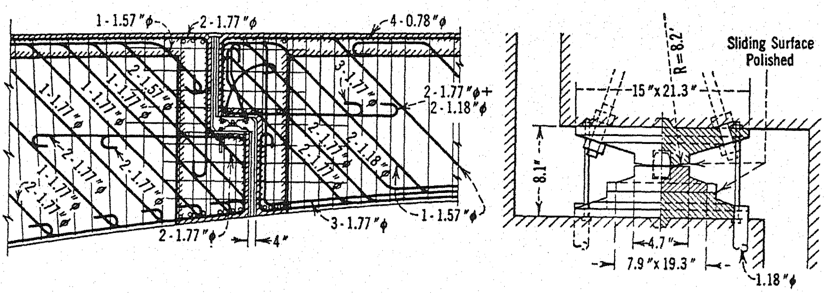


FIG. 57.—Expansion Bearing for Suspended Span. Bridge across Danube at Dillingen. (*See p.* 133.)

To take care of the shears, the widths of the two brackets at each hinge are usually made greater than the width of the cantilever or the girder. To stiffen the construction, cross beams are used between the brackets both in the cantilever and in the short span. Reinforcement of brackets consists of several bent bars extending from the member into the bracket, horizontal bars, and stirrups. See p. 135.

Provision for Expansion and Contraction at Hinges. — It is important to provide efficient but inexpensive arrangements at the hinges to take care of expansion and contraction of the short span. At one end of the suspended short span, fixed bearings should be used, but these should permit rotating movements of the end of the girder in a vertical plane.

Expansion bearings should be provided at the other end, and these also should have provision for rotation of the girder ends. The most effective expansion bearing is the roller bearing shown in Fig. 63, p. 147, and used at the end of a 60-ft. suspended span. Such bearings are too expensive for bridges of smaller spans. A somewhat less expensive expansion bearing, shown in Fig. 57, p. 133, was used in the bridge across

Danube at Dillingen.[1] The short span in this bridge is 60 ft. long, and each of the supporting cantilevers is 31 ft. long. The reaction at each bearing is 66.5 metric tons (1 metric ton = 2 205 lb.). This bearing permits sliding motion for expansion, and rocking motion for rotation of the girder end.

Simple expansion bearings may consist of two metal plates with a lead sheet between, as shown in Fig. 62, p. 146.

Determination of Bending Moments and Shears

The component parts of structures of this multi-span type may be: simple spans suspended from cantilevers; girders with one cantilever; and girders with two cantilevers.

Simply Supported Spans. — Simply supported spans do not need to be considered here because their design is essentially the same as that of ordinary simply supported girder bridges described in Chapter V, with the exception of the arrangements at the hinges. To reduce the weight on the cantilevers, it is advisable to use the lightest possible floor system. Also the use of light-weight aggregates may be of advantage.

Girders with Cantilevers. — Bending moments in girders with cantilevers are found separately for the loads on the cantilevers and for those on the main span; and the results are then combined. Bending-moment diagrams are shown in Fig. 58, p. 137, for a girder with two cantilevers and also for a girder with one cantilever.

Bending Moments in Cantilevers. — The bending moments in cantilevers for girders with one and two cantilevers may be found by means of the following formulas and rules.

For dead load and for uniformly distributed live load, maximum bending moments are obtained considering the short span and the cantilever as fully loaded. The cantilever is then subjected to a concentrated load acting at the end of the cantilever and equal to the dead-load reaction of the short span, and to the loading on the cantilever. Bending moments are easily found by the formulas on p. 119.

For concentrated wheel loads, the most unfavorable loading occurs when a heavy load is placed at the end of the cantilever, and the short span and the cantilever are loaded with as many loads as can be accommodated there.

Bending Moments for the Main Span. — For dead load, all positive bending moments due to the loads on the main span may be considered as reduced by the cantilever bending moments at the supports in the

[1] See Dr. O. Muy, "Die neue Dillinger Donau Brücke," *Bau-Ingenieur*, 1926, Heft 10.

manner explained on p. 122 in connection with girders with one cantilever, and on p. 123 for girders with two cantilevers.

For live loads, positive bending moments are the same as for a simply supported span. Negative bending moments in the main span are produced when the cantilevers are fully loaded and the span is not loaded. See Figs. 48 (*c*), p. 121, and 49 (*b*), p. 123.

Bending moments for dead load and live load are combined as explained on p. 168. Combined bending-moment diagram for a girder with two cantilevers is shown in Fig. 48 (*d*), p. 121; and for a girder with one cantilever in Fig. 49 (*c*), p. 123.

External Shears. — For dead load and for uniformly distributed live load, external shears in cantilevers vary from a minimum at the end of the cantilever, where it equals the reaction of the short span, to a maximum at the support where it equals the reaction of the short span plus all the loads on the cantilever.

In the main span of a girder with two symmetrically loaded cantilevers, the shears for dead loads are the same as for a simply supported span. For a girder with one cantilever, the shears due to dead load in the end next to the cantilever are increased by $\frac{M_{c(d)}}{l}$, where $M_{c(d)}$ is the cantilever bending moment at support for dead load, and l is the span length. At the free end, the shears are decreased by the same amount.

For a live load, the shears in the main span are considered as equal to the shears in a simply supported span plus $\frac{M_{c(l)}}{l}$, where $M_{c(l)}$ is the bending moment for live load in the cantilever next to the half of the span under consideration. In a girder with one cantilever, there is no increase in live-load shears in the half of the girder next to the free end.

Brackets at Hinges. — Brackets at the hinges are vital parts of the construction, and great care should be taken in their design and construction. Owing to paucity of experimental data there is an excusable tendency to over-reinforce this part of the structure. Any excess of steel used here forms only a very small proportion of the total steel tonnage in the structure.

Brackets at the ends of cantilevers should be reinforced with horizontal bars near their top and bottom surfaces; with diagonal bars extending from the cantilevers into the brackets; and with stirrups. All this reinforcement is intended to resist not only stresses due to the loads and to contraction but also the tendency to cracking caused by the sudden reduction in the cross section at the bracket, which concrete shares with other cast materials. Although the existence of this tendency is

known, its effect cannot at present be computed with any degree of accuracy.

First, the bracket itself should be considered as a short cantilever and investigated for bending stresses and for diagonal tension in the ordinary manner. The allowable tensile stress used here, however, should be not more than one-half of the specified stresses in steel used elsewhere in the structure. The width of the bracket should be enlarged to reduce the unit shears. Stirrups should be employed to resist diagonal tension without taking into account the bent bars extending into the cantilever. In many designs, both horizontal and vertical stirrups are used.

Second, diagonal bent bars should be provided extending from the cantilever into the bracket, and properly anchored therein, to resist the tension produced by the reaction of the suspended span. No reliance should be placed upon concrete in resisting this tension. The diagonal bars cut by a section drawn from the corner of the bracket downward at 45° may be considered as effective in resisting this tension.

Third, horizontal reinforcement should be provided in the bracket to resist the tension produced by contraction of the short span, and caused by imperfect action of the expansion bearings. This tension is equal to the reaction of the suspended span multiplied by the coefficient of friction of the expansion bearing. Low allowable unit stress should be accepted in determining the cross section of tension bars to resist this tension. Similar reinforcement also should be provided in the cantilever.

The brackets at the ends of the suspended spans should be designed in the same manner as the cantilever brackets, except that the position of the reinforcement should be reversed.

Numerical Example

In the following numerical example, bending moments and shears are determined for girders with one and with two cantilevers, respectively, as used in a multi-span structure, where ends of the cantilevers support short spans. The reinforcement of a main girder is also shown here.

Example. — Use cantilever design in a bridge of five openings, in which the end openings are 60 ft. each; and the center openings are 90 ft. each. Select arrangement of members, compute bending moments and shears in main girders, and show reinforcement in the girder with one cantilever.

Cross section of the bridge is as shown in Fig. 58, p. 137.

Live load: trains of 20-ton trucks as shown in Fig. 23, p. 60.

Solution. — Hinges are placed in the second and fourth spans, so that longitudinally the structure is divided into two girders with one cantilever each, one girder with two cantilevers, and two short spans suspended from the cantilevers. This is shown in Fig. 58.

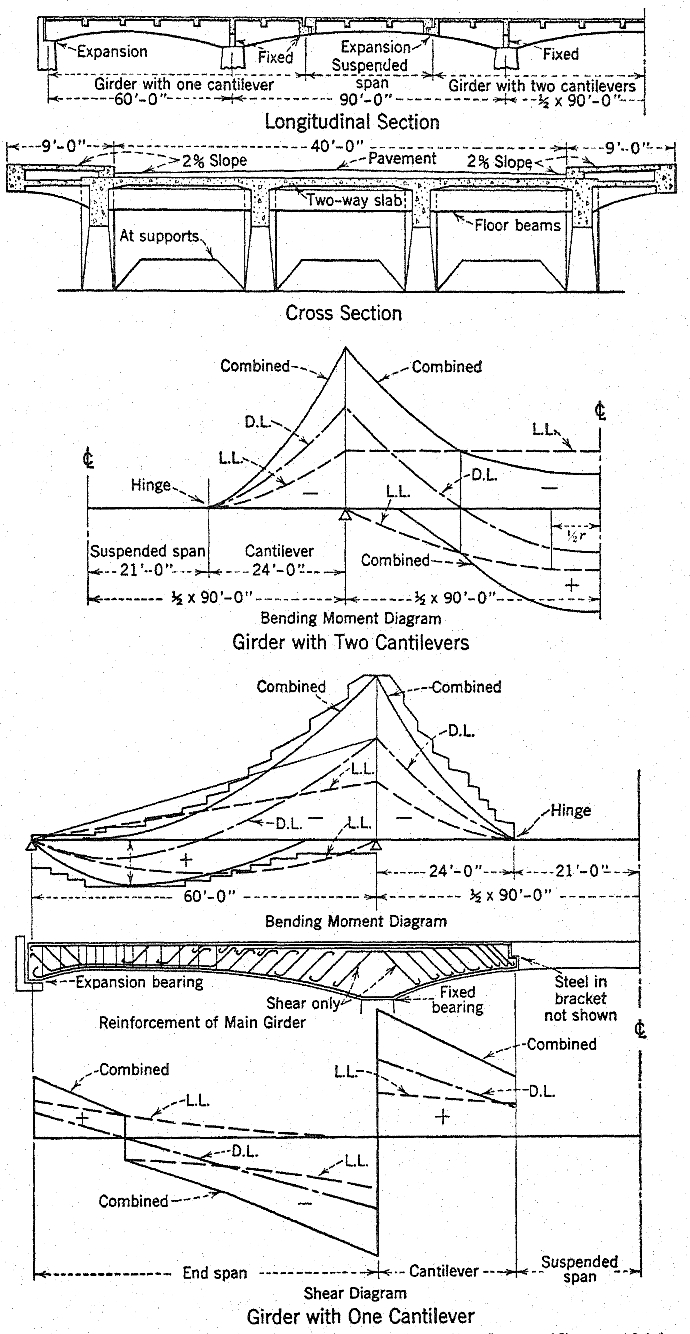


FIG. 58.—Example of Design of Cantilever Bridge. (*See p.* 136.)

The floor system is designed first, and the slab thicknesses determined. This work is not shown here. The floor design used here consists of slabs reinforced in two directions, and supported by the girders and by floor beams. On the basis of the assumed cross sections of the main girders the following unit dead loads are accepted: End spans, $w_{d1} = 3\,600$ lb.; cantilevers, $w_{d2} = 4\,600$ lb.; short spans, $w_{d3} = 3\,100$ lb.; center span, $w_{d4} = 4\,000$ lb.; each per lineal foot of girder. After the dimensions are found, the assumed dead loads should be checked. Since the dead load of the girders is variable, it is advisable in important structures to subdivide each girder into a number of sections, to compute the dead load for each section, and to consider these as loads concentrated in the centers of gravity of the sections. The reactions of the floor beams also should be considered as concentrated loads. When the revised bending moments and shears do not agree with those for the assumed unit loads, the amounts and disposition of reinforcement should be changed. This work is not performed in this example.

Dead Loads. —

All cantilevers:

		End shears	Moment arm	Bending moments
Short span	$3\,100 \times 21$	65 100	24×12	18 750 000
Cantilever	$4\,600 \times 24$	110 400	12×12	15 900 000

Total at support of cantilever $V_2 = 175\,500$ lb. $M_{c(d)} = 34\,650\,000$ in-lb.

End span:

Static bending moment in end span

$$M_{\text{max.}} = \tfrac{12}{8} \times 3\,600 \times 60^2 = 19\,440\,000 \text{ in-lb.}$$

Shears in end span

		Free end	End next to cantilever
Static end shear	$3\,600 \times 30$	108 000	108 000
Due to cantilever bending moment	$\dfrac{34\,650\,000}{12 \times 60}$	$-48\,000$	48 000
Total end shear		60 000 lb.	156 000 lb.

Center span:

Static bending moment in center span

$$M_{\text{max.}} = \tfrac{12}{8} \times 4\,000 \times 90^2 = 48\,600\,000 \text{ in-lb.}$$

Static end shear: $V = 4\,000 \times 45 = 180\,000$ lb.

Cantilevers do not affect shears in the center span due to dead loads.

The cantilever bending moments for dead loads are plotted at the supports of the girders with cantilevers. In the girder with one cantilever, connect the apex of the plot with the other support; and using this inclined line as a closing line, draw the parabola representing the dead-load bending moment in the main span. In the girders with two cantilevers, the closing line is parallel to the span, and all positive bending moments there are reduced by the cantilever bending moment at the support.

Live Loads. — Three lines of trucks give a lateral reaction on one girder of 1.4 truck lines. Since three lines of trucks are necessary to produce the maximum results, the loading may be reduced by 10 per cent for the reasons explained on p. 11. An

impact ratio of 0.18 is used. The truck load carried by one girder is $W = 1.4 \times 0.9 \times 1.18 \times 40\,000 = 59\,500$ lb.; of which 47 600 lb. is on the rear axle, and 11 900 lb. on the front axle.

End Span. — For loads on the end span, find the maximum bending moment, using the formula in the table on p. 20, item 4, $r = 13.7$ ft.

$$M_{\text{max.}} = \left[\left(0.707 - \frac{9.694}{60}\right)^2 - \frac{2.8}{60}\right] \times 59\,500 \times 60 \times 12 = 10\,730\,000 \text{ in-lb.}$$

End shear: use table on p. 24, item 4

$$V = \left(2 - \frac{38.6}{60}\right) 59\,500 = 80\,700 \text{ lb.}$$

In the same manner, shears at intermediate points are found.

Ninety-Foot Center Span. — For live loads on the center span, find the maximum bending moment, using the table on p. 20, item 5, $r = 17.18$ ft.

$$M_{\text{max.}} = \left[\left(0.732 - \frac{12.75}{90}\right)^2 - \frac{2.8}{90}\right] \times 59\,500 \times 90 \times 12 = 20\,320\,000 \text{ in-lb.}$$

Maximum end shear, using the table on p. 24, item 6

$$V = 3\left(1 - \frac{35.8}{90}\right) \times 59\,500 = 107\,500 \text{ lb.}$$

Shears at intermediate points are also found from the formulas given there.

Cantilevers. — For maximum bending moment at the support of the cantilever, place the rear axle of a truck at the end of the cantilever and its front axle on the cantilever. On the short span, place another truck with the front axle 19 ft. from the cantilever end. This gives at the support

$$M_{c(l)} = 19\,950\,000 \text{ in-lb.}$$

For maximum end shear, place one rear axle at the support, and cover the cantilever and the short span with trucks. Maximum end shear is

$$V = 102\,500 \text{ lb.}$$

At the end of the cantilever, the shear equals the maximum end shear in the short span obtained from the formula in the table on p. 24, item 3

$$V = \left(1.8 - \frac{29.2}{42}\right) \times 59\,500 = 65\,700 \text{ lb.}$$

Effect of Live Loads on Cantilevers upon Main Spans. — Live loads on cantilevers produce negative bending moments in the main spans. These, for a span with one cantilever, vary from a maximum at the cantilever end to zero at the other end. For a girder with two cantilevers, bending moments are constant throughout the span.

In the 60-ft. girder with one cantilever, only the shears in the half next to the cantilever are increased by the cantilever loads. The increase equals:

$$V = -\frac{19\,950\,000}{60 \times 12} = -27\,700 \text{ lb.}$$

In the 90-ft. girder with two cantilevers, shears in the girder are increased in both halves by

$$V = \pm \frac{19\,950\,000}{90 \times 12} = \pm 18\,500 \text{ lb.}$$

Uplift at End of Span with One Cantilever. — Since the end shear for dead load at the free end of the girder with one cantilever, $V = 60\,000$ lb., is more than twice the uplift produced there by the cantilever live loads, $V = -27\,700$ lb., there is sufficient factor of safety against uplift.

Bending-Moment and Shear Diagrams. — Bending-moment and shear diagrams shown in Fig. 58, p. 137, are used to determine the cross sections of the girder, the required amounts of reinforcement, the points of bending of reinforcement, and finally the amount and disposition of the web reinforcement.

Dimensions of Girders and Amounts of Reinforcement. — In the center of the span, each girder is a T-beam, therefore a small depth is sufficient there to take care of the positive bending moments. Toward the supports, not only the depth of the girder but also the width of the stem is increased to take care of the increasing shears and of the compression stresses due to negative bending moments. The widths of stem vary according to a straight line from a minimum in the central portion of the span to a maximum at the support.

In the diagram for the end girder are shown by broken lines the resisting moments of the girder obtained by multiplying at each section the area of the available reinforcement by the effective depth of the girder and the allowable unit stress in steel. The design is satisfactory because the diagram of the resisting moments lies outside of the diagram of the combined bending moments, which signifies that at all points the resisting moments are larger than the bending moments to be resisted.

The reinforcement is arranged so as to use comparatively short bars, which facilitates handling and placing of reinforcement. At some points the bent bottom reinforcement, and at other points the bent top reinforcement, is made to serve as diagonal tension reinforcement.

In accordance with American practice, in this example the concrete of the girder is assumed to resist diagonal tension at the rate of 40 lb. per sq. in. The balance of the diagonal tension is provided for by bent bars in the portion of the girder where bent bars are available, and by stirrups in the balance of the girder.

(*b*) Single Openings with Concealed Counterweighed Spans

In a bridge with one opening, the depth of the girder in the center may be reduced without resorting to statically indeterminate designs by using a construction consisting of two short heavy concealed spans with cantilevers, and a short center span supported by these cantilevers. This arrangement is shown in Fig. 59, p. 141. The short anchoring spans are concealed behind the abutment walls, one at each side of the bridge. The visible construction then consists of two cantilevers extending into the center opening, and of the short suspended span.

This design serves the same purpose as the design with concealed counterweighed cantilevers described on p. 125. Both designs are statically determinate.

Design of Girders. — The design of the structure should begin with the design of the suspended span and of the cantilevers. The transverse spacing of the girder is determined in the same manner as for

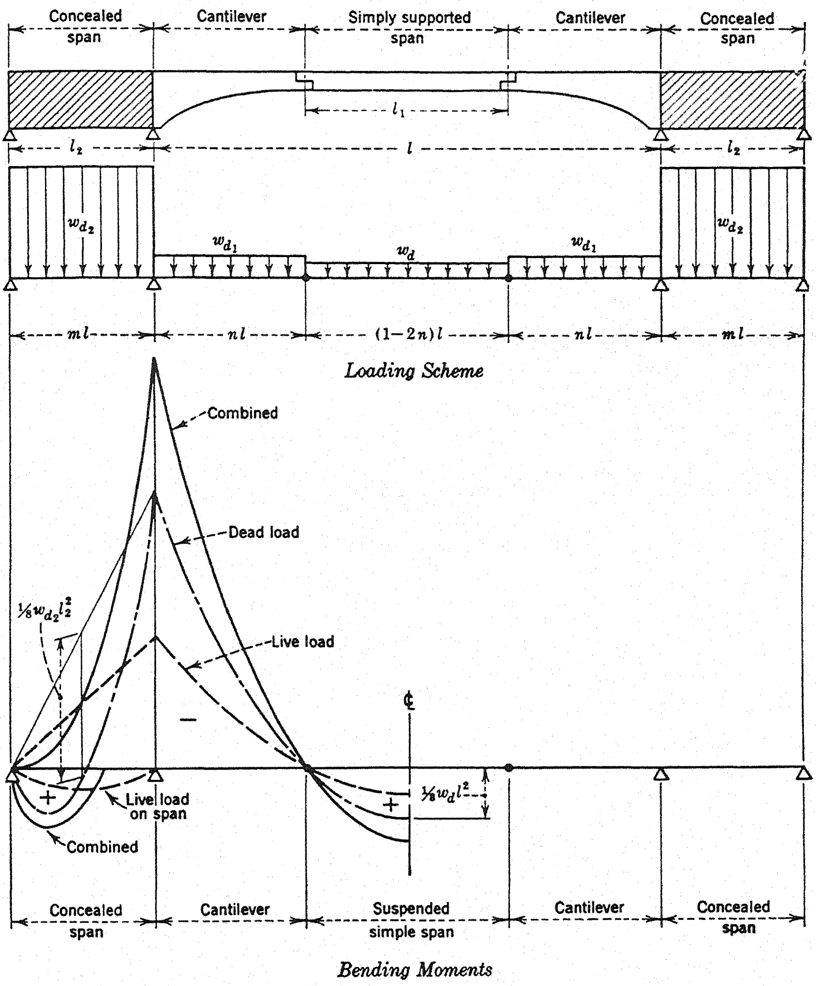


FIG. 59.—Concealed Counterweighed Spans with Cantilevers. (*See p.* 140.)

other girder bridges. In this case, however, it is advisable to adopt a floor system having the smallest dead load, even if, by itself, such design would not be economical, because, by reducing the dead load, the required balancing dead loads on the anchoring spans are also reduced.

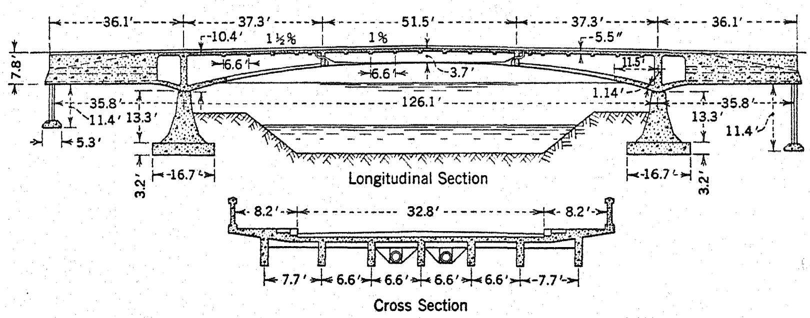
After the floor system is designed, bending moments are computed

in the cantilevers for live and dead loads. The negative shear or uplift produced by this bending moment in the anchoring span must be resisted by the dead loads on the anchoring span. The length of the anchoring span is then assumed and the necessary balancing dead load found from the requirement that there must be no uplift when the center opening is fully loaded with double the live load, and there is no live load on the anchoring span. Several lengths of anchoring span should be tried before making the final selection.

The anchoring span may consist of boxes filled with well-tamped fill, or of a solid concrete block of proper depth.

Example from Practice of Bridges with Concealed Anchoring Spans

Bridge in Temesvar, Hungary. — A good example of a design using concealed counterweighed anchoring spans is furnished by the bridge in Temesvar illustrated in Fig. 60, p. 142. The span of the center opening,



From Beton und Eisen, 1909, p. 359.

Fig. 60.—Bridge at Temesvar, Hungary. Concealed Counterweighed Spans. (*See p.* 142.)

from center to center of supports, is 126.1 ft.; the concealed spans are 36.1 ft. each; the cantilevers are 37.3 ft. each, giving the ratio of cantilever to span length of 1 : 3.4. The suspended span is 51.5 ft. long.

The depth of the main girder in the center is 3.7 ft., giving a ratio of depth to span of 1 : 34.3. At the supports the depth increases to 10.4 ft., and the underside of the girders is parabolic. The width of the stem of the cantilevers increases from about 10 in. at the ends to about 20 in. at the supports. Compression stresses at the support are resisted by a bottom slab.

The girders of the concealed span carry a counterweight consisting of a solid concrete block. They rest at one end on the heavy center piers, and at the other end on ten reinforced-concrete columns, provided with a continuous footing.

Examples from Practice of Cantilever Bridges

Cantilever bridges consisting of combinations of cantilevered girders and short spans have been used to a considerable extent for structures of moderate as well as of long spans. Several examples from practice are described and illustrated here.

Bridge de la Madelaine, across the Loire at Nantes, France.[2] — The bridge across the Loire at Nantes is of unusual design. It has three openings, 141.6 ft., 220 ft., and 141.6 ft. In the longitudinal section the structure consists of two anchoring end spans, each 141.6 ft. long and provided with one cantilever 90.3 ft. long extending into the 220-ft. center opening. The short suspended span is 39.4 ft. long.

In cross section, the bridge is composed of four girders of very unusual design. The lower flange of the girder, which in this case acts in compression, is octagonal in cross section and is very heavily reinforced with longitudinal bars and spirals. At the points of the largest stresses, longitudinal reinforcement consists of thirty-four 2-in. round bars, and forms 22.7 per cent of the concrete section of the flange. The spirals consist of wires from 0.47 to 0.79 in diameter with a pitch of 7 in.

A thin concrete web connects the bottom flange with the top flange which here is a part of the roadway. This web is reinforced by two sets of diagonal bars placed at right angles to each other, and anchored in the flanges.

As might have been anticipated, difficulty was experienced during construction in properly encasing the bars with concrete. The repairs necessary after removal of the forms are described in the article referred to in the footnote.

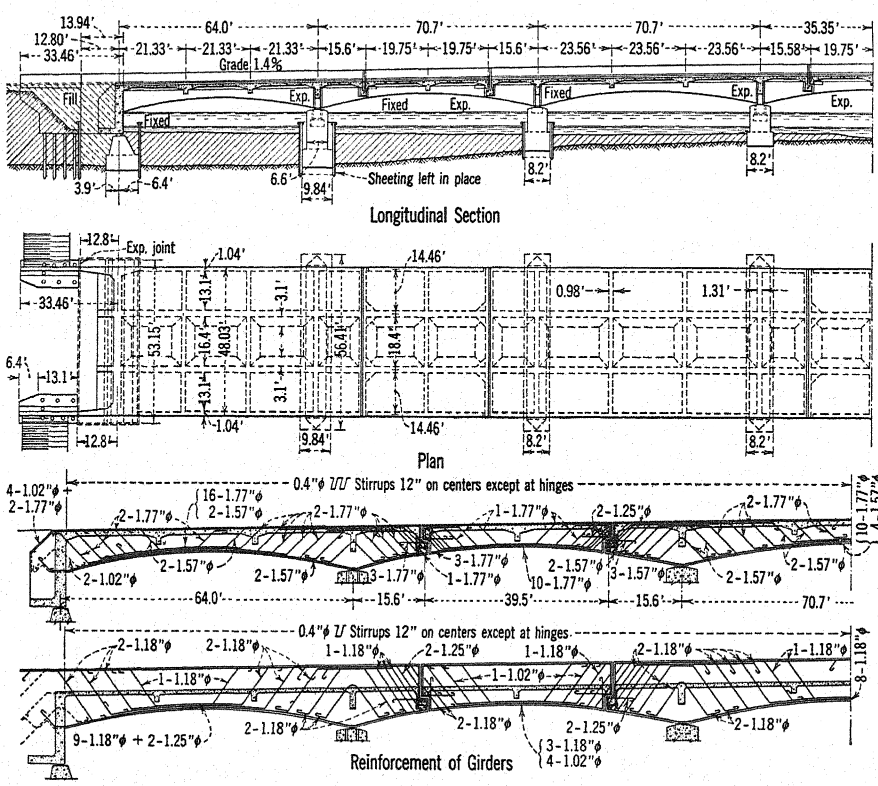
South Esk River Bridge in Scotland. — Another cantilever bridge of unusual design, and unusual length of span, is the bridge across South Esk River designed by Sir E. Owen Williams.[3] It has three openings: each of the two end openings is 150 ft. long, and the center opening is 216 ft. long. Each end span is provided with one cantilever 87 ft. $2\frac{3}{8}$ in. long extending into the center opening. The short center span of 21 ft. $7\frac{1}{4}$ in. is supported by the cantilevers.

The bridge is a through structure with a 20-ft. roadway and two sidewalks, each 8 ft. wide. The girders extend above the roadway, and their height is largest at the river piers. The top of each girder on each side of the pier is a concave curve with minimum ordinates at the center of the end span and at the cantilever end. In the high portion of the girders open web is used, consisting of verticals and diagonals.

[2] See *Annals des ponts et chaussées Français,* 1931, May and June, p. 151.

[3] See *Engineering News-Record,* Aug. 10, 1931.

Bridge at Lindau, Germany.[4] — This bridge is 479 ft. long between abutment walls, and consists of two 64-ft. end openings, and five 70.7-ft. interior openings. Hinges are provided in the second, fourth, and sixth



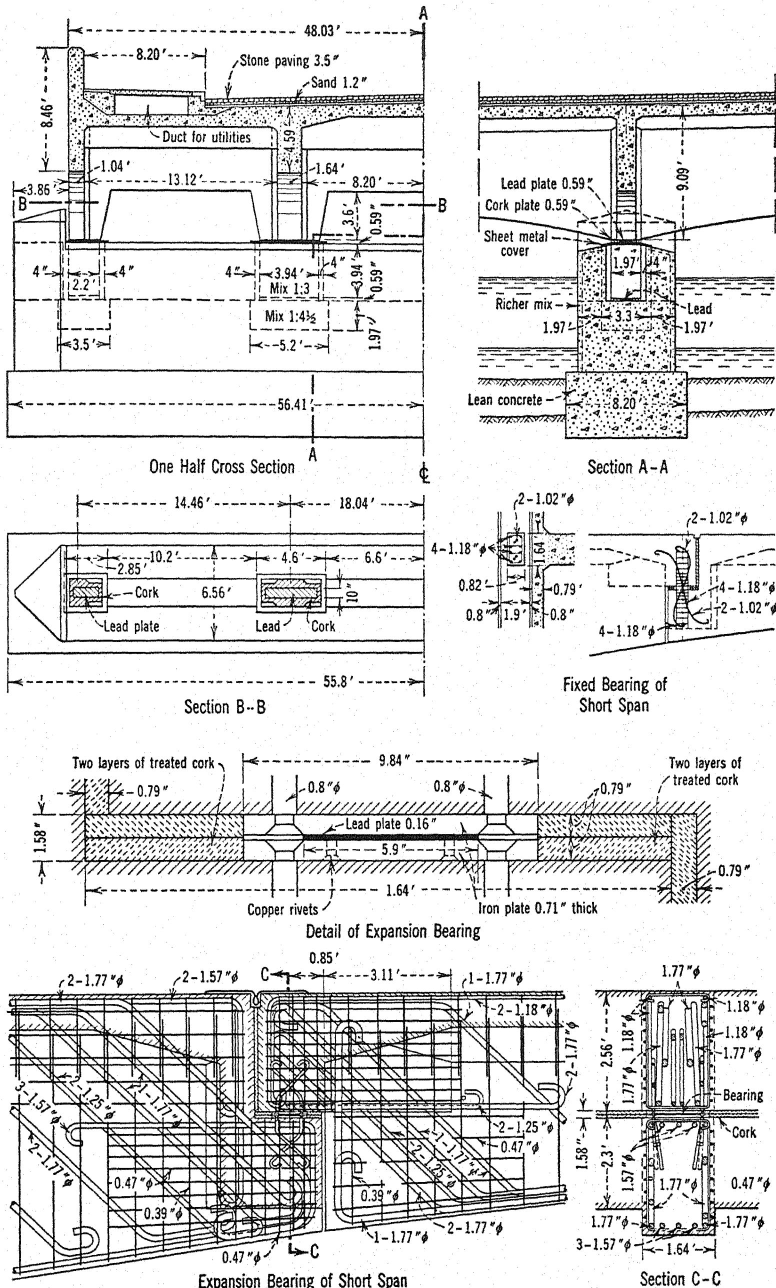
From Beton und Eisen, 1929.

FIG. 61.—Cantilever Bridge at Lindau, Germany. (*See p.* 144.)

opening. Therefore, in the longitudinal section the structure consists of two girders with one cantilever, two girders with two cantilevers, and three short spans supported by cantilevers. The length of cantilevers is 15.6 ft., and of the short spans 39.5 ft. The ratio between the length of the cantilever and the length of the interior span is 0.22. (See Figs. 61 and 62.)

In cross section, the bridge consists of two interior and two exterior girders. The exterior girders extend above the top of the sidewalk and serve as railings of the bridge. Interior girders are 4.59 ft. deep in the center, and 9.09 ft. at the supports, giving ratio of minimum depth to

[4] For full description see *Beton und Eisen*, 1929, p. 84.



From Beton und Eisen, 1929.

FIG. 62.—Cantilever Bridge at Lindau, Germany. (*See p.* 144.)

span of 1 : 15.4; and a ratio of minimum to maximum depth of girder of 1 : 1.98. The underside of the girders is segmental. The floor system consists of slabs reinforced in two directions, which makes possible the use of a slab 10.2 in. thick for the wide spacing of the girders and for the heavy loading. (German Class I loading was used in this design.) The bridge is illustrated in Fig. 61, p. 144 and Fig. 62, p. 145.

The cross section of the bridge is of interest. The curbs were poured after the forms were removed. The sidewalk slabs are removable. Space is provided under the sidewalks for pipes and ducts.

Each main girder has at one end a fixed bearing and at the other end an expansion bearing consisting of concrete rockers, more fully described on p. 382.

Each suspended span is provided at one end with an expansion bearing consisting of two iron plates, each 0.71 in. thick, one anchored to the bottom of the girder bracket and the other to the top of the cantilever bracket. Between these plates, a 0.16-in.-thick lead plate is placed and riveted by copper rivets to the bottom plate. This arrangement permits both longitudinal and rotating motions of the end of the suspended span. Design of the fixed bearing at the other end is shown in Fig. 62. It permits small rotating motions of the girder end.

At the ends of the bridge, abutment walls are connected with the girders, which produces a degree of restraint at the ends of the end girders.

Details of girder reinforcement are shown in the figure. Since, according to the German practice, all diagonal tension is resisted by web reinforcement, stirrups extend the whole length of the span.

Bridge across the Danube at Thalfingen.[5] — This bridge is an example of a cantilever through bridge; it consists of three openings: 79.1 ft., 110.9 ft., and 79.1 ft. long. The supports of this bridge rest on piles; therefore, cantilever design was used in preference to a continuous girder design.

In longitudinal section, two one-span end girders, each with one cantilever, were used, and the center opening is taken care of by two 29.5-ft. cantilevers and by the 51.9-ft. suspended span. The ratio of the cantilever length to the length of the center span is 1 : 3.75. The ratio of the depth in the center to the span length is 1 : 14.1; and the ratio of minimum to maximum depth of girder is 1 : 1.6. The width of the girder is 1.64 ft. in the central portion, and at the supports the width below the slab was increased to 2.3 ft. by widening the stem on the inside, while the exposed outside surface was kept plane. The portion of the girders extending above the roadway serves as railing.

[5] See Nakonz, "Einige neuere Ausführungen grösserer Eisenbetonbrücken," *Bau-Ingenieur*, 1928, p. 463.

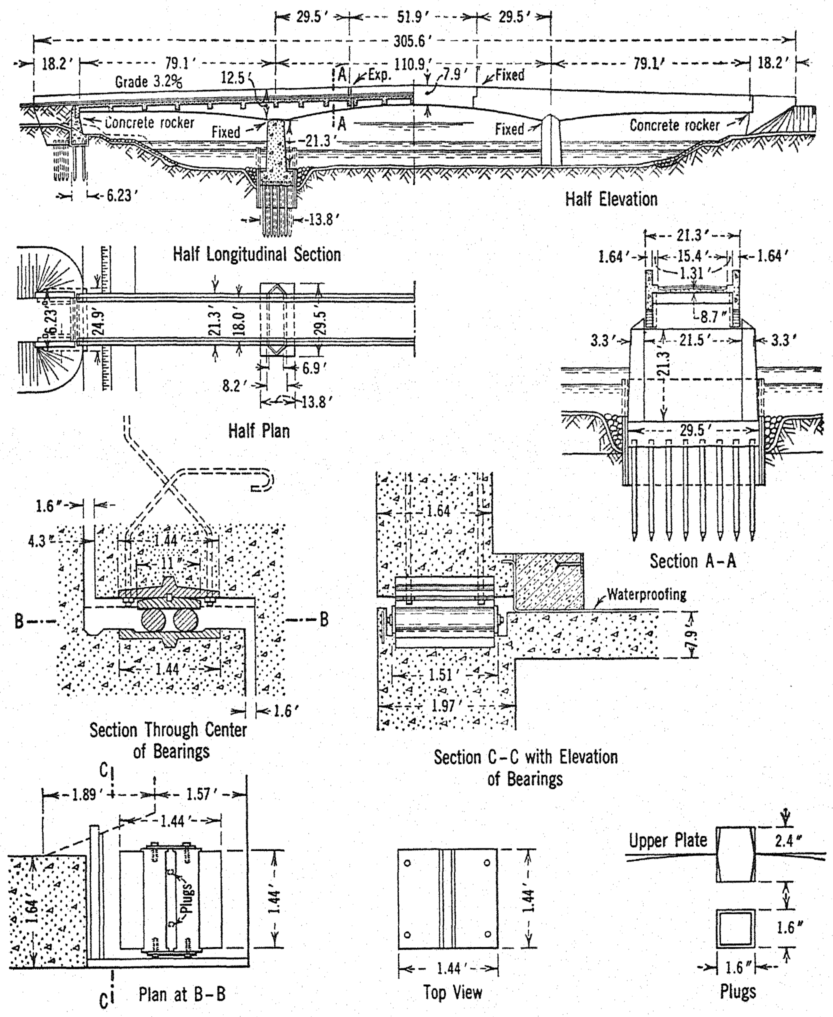


FIG. 63.—Bridge across Danube at Thalfingen. (*See p.* 146.)

The main girders have fixed bearings at the piers and are provided with rockers at the abutments. The suspended span is provided at the expansion end with a roller bearing shown in the figure.

CHAPTER VIII

DESCRIPTION OF CONTINUOUS GIRDER AND MULTI-SPAN FRAME BRIDGES

In previous chapters, structures have been considered in which the main girders are statically determinate. Such structures, while relatively easy to design, present very seldom the most economical solution of the problem.

For a great majority of cases, much more economical structures are obtained by making the main girders continuous and, where possible, connecting them with their vertical supports to form rigid frames. Such designs are not only more economical but also they are lighter and more rigid. The underside of the girders in each span may be arched, thus contributing further to the economy and giving the structure the graceful appearance of a series of arches.

The purpose of this chapter and those which follow is to acquaint the designer fully with these economical types of bridge design by discussing fully the design details, supplying comparatively simple rational formulas for design and finally illustrating them by numerical examples. The design formulas, while derived by means of higher mathematics, in their final form consist only of simple, algebraic expressions.

The formulas and methods given in Chapters IX to XII are not restricted to bridge design. They will be found useful in the design of buildings and other structures where the spans of girders are unequal, where the loadings are unusual, or where for any reason the use of rational design method is desired.

The design details and the suggestions as to arrangement and bending of reinforcement also can be adapted to any type of reinforced concrete structure.

This chapter, then, is devoted to the description of continuous girder bridges and multi-span frame bridges. The term " continuous girder bridges " designates structures consisting of a number of longitudinal girders extending over two or more spans, in which the floor system is not connected with the supports. Strictly speaking, a continuous girder should be provided with expansion bearings over all supports but one. However, girders running into floor beams, and girders connected by

dowels with the supporting columns, but not rigidly connected with them, also may be classed as continuous girders. (See Fig. 73 (*b*) and (*c*), p. 164.)

The term " multi-span frame bridges " applies to structures consisting of a number of parallel longitudinal girders, each of which is rigidly connected with the supporting columns or piers.

Since these two types of bridges have many characteristics in common, particularly so far as the design details of the floor system are concerned, their description and examples from practice are given in one chapter. Each type, however, requires different design formulas for determining bending moments and shears; therefore, formulas for continuous girders are treated in Chapters IX and X, and those for multi-span frames in Chapters XI and XII. One-span rigid-frame bridges are treated separately in Chapter XIII. Continuous flat slab bridges are also treated in a separate chapter.

Use of Continuous Girder Designs. — Continuous girder and multi-span frame designs are used to a great extent for complete bridges, for floor construction of arch bridges with open spandrels, and for approaches to long-span arches and trusses. Span lengths ranging from 30 to 100 ft. are common, while long-span girder bridges have been built with spans up to 202 ft. (See p. 2.) A design of a girder bridge with an unprecedented span length of 347.8 ft. won one of the prizes in the competition for bridge designs across the Rhein at Basel. Although the bid based on this design was somewhat higher than the low bid for the adopted steel design, the difference in cost was not sufficiently large to discourage similar attempts in the future. (See p. 154.)

Advantages of Continuous Girder Designs for Bridges. — Continuous girder bridges not connected with supporting piers have the following advantages over simply supported girder designs.

1. They require a smaller amount of steel and concrete. This reduces the cost materially below that of simply supported structures.

2. A much smaller ratio of the depth at the center of span to the span length may be used than is possible with simply supported spans. This is of great importance where the available headroom is restricted.

3. They require fewer bearings. At any pier only one bearing is needed for two adjoining spans of a continuous girder; for simply supported girders two bearings are required at each pier. Therefore narrower piers are possible for continuous girder bridges.

4. They require fewer expansion joints, because with continuous girders expansion joints are required only at the ends of each group of spans, whereas with simply supported designs expansion joints are required at the expansion end of each span.

5. The reactions of the continuous girder are transmitted centrally to the pier because the bearing is placed on the center line of the pier.

6. The deflection and vibration of continuous girder bridge are less than for simply supported designs.

Advantages of Multi-Span Rigid Frame Bridges. — In addition to the advantages just enumerated, multi-span rigid frames have the following advantages over simply supported designs which they do not share with the continuous girder designs.

7. Multi-span frames do not require any bearings at the supports.

8. Owing to rigid connections between the vertical supports and the horizontal members, the stability of the vertical supports in rigid frames is much greater than that of independent piers.

Disadvantages of Continuous Girders and Multi-Span Frames. — Continuous girders and rigid frames have the following disadvantages in comparison with simply supported girder designs.

1. Uneven settlement of foundations has a bad effect upon the strength of continuous girders as well as of rigid frames. Therefore, structures of these types should not be used where fairly unyielding foundations cannot be obtained at reasonable cost. The effect of expected small yielding of the foundations, however, can be provided for in the design of continuous girders.

2. Placing of reinforcement in continuous girders is somewhat more complicated than in simply supported spans. This objection, however, disappears where workmen experienced in placing reinforcing steel are obtainable.

3. The sequence of placing concrete in continuous girders must be more carefully worked out than for simply supported girders. Also more care is required when removing formwork.

4. Continuous girders and rigid frames are statically indeterminate structures, and their design is more complicated than that of simply supported girders. This disadvantage disappears where the design of the structure is in the hands of a competent engineer. The methods of design given in this volume can be used without difficulty.

Continuous Girders vs. Multi-Span Frames. — The following discussion will be helpful in deciding whether to use a continuous girder design or a rigid-frame design.

Continuous girders not connected with the supporting piers should be used: (1) Where the structure is to be supported on already existing massive piers. (2) Where the nature of the structure requires heavy piers having considerable stability. No attempt should ever be made to connect rigidly the superstructure to heavy substructure, because such piers are not elastic enough to form an integral part of a rigid frame.

In other instances, either continuous girders or rigid frames may be used, and the selection is governed by economy. For instance, in countries where stone is abundant and cheap, and mason labor inexpensive, it may be more economical to use masonry piers; and as a consequence continuous girders with masonry piers may be preferable to rigid-frame designs which would require vertical supports of reinforced concrete.

Where reinforced-concrete vertical supports are used, and no mass is required for their stability, rigid frames may be found preferable. They are particularly desirable where the available width for vertical supports is small, as in crossings over railroad tracks. The stability of vertical supports is much greater when they are connected rigidly with the horizontal members.

The effect upon bending moments of rigid connections between the girder and the vertical supports is clearly evident from the discussion on p. 220.

Arrangement of Spans. — It is of advantage to adopt a symmetrical arrangement of spans in a multi-span continuous structure because this simplifies not only the design but also the construction of the structure. In structures consisting of two spans, preferably both spans should be equal. In three- and four-span structures, it is often advisable to make the end spans somewhat shorter than the interior spans. This is particularly desirable with continuous girders where otherwise the bending moments in the end spans would be appreciably larger than in the interior spans. Compare bending moments in continuous girders of equal spans with bending moments in the example on p. 210, where the end spans are 20 per cent shorter than the center span.

Use of Longitudinal Cantilevers. — Longitudinal cantilevers of continuous girders and rigid frames serve the following purposes. They furnish restraint for the end spans, thus equalizing to some extent bending moments in the interior and in the exterior spans. Also cantilevers may be used to replace heavy abutments, thus reducing the cost of the structure.

Where it is desirable to get an appreciable restraint at the end supports, counterweight cantilevers may be used. (See Fig. 67, p. 156.)

Arrangement of Longitudinal Girders. — In cross section, a continuous deck girder bridge, as well as a deck rigid-frame bridge, is similar to simply supported girder bridges described in Chapter V; and the floor of such structures should be designed in the same manner as outlined in that chapter. Three types of floor design are most common: (1) floor slab spanning between closely spaced longitudinal girders; (2) floor slab

supported by closely spaced floor beams spanning between longitudinal girders; (3) floor slab reinforced in two directions and supported by longitudinal girders and by widely spaced floor beams. In type 3 the floor beams are usually spaced so as to give square or nearly square panels of the slab. The advantages of this type of floor construction are discussed on p. 88.

In continuous through girder bridges and through rigid frames, the floor design is the same as explained in Chapter VI in connection with simply supported through girder bridges.

Main Girders of Continuous Girder Bridges. — The depth of continuous girders may be made much smaller than the depth of simply supported girders for the same spans and loadings, because the positive bending moments for continuous girders are appreciably smaller than for simply supported spans. The depth in the central portions of the spans may be still further reduced by using girders with variable moments of inertia, but in such designs the depths at and near the supports must be correspondingly increased.

In deck girder designs, the cross section of the main girders in the center of each span is usually a T-beam, the floor slab forming the compression flanges. No difficulties are there encountered so far as compression stresses are concerned. At the support, however, unless other provisions are made, the girder in effect is a rectangular beam. The depth of the girder used in the center is usually not sufficient to take care of the large compression stresses produced by the negative bending moments at the supports, and it may be necessary to resort to one or several of the following means of increasing there the compression resistance of the girder. Compression reinforcement may be used; the depth of the girder at the support may be increased either by introducing haunches or by making the underside of each span of the girder parabolic or segmental; the width of the girder stem may be increased, with or without a simultaneous increase in the depth; finally, a bottom slab may be used as shown in Figs. 65, p. 154, and 66, p. 155. In Fig. 64, p. 152 the depth of the girder was increased from 2 ft. 6 in. in the center to 9 ft. at the supports. Also the width of stem of the girder was increased gradually from 16 in. to 24 in.

The bottom slab used for resisting compression stresses due to negative bending moments may extend only for short distances at each side of the support; or it may extend the whole length of the bridge, in which case the cross sections of the girders are cellular. The thickness of the bottom slab is usually largest at the supports and decreases toward the center of each span. In the cellular design, no cross beams are necessary because the bottom flange braces the girders satisfactorily. It has been

found by tests that cellular designs are very effective in distributing concentrated wheel loads over the whole width of the bridge. In river crossings the bottom slab protects the girder stems from injury by driftwood and ice floes.

Girders with Constant Moments of Inertia. — Moments of inertia of a girder may be considered as constant when the depth of the girder and the width of its stem are constant. Short and shallow haunches at

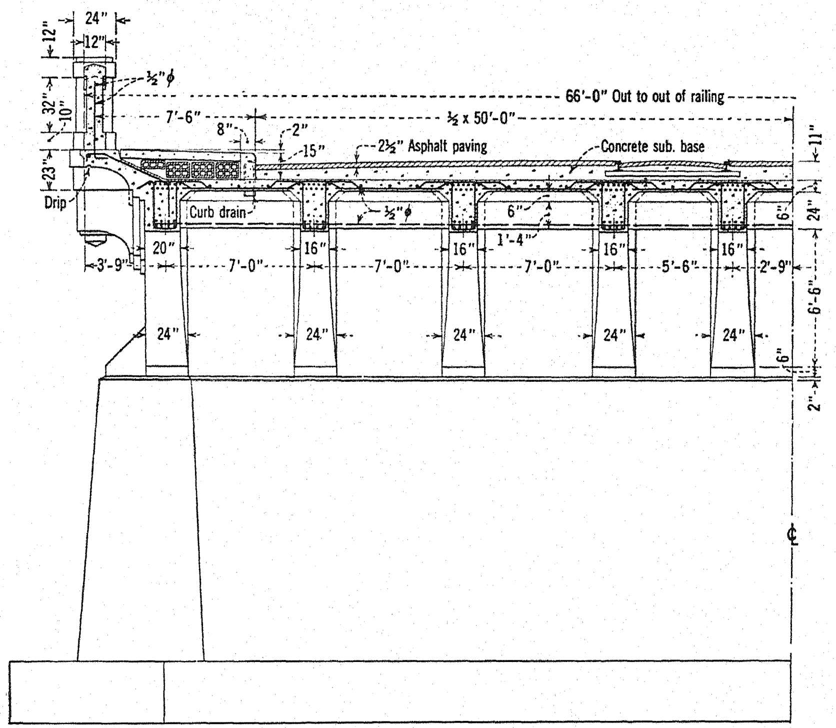


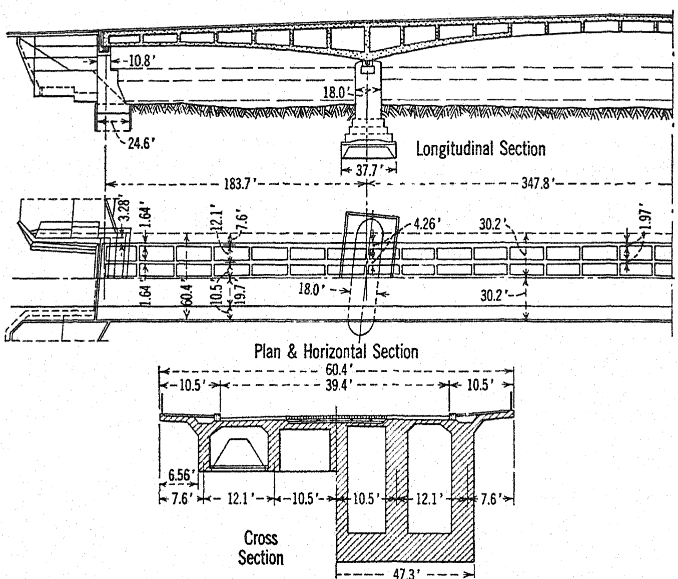
FIG. 64.—Cross Section across Kalamazoo River, Michigan. (*See p.* 152.)

the supports do not affect the bending moments in a girder to any considerable extent, so that the girder may be considered in design as having constant moments of inertia.

Girders with Variable Moments of Inertia. — When a girder is provided with large haunches or with a widened girder stem, when the intrados of the girder is curved, or when a bottom slab is used to resist compression stresses, the moments of inertia of the girder should be considered as variable. The variation of the moments of inertia is determined as discussed on p. 208.

EXAMPLES OF CONTINUOUS BRIDGES

Examples from practice of continuous girder bridges not connected with the supports have been selected so as to cover the whole field. Some of the structures are provided with longitudinal cantilevers, which in two cases form heavy counterweights to get greater restraint at the ends.



From Beton und Eisen, year 1931.

FIG. 65.—Prize Design for Bridge across Rhein at Basel. *(See p. 154.)*

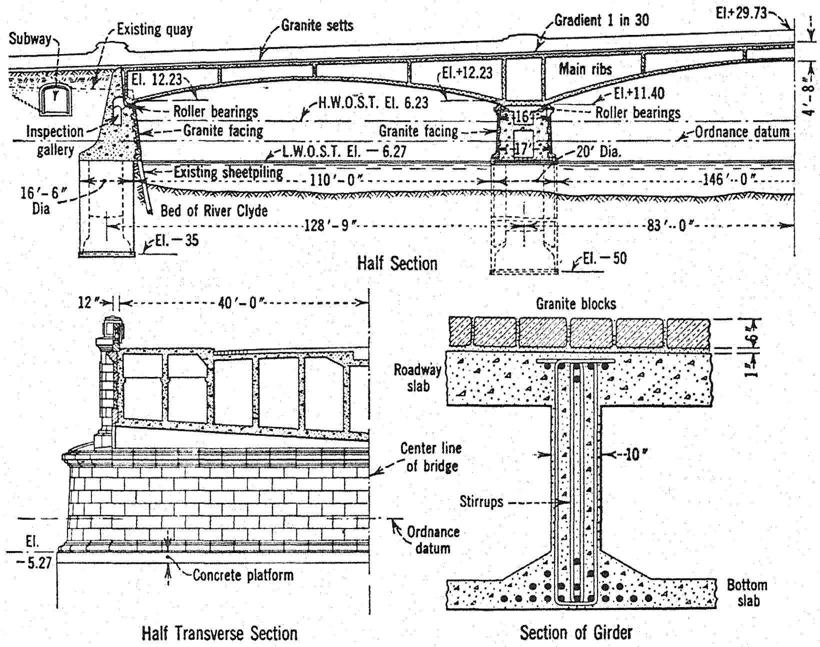
Prize Design of Bridge across Rhein at Basel.[1] — This design consists of three spans, the end spans of which are 183.7 ft. long, and the center span has the unprecedented length of 347.8 ft. The girders have variable moments of inertia, with a ratio of minimum to maximum moment of inertia of $\frac{1}{43}$. The compression zone of the girders is strengthened by bottom slabs, which at each end of the girder extend the whole length of the end span and for 80 ft. into the center span. The girders are provided with fixed bearings at one pier and with concrete rocker bearings at all other supports.[2] See Fig. 65, p. 154.

[1] This design was prepared by Professor E. Mörsh and received third prize in the competition held for selecting a design for this structure.

[2] For full description see: A. Bühler, "Der internationale Wettbewerb zur Erlangung von Entwürfen und Angeboten für eine neue Strassenbrücke über den Rhein," in *Beton und Eisen*, 1931, p. 226.

Herval Bridge across Rio do Peixe, Brazil. — This bridge consists of three spans, 88 ft., 224 ft., and 88 ft. long. The end spans are anchored to the abutments to take care of uplift. The depth of girders at the center is 5 ft. 7 in., giving a ratio of depth to span of 1 : 40. At the supports the girders are 13 ft. 4 in. deep.

In addition to the unusually long span, the bridge is remarkable for the method of erection there employed. The center span was erected by the cantilever method in 5-ft. sections.[3] After erection of the girders, a rigid connection was effected between the girders and the piers, changing the structure to a rigid frame.



Scott, Chief Engineer.

FIG. 66.—Oswald Street Bridge at Glasgow. (*See p.* 156.)

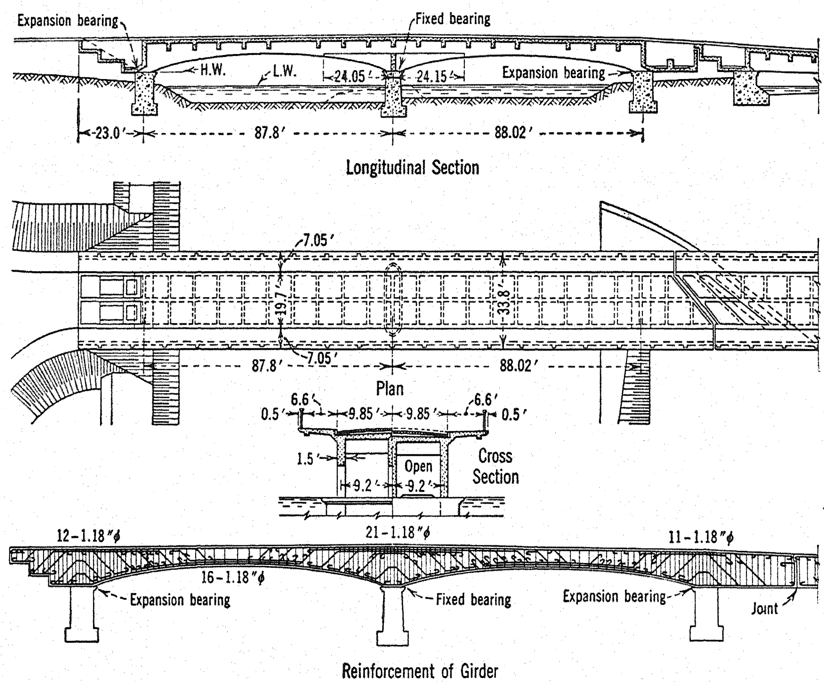
New Oswald Street Bridge, Glasgow, Scotland.[4] — This bridge has one of the longest spans for which reinforced-concrete continuous girders have been used. It consists of two equal end spans, 128 ft. 9 in. long each, and of a center span 166 ft. long. The depth of the girders at the center is 4 ft. 1 in., giving a ratio of depth to span of 1 : 40.

[3] For description and illustration, see: Rolf Schjodt, " Long Rigid Frame Erected by Cantilever Method," *Engineering News-Record*, 1931, p. 208.

[4] For description of this bridge, see *Engineering News-Record*, Vol. 95, p. 536. The construction of the bridge is described in *The Engineer* (London), 1926, p. 62.

As evident from the illustration, the girders are of cellular design. Attention is called to the small width of the girder stem and to the heavy stirrups. The bridge is illustrated in Fig. 66, p. 155.

Continuous Bridge of Two Spans with Concealed Counterweighed Cantilevers.[5] — In the two-span girder bridge across the Murg at Gernsbach, Germany, shown in Fig. 67, p. 156, counterweighed cantilevers



From Beton und Eisen, year 1928.

FIG. 67.—Highway Bridge across the Murg. *(See p. 156.)*

at both ends provide end restraint which reduces the positive bending moments in the girders. The fixed-point method was used in determining the bending moments.

In cross section, three longitudinal girders are used, spaced 9 ft. on centers. The floor beams are spaced 8.9 ft. on centers, and the slab is reinforced in two directions. The sidewalk slab is cantilevered out. The depth of the girder in the center is 5.0 ft., giving a ratio of depth to span of 1 : 17.5.

[5] See W. Stortz, " Die Strassenbrücke über die Murg bei Gernsbach. *Beton und Eisen*, 1928, p. 344.

Hindenburg Bridge across Nagold River at Pforzheim. — An interesting example of the use of counterweighed cantilevers in connection with continuous girders is furnished by the design shown in Fig. 68, p. 157. The long span of this bridge is 145.6 ft., and it is continuous on one end with a short 70.9-ft. span, and on the other end it is provided with a counterweighed cantilever 35.1 ft. long.

The structure is cellular in cross section. The interior girders of the long span are 5.7 ft. deep, giving a ratio of depth to span of 1 : 25.6. At the supports the girder is 9.8 ft. deep, and its width is increased from 1.3 to 2.3 ft. in the compression zone.

Each girder is provided with a fixed bearing at the end next to the cantilever, and with reinforced-concrete rockers at the other supports.[6]

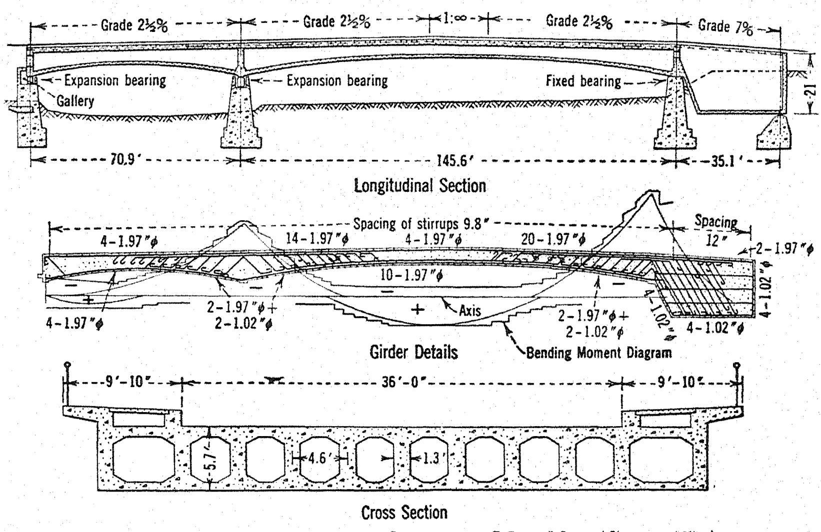
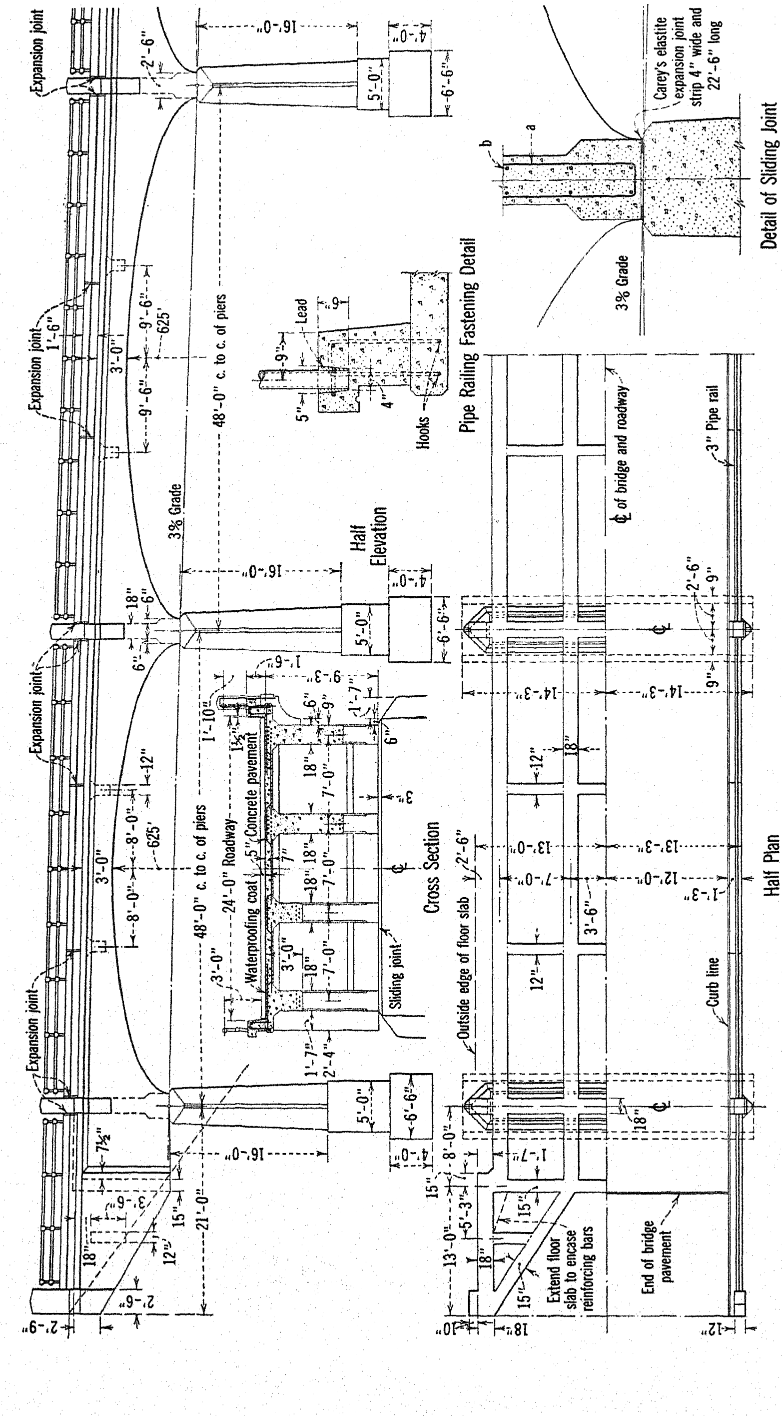


FIG. 68.—Hindenburg Bridge across Magold. *(See p. 157.)*

Salmon River Bridge at Port Ontario, New York. — This structure, illustrated in Fig. 69, p. 158, consists of four spans, of which the end spans are 48 ft. long each, and the interior span 55 ft. long. At each end 21-ft. cantilevers are used, which in conjunction with the end pier replace the abutment.

The minimum depth of the girders is 3 ft., giving a ratio of depth to span of 1 : 18.4. At the supports the girders are 9 ft. 3 in. deep, and the underside of the girder is curved, giving the pleasing appearance of an arched bridge. In design the girders were considered as having variable

[6] See *Beton und Eisen*, 1931, p. 41.



Concrete Steel Engineering Co.

FIG. 69.—Salmon River Bridge. (See p. 157.)

moments of inertia, and the fixed-point method was used for determining the bending moments.

At the supports, the girders are braced by heavy cross beams extending to the bridge seats. Two intermediate cross struts are used in each span. The girders are anchored to the center pier, and at all other piers they are provided with sliding bearings.

The unusually deep curbs anchored by U-stirrups to the slab were built after the structural concrete had hardened and the formwork had been removed. Expansion joints in the curbs are provided at each pier and at two intermediate points in each span. The railing consists of 3-in. iron pipe. The end posts and the posts at each pier are of reinforced concrete, but they are not connected with the railing.

The concrete pavement, placed over a waterproofing coat, is 5 in. thick at the crown and 4 in. at the curbs. Expansion joints, provided in the pavement along the curbs, are filled with poured waterproofing.

APPROACHES TO LONG-SPAN STRUCTURES

Reinforced-concrete continuous girder bridges are often used for approaches to long-span bridges such as trusses and arches. Two such designs are here illustrated.

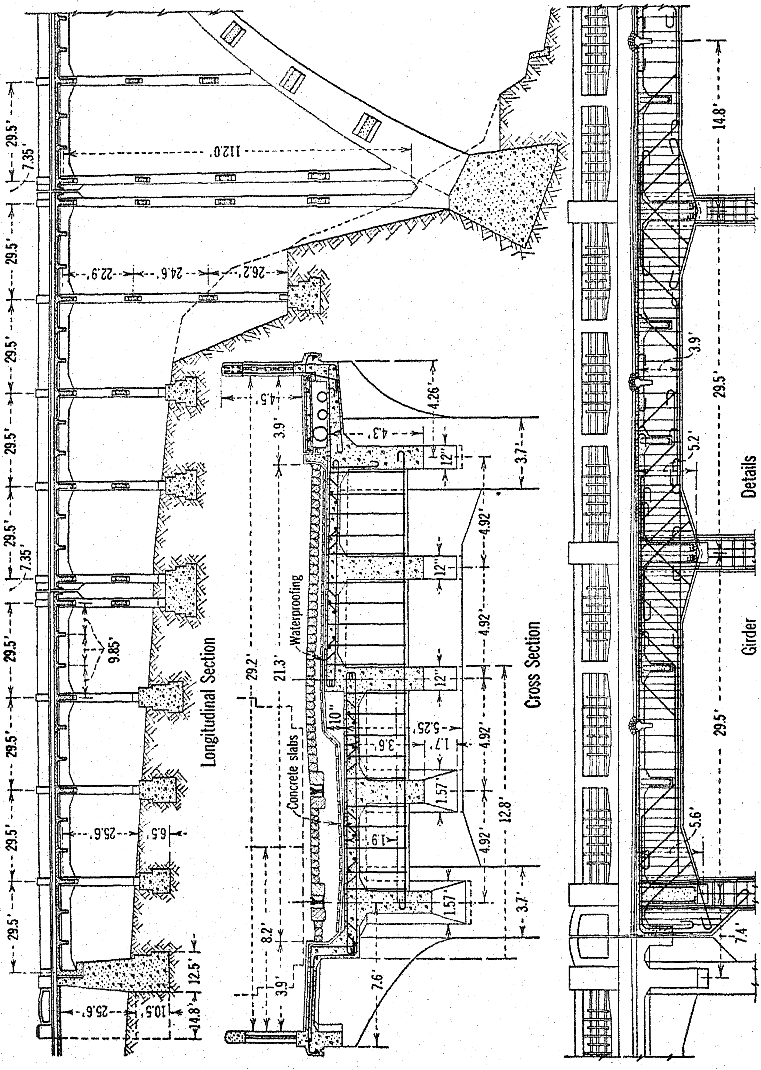
Approaches to Arch Bridge across Luznitz River. — The design shown in Fig. 70, p. 160, was made by Dr. Ing. Eduard Victora of Prag. The left approach to the arch bridge consists of two four-span rigid frames; the right approach is composed of one four-span and one two-span frames. At each expansion joint between the adjoining frames double columns are used as shown in the figure, in which the left approach is illustrated.

The spans of the approaches, as well as of the floor system of the open spandrel arch, are 29.5 ft. long. This constant span length was adopted for architectural reasons. The vertical supports consist of two column bents connected at the top by a heavy cross beam. The heights of the bents vary from 26 to 125 ft.

The cross section of the bridge is unsymmetrical because one half of it is designed for highway traffic only, the other half carrying a railroad track. To take care of the specified ballast under the ties, the top of the concrete slab under the tracks was depressed.

The design was worked out by considering the construction as a rigid frame. The effect of temperature changes as well as of the unequal compression of the vertical supports was taken into account in design. The reinforcement of the girders is shown in the figure.

Approaches to Pulaski Bridge in Oswego County, New York. — The design shown in Fig. 71, p. 161, was used as the left approach to a one-span arch bridge over the Salmon River.



Dr. Ing. Eduard Victora, Engineer.

FIG. 70.—Approaches to Arch Bridge across Luznitz River. (*See p.* 159).

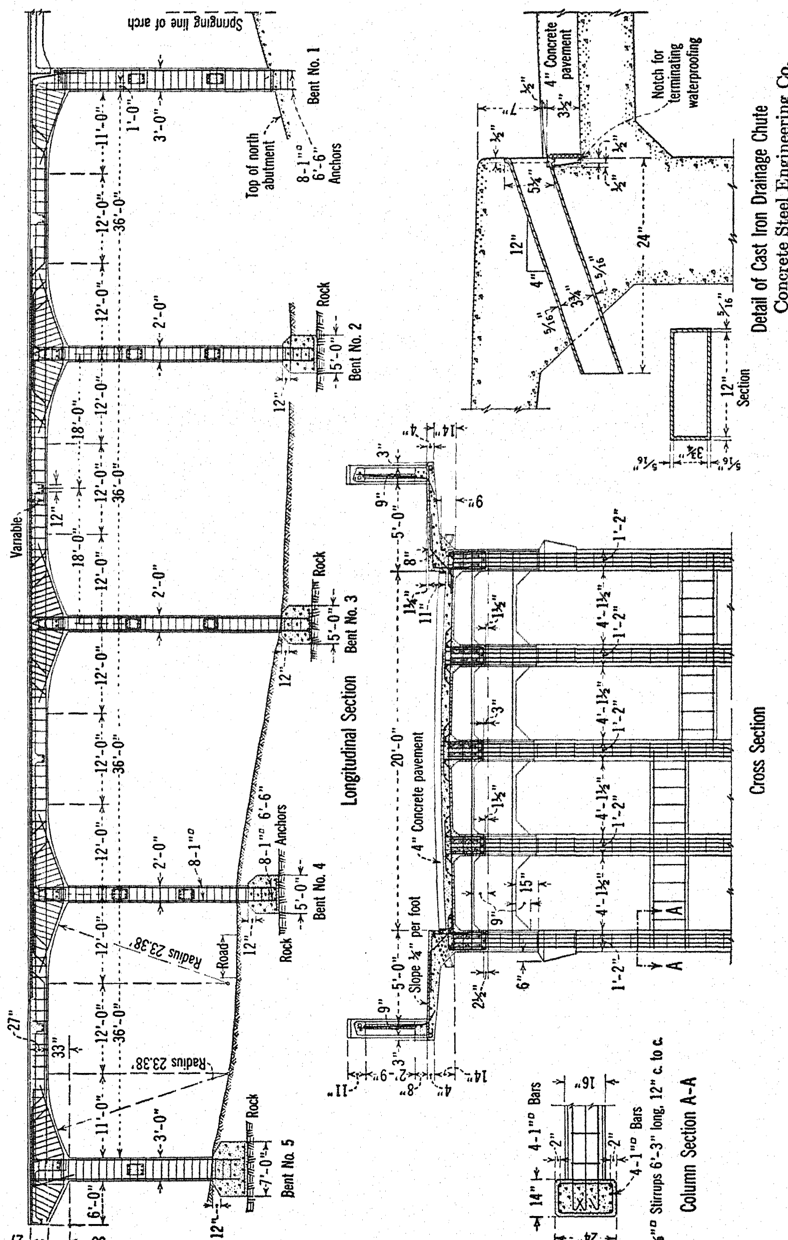


FIG. 71.—Approaches to Pulaski Bridge. (*See p.* 159.)

In design, the structure was considered as a rigid frame, and the fixed-point method was used to determine the bending moments. The moments of inertia of the girder were considered as variable, because in each span the girders are provided with deep segmental haunches.

MULTI-SPAN RIGID FRAMES

Overhead Bridge at Olivia, Minn. — The structure built by the C. M., St. P. & P. R. R. was designed as a rigid frame of three equal 42-ft. spans. The bridge consists of five longitudinal rigid frames spaced 7 ft. on centers. The end vertical members are imbedded in the fill, and in conjunction with an 8-ft. deep apron they take the place of the abutments. See Fig. 72, p. 163.

The interior girders are 1 ft. 3 in. wide and 2 ft. 10 in. deep in the center of the span, so that the ratio of depth to span is 1 : 14.9.

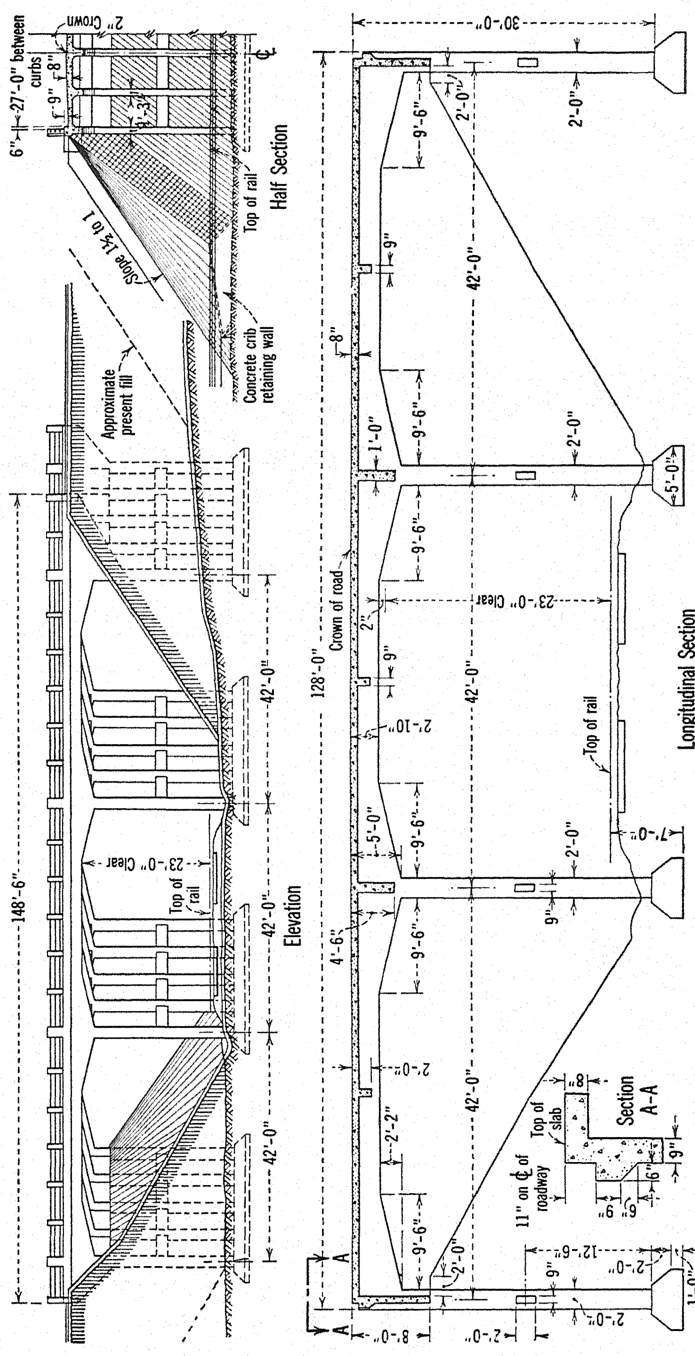
CONTINUOUS THROUGH GIRDER BRIDGE

Continuous Through Girder Bridge across River Murg. — A good example of a continuous through girder bridge is the railroad bridge across the River Murg designed by Dr. Ing. K. Schaechterle.[7] It is a one-track structure of five spans of which the two short end spans are simply supported, while the three interior spans form a three-span continuous girder with spans 41.2 ft., 49.9 ft., and 41.2 ft., all measured between the center lines of the piers. In cross section the bridge consists of two through girders spaced 11.2 ft. between their inside faces, and a solid slab spanning between them.

The depth of each girder is 6.2 ft., and its width is increased at the top to 3.6 ft. The ratio of depth to span is 1 : 8. The girders were designed as continuous girders with free ends by a method similar to those given in this volume.

Each girder is supported by independent columns, reinforced by vertical bars and by closely spaced hoops. The foundation of each column is carried to rock. The girder is anchored at one support and is supplied at all other supports with sliding bearings consisting of two plates, of which the bottom plate has a curved area of contact permitting rotating in addition to sliding motions.

[7] See Dr. Ing. K. Schaechterle, "Zwei Bahnbrücken in Eisenbeton über die Murg," *Beton und Eisen*, March 20, 1926, p. 109.



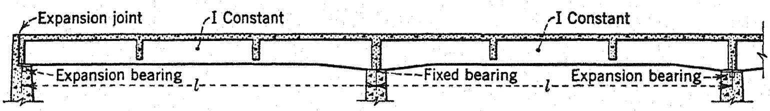
Chas. W. Bainbridge, Engineer of Design.

FIG. 72.—Highway Bridge across C. M., St. P. & P. R. R. Tracks. (*See p.* 162.)

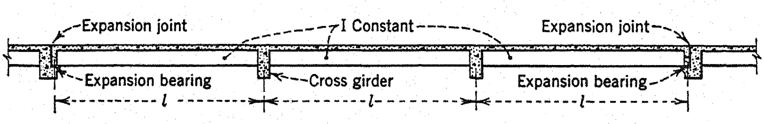
CHAPTER IX

DESIGN OF CONTINUOUS GIRDERS OF EQUAL SPANS

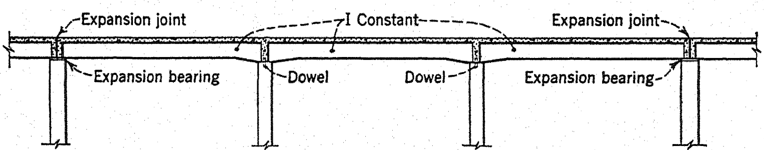
Girder bridges in which continuous longitudinal girders are used are described and illustrated in the previous chapter; this chapter gives information and formulas needed for the determination of dimensions and of the required amount of reinforcement for continuous girders of two, three, and four equal spans with free ends. The information in this chapter is sufficient for logical and safe designs of continuous girders of equal spans with free ends. For other arrangements of spans, and different conditions of restraint at the ends, the method given in Chapter X should be followed.



(*a*) *Girders on Independent Piers*



(*b*) *Girders between Cross Girders*



(*c*) *Girders Dowelled to Columns*

FIG. 73.—Types of Continuous Girders of Equal Spans. (*See p.* 164.)

Types of Continuous Girders. — Strictly speaking, formulas and rules here given apply only to continuous girders which rest on piers without being connected with them, and which are provided with expansion bearings at all supports but one. Such an arrangement is shown in Fig. 73 (*a*), above.

For the two conditions shown in Fig. 73 (*b*) and (*c*), the use of formulas in this chapter, though not exact, is accurate enough for practical purposes. In the first case, where the girders run into cross beams, the torsional resistance of the cross beams affects to some extent bending moments in the girders; and its effect increases with the increase in the stiffness of the cross beams. Ordinarily this effect may be neglected. In the second case, the girders are doweled to the columns by short dowels placed so as to permit small angular movements of the girders at the supports. Here, also, bending moments in the girders are affected by the connection of the girders and the columns, but the effect is negligible.

Assumptions. — All formulas given in this chapter are derived from the well-known three-moment equations[1] and are based on the assumption of constant moments of inertia. These formulas may be used where haunches, if any, are shallow and do not exceed in length 0.2 of the span length. For girders with variable moments of inertia, the fixed-point method given in Chapter X should be used.

How to Design Continuous Girders of Equal Spans. — To design a continuous girder bridge, proceed as follows.

Decide upon the span lengths.

Select arrangement of girders in cross section of bridge as explained on p. 44. Design the floor system. Compute all dead loads carried by the girder. Assume the weight of girder stem. Compute total unit dead load of girder.

Find equivalent uniformly distributed live load, including impact, as explained on p. 27. Equivalent unit live load for bending moments may be different from that for shear.

Using bending-moment and shear coefficients for the appropriate number of spans from the table on p. 166, compute bending moments and shears, separately for dead load and for live load. Combine bending moments for dead and live loads. The sum of bending moments at each critical section is used to determine the main dimensions of the girder and the maximum amounts of reinforcement.

Length of bars and points of bending of reinforcement are determined according to the combined bending-moment diagram for the girder. Shear diagrams should be used to determine the spacing of the web reinforcement.

The design of a continuous girder bridge of equal spans is illustrated by a numerical example on p. 173. A separate example is given on p. 186 for a bridge with cantilevers.

[1] See "Concrete, Plain and Reinforced," Vol. II, p. 17.

TABLE I

CONTINUOUS GIRDERS OF EQUAL SPANS, FREE ENDS

MAXIMUM BENDING MOMENTS AND END SHEARS

w_d = unit dead load; w_l = unit live load; l = span

Location	Mark	Two Equal Spans		Three Equal Spans		Four Equal Spans	
		Dead Load	Live Load	Dead Load	Live Load	Dead Load	Live Load
				Maximum End Shears			
1st span Left	V_1	$0.375w_dl$	$0.4375w_ll$	$0.4w_dl$	$0.45w_ll$	$0.393w_dl$	$0.446w_ll$
1st span Right	V_{2l}	$0.625w_dl$	$0.625w_ll$	$0.6w_dl$	$0.617w_ll$	$0.607w_dl$	$0.620w_ll$
2nd span Left	V_{2r}	$0.625w_dl$	$0.4375w_ll$	$0.5w_dl$	$0.583w_ll$	$0.536w_dl$	$0.603w_ll$
2nd span Right	V_{3l}	$0.375w_dl$	$0.625w_ll$	$0.5w_dl$	$0.583w_ll$	$0.464w_dl$	$0.572w_ll$
				Maximum Bending Moments			
				Negative at Supports			
2nd support	M_2	$-0.125w_dl^2$	$-0.125w_ll^2$	$-0.1w_dl^2$	$-0.117w_ll^2$	$-0.107w_dl^2$	$-0.121w_ll^2$
3rd support	M_3			$-0.1w_dl^2$	$-0.117w_ll^2$	$-0.071w_dl^2$	$-0.107w_ll^2$
				Positive Bending Moments			
1st span	$M_{1\max.}$	$0.07w_dl^2$	$0.096w_ll^2$	$0.08w_dl^2$	$0.101w_ll^2$	$0.077w_dl^2$	$0.099w_ll^2$
	x_1	$0.375l$	$0.4375l$	$0.4l$	$0.45l$	$0.39l$	$0.446l$
2nd span	$M_{2\max.}$	$0.07w_dl^2$	$0.096w_ll^2$	$0.025w_dl^2$	$0.075w_ll^2$	$0.036w_dl^2$	$0.080w_ll^2$
	x_2	$0.625l$	$0.5625l$	$0.5l$	$0.5l$	$0.64l$	$0.518l$

In girders with three and four equal spans, values in right half are symmetrical with values in left half of girder.

Distances x_1 and x_2 are measured from left support of span under consideration.

When unit loads are in pounds per square foot and the span in feet, the bending moments are in foot-pounds. Multiply coefficients by 12 to change bending moments to inch-pounds.

Formulas for Bending Moments and Shears. — The table on p. 166 gives formulas for bending moments and shears at critical sections for continuous girders of two, three, and four spans, separately for dead load and for live loads.

For dead load, formulas are based on the assumption that all spans are loaded simultaneously with uniformly distributed loading, w_d, per unit of length.

For live load, formulas are based on the most unfavorable positions of the uniformly distributed live load, w_l. For concentrated wheel loads equivalent uniformly distributed loading should be determined before using the table.

Bending-Moment Diagrams. — Bending-moment diagrams are given in Figs. 74 to 76 for continuous girders of two, three, and four equal spans.[2] In each case, separate diagrams are given for dead load and for live load. Also, combined bending-moment diagrams are given for several ratios of the unit live load w_l to the total load $w = w_d + w_l$.

Use of Combined Bending-Moment Diagrams. — Combined bending-moment diagrams are necessary for determining the length and the disposition of the tension and compression reinforcement, and also for determining the length and depth of haunches, when they are governed by bending moments.

For main girders of important structures, it is always advisable to prepare bending-moment diagrams. (See example, p. 177.) For less important members, an experienced designer may be able to determine the positions of the points of bending of reinforcement by reference to the appropriate combined bending-moment diagram, Figs. 74 to 76, using the curves for the proper ratio of unit live load, w_l, to total unit load w. These diagrams may be used in preparing preliminary designs for estimating purposes even for girders of large spans.

Special attention is called to the fact that in bridge design negative bending-moment reinforcement should be extended much farther on both sides of the supports than is ordinarily done in building construction. In some cases, it is even necessary to extend some negative bending-moment reinforcement the whole length of the span, particularly in members with dead load small in comparison with the live load. (See the numerical example on p. 177.)

[2] In the diagrams for four equal spans at intermediate points next to the supports, slightly larger negative bending moments for live loads could be obtained by using partial loadings in which the portions of the span near the points under consideration are not loaded. Such loadings were used for girders of two and three spans. The occurrence of such complicated loadings in a girder of four spans is not likely, hence they were not used in the diagrams for four spans.

To draw a combined bending-moment diagram, proceed as follows:

1. Compute dead-load bending moments at critical sections and at several intermediate sections, using formulas from the table on p. 166 and from proper bending-moment diagrams, pp. 168 to 170.

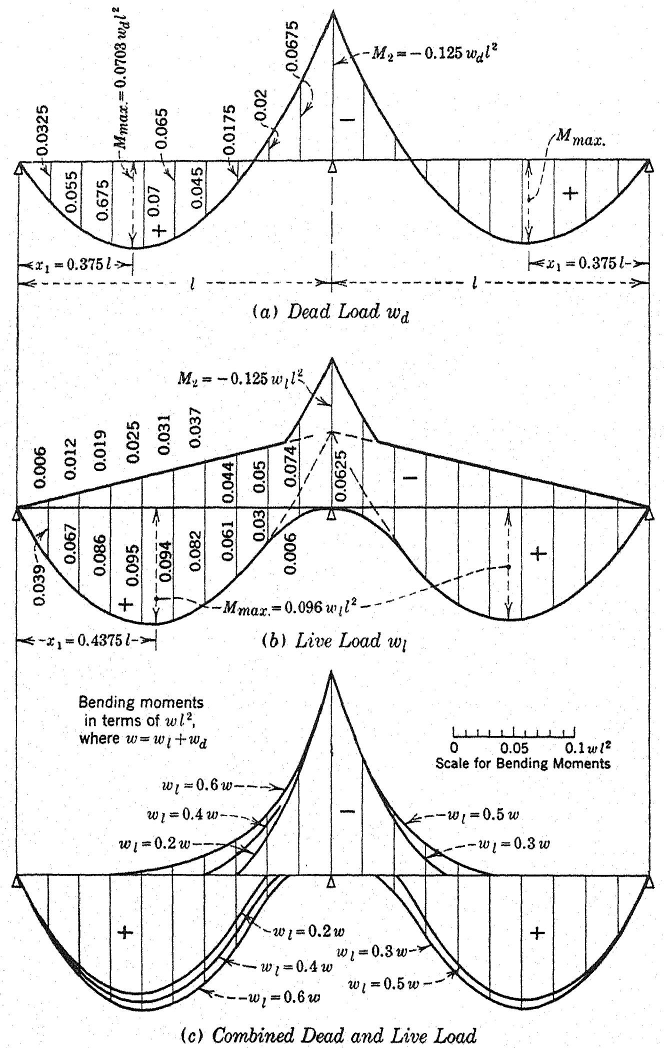


FIG. 74.—Two Equal Spans, Free Ends. Bending-Moment Diagrams. (See p. 167.)

2. Similarly, compute bending moments for live load at the same sections.

3. Combine at each section the values for dead load with the values

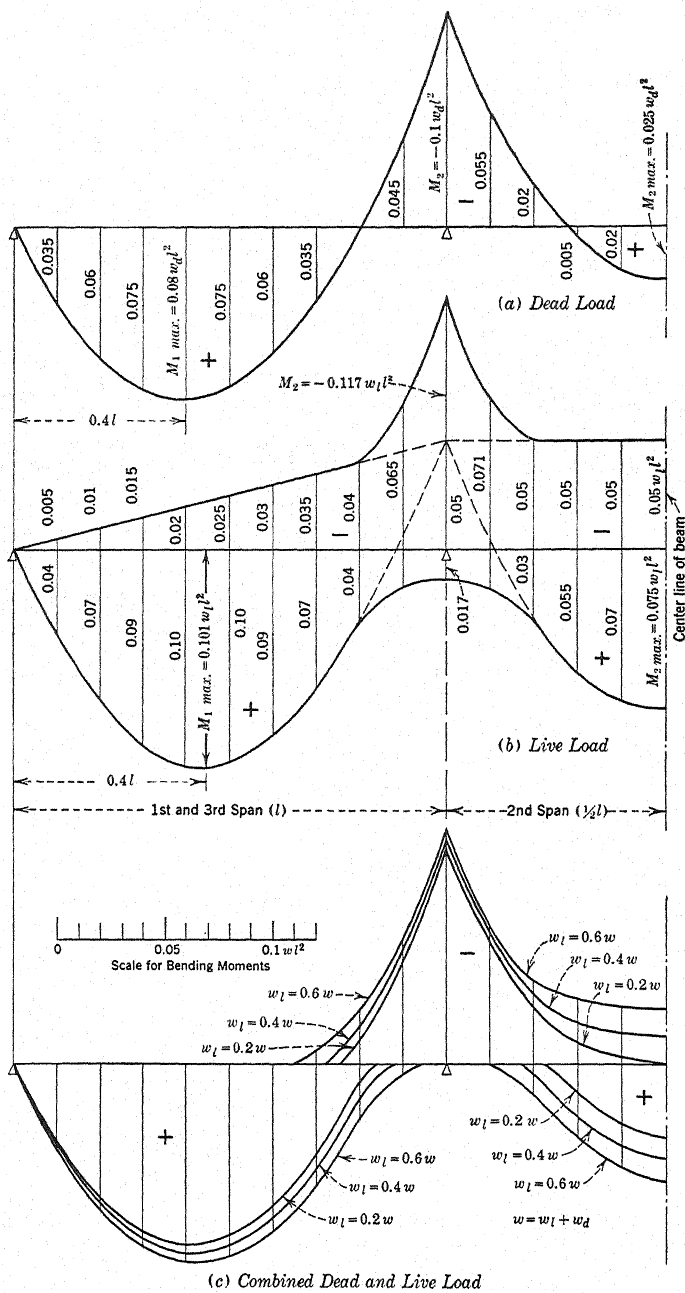


FIG. 75.—Three Equal Spans, Free Ends. Bending-Moment Diagrams. (*See p.* 167.)

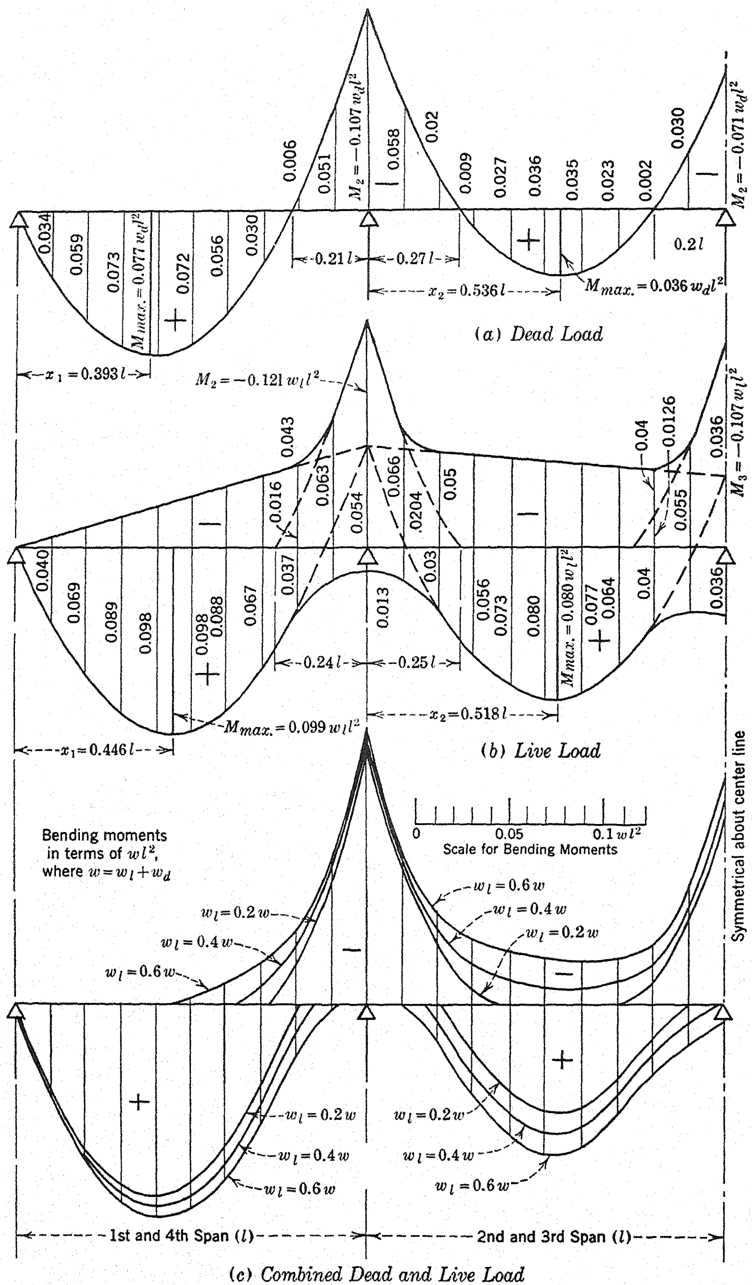


FIG. 76.—Four Equal Spans, Free Ends. Bending-Moment Diagrams. (See p. 167.)

for live load, and plot the results. The curves obtained by connecting the plots represent the combined bending-moment diagram.

3a. Instead of computing bending moments at intermediate points, the method used in the numerical example on p. 177 may be employed.

4. Where at any point the bending moment for dead load is of the same sign as the bending moment for live load to be used for combining, the bending moment for dead load is simply added to the appropriate bending moment for live load. Where the two values are of opposite signs, and, therefore, the bending moment for dead load balances partly or wholly the bending moment for live load, the dead-load bending moment should be divided by a factor of safety before it is added, with its sign, to the bending moment for live load.

This requirement is made clear by the following explanation: Assume that at a certain section the dead load produces a positive bending moment $M_d = 500\,000$ in-lb.; and that at the same point for the most unfavorable position of the live load a negative bending moment is produced $M_l = -500\,000$ in-lb. Since the two values balance, it would seem that at that section no negative bending moment reinforcement is required. This conclusion would be erroneous.

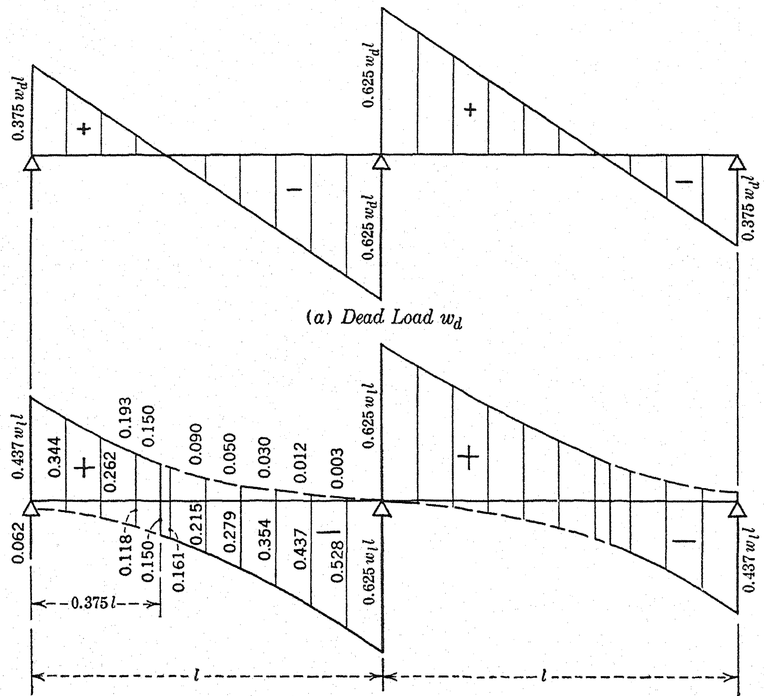
To provide for unusual conditions, all structures must be designed with a proper factor of safety, which means that, in case of overloading, if the design loads were doubled the stresses in steel should not exceed the elastic limit in steel. In this case the factor of safety is 2. Assume that in the example just cited a factor of safety of 2 is required. When the live load is doubled the bending moment for live load becomes $M_l = -2 \times 500\,000 = -1\,000\,000$ in-lb. Since the dead load remains the same, the balancing dead-load bending moment remains as before, $M_c = 500\,000$ in-lb. When combined the total bending moment for double the live load and the dead load is $M = -1\,000\,000 + 500\,000 = -500\,000$ in-lb. Therefore enough reinforcement is required at that section to resist a bending moment of $M = -500\,000$ in-lb. with a stress equal to the allowable unit stress multiplied by the factor of safety. When reduced to the basis of the working stresses the combined bending moment to be provided for at the section under consideration is equal to the negative bending moment for live load minus the positive bending moment for dead load, divided by the factor of safety.

EXTERNAL SHEARS

The table on p. 166 gives formulas for maximum end shears separately for dead load and for live load. The diagrams on pp. 172 to 174 give external shears at all intermediate points, also separately for dead load and for live load.

For dead loads, the shears are computed for a condition of loading when all spans are loaded simultaneously with uniformly distributed loading.

For moving live loads, the shears are computed for the most unfavorable positions of the loading. When the loading consists of concentrated wheel loads, it is necessary to determine the equivalent uniformly dis-



(b) Live Load w_l

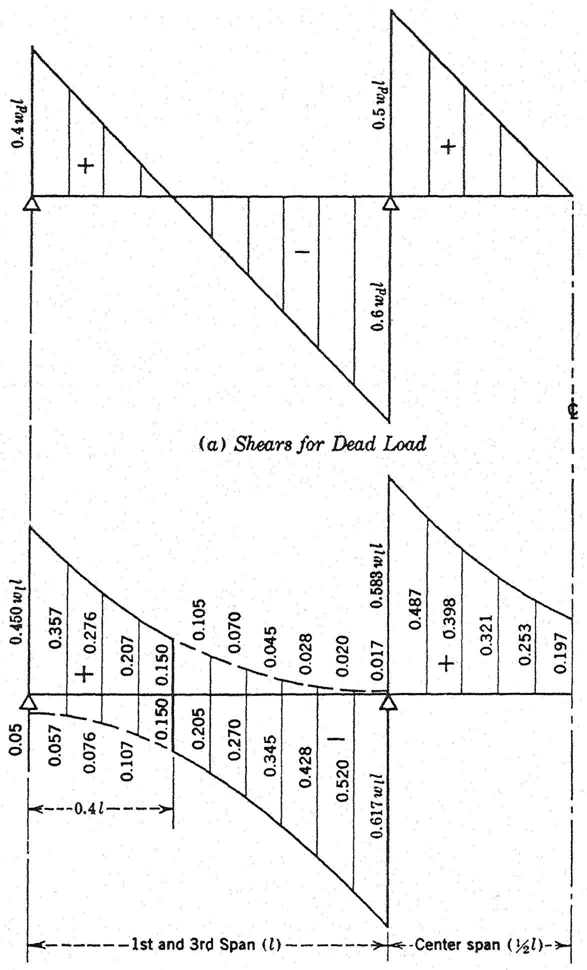
Live Load is Uniformly Distributed Moving Loading.

FIG. 77.—Two Equal Spans. Shear Diagrams. (*See p.* 171.)

tributed loading before the formulas and diagrams can be used. It should be remembered that the equivalent uniformly distributed loading for shears is not necessarily equal to the equivalent loading for bending moments.

To use the diagrams, compute the values w_dl and w_ll, and multiply them by proper coefficients from the appropriate diagram.

After the shears are computed, add the values for dead load to the appropriate values for live load, including impact, and plot the combined shear diagram to be used for determining the amount and the disposition of the web reinforcement.



(b) *Shears for Live Load*

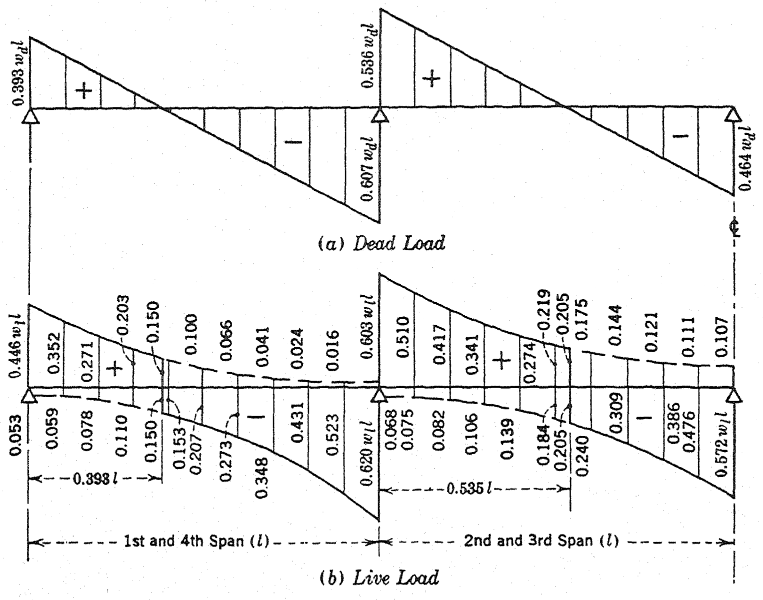
Live Load is Uniformly Distributed Moving Loading.

FIG. 78.—Three Equal Spans. Shear Diagrams. (*See p.* 171.)

EXAMPLE OF DESIGN OF CONTINUOUS GIRDER BRIDGE

The use of the formulas and suggestions given in the preceding pages is illustrated by the following numerical example.

Example. — Design a continuous girder bridge of three equal spans, with span lengths of 60 ft. measured between centers of bearings. The girders are not connected with the supporting piers. The requirements are:



Live Load is Uniformly Distributed Moving Loading.

FIG. 79.—Four Equal Spans. Shear Diagrams. (*See p.* 171.)

Live loads: For roadway, 20-ton truck; for girders equivalent uniformly distributed loading of 150 lb. per sq. ft. of roadway, selected arbitrarily.

Width of roadway,[3] 21 ft. between curbs. No sidewalks.

Pavement consists of concrete slab of average thickness of 5 in.

Reinforced-concrete railing is assumed to weigh 250 lb. per lin. ft.

Solution. — Adopt cross section shown in Fig. 80, p. 175, in which two longitudinal girders support a slab cantilevered on each side of the bridge. The slab is designed first as recommended on p. 49, although the computations are not given here. In actual design, make comparative estimates of several arrangements of girders in cross section based on the unit costs of materials and formwork in the locality where the bridge is to be built. Consider also all other local conditions. The use of slabs reinforced in two directions may be found more economical than the slab reinforced in one direction used here.

Dead Load. — Compute the dead load of the slab, pavement, curb, and railing per foot of girder. This is found to be 2 590 lb. per lin. ft. Assume dead load of girder stem below slab as 1 350 lb. per lin. ft.

$$\text{Total dead load } w_d = 2\,590 + 1\,350 = 3\,940 \text{ lb. per lin. ft. of girder}$$

Hence $w_dl = 3\,940 \times 60 = 236\,400$ lb.; $w_dl^2 = 14\,184\,000$ ft-lb.; and maximum static bending moment, $\frac{1}{8}wl^2 = 1\,773\,000$ ft-lb.

Live Load and Impact. — Using the specified unit live load of 150 lb. per sq. ft.,

[3] Note that, in recent practice, 22-ft. or, better still, 24-ft. width is considered as desirable for a two-lane bridge. See p. 5.

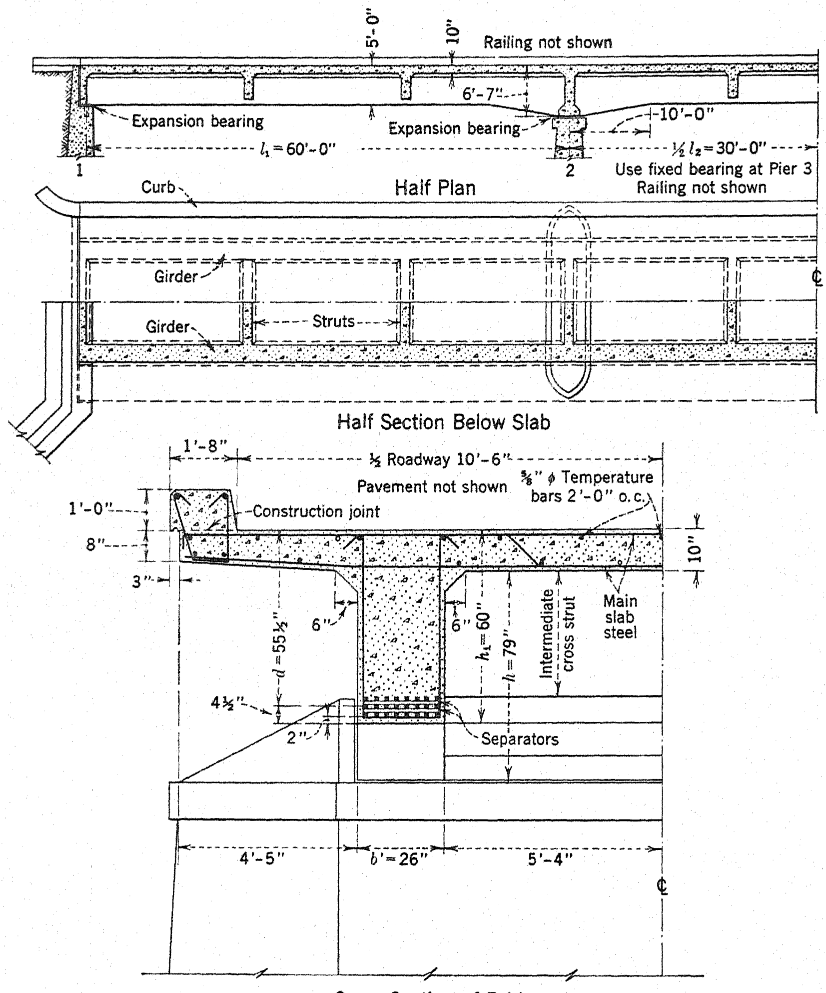


FIG. 80.—Numerical Example. General Concrete Dimensions. (*See p.* 174.)

the live load and impact are $w_l = 150 \times \frac{21}{2} = 1\,575$ lb. per lin. ft. of girder. $w_l l = 94\,500$ lb.; $w_l l^2 = 5\,670\,000$ ft-lb.; and $\frac{1}{8} w_l l^2 = 708\,700$ ft-lb.

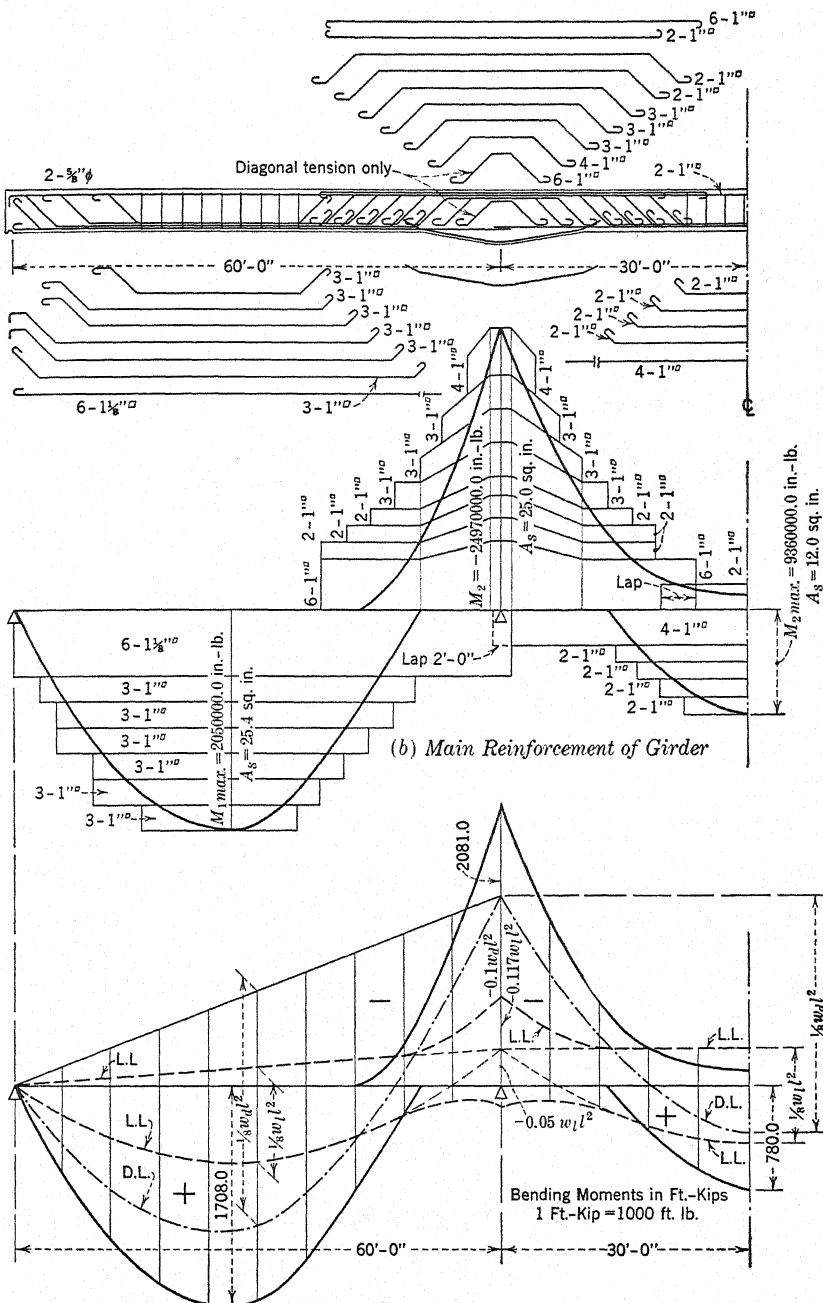
Bending Moments and Shears. — Bending moments and shears at the critical sections are found separately for dead load and for live load, using formulas in the table on p. 166 for three equal spans. The values of negative bending moments at intermediate points next to the interior piers are taken from the bending-moment diagram for live load on p. 169. On p. 176 these bending moments are tabulated; also combined bending moments are given for dead and live load.

Span	Location	Mark	Dead Load: Formula	Dead Load: Computed Value, lb. or ft-lb.	Live Load: Formula	Live Load: Computed Value, lb. or ft-lb.	Dead Load plus Live Load
				Maximum End Shears			
1st	Left	V_1	$0.4w_dl$	94 600	$0.45w_ll$	42 500	137 100 lb.
	Right	V_{2l}	$0.6w_dl$	141 800	$0.617w_ll$	58 300	200 100 lb.
2nd	Left	V_{2r}	$0.5w_dl$	118 200	$0.583w_ll$	55 100	173 300 lb.
				Negative Bending Moments			
1st	$x_1 = 0.8l$				$-0.04w_ll^2$	−227 000*	
	$x_2 = 0.9l$				$-0.065w_ll^2$	−369 000*	
	$x_3 = l$	M_2	$-0.1w_dl^2$	−1 418 000	$-0.117w_ll^2$	−663 000	−2 081 000 ft-lb.
2nd	$x_1 = 0.1l$				$-0.071w_ll^2$	−403 000*	
	$x_2 = 0.2l$				$-0.05w_ll^2$	−284 000*	
				Positive Bending Moments			
1st	At x_1	$M_{1max.}$	$0.08w_dl^2$	1 135 000	$0.101w_ll^2$	573 000	1 708 000‡ ft-lb.
		x_1	$0.4l$	24.0 ft.	$0.45l$	27 ft.	
2nd	At x_2	$M_{2max.}$	$0.025w_dl^2$	355 000	$0.075w_ll^2$	425 000	780 000 ft-lb.
		x_2	$0.5l$	30.0 ft.	$0.5l$	30 ft.	
				Corresponding Negative Bending Moment†			
		M_2			$-0.05w_ll^2$	−284 000	

*These values are used to draw negative bending-moment diagram for live load.

† This value at support is used for drawing bending-moment curve for maximum positive bending moments.

‡ x_1 for the maximum positive bending moment for dead load does not coincide with that for live load. Therefore it is not exact to add the maximum bending moments as was done in the table; but the difference between this sum and the exact value, which may be scaled from the bending-moment diagram, is negligible.



(a) *Bending Moment Diagrams*

FIG. 81.—Example of Design of Continuous Girders. (*See p.* 178.)

Bending-Moment Diagrams. — Using the bending moments computed in the table on p. 176, bending-moment diagrams are drawn, separately for dead load, live load, and for combined values, as shown in Fig. 81 (*a*), p. 177. Instead of computing bending moments at intermediate points, as outlined on p. 168, under 3, bending-moment curves are constructed graphically. In the center span the negative bending moments for dead load are plotted at the supports above the axis; the closing line for the string polygon is drawn, which in this case is a line parallel to the axis; starting from this closing line, the maximum static bending moment is plotted in the center of the span. Through these three points a parabola is constructed graphically, using the method shown in Fig. 95, p. 213; and the result gives bending moments for dead load in the center span, with positive values below the axis and negative values above the axis. In the same manner a parabola is drawn for live load for the condition of loading giving maximum positive bending moment in the center span.

In the end span, the negative bending moment for each loading is plotted at the support; the closing line is drawn by connecting this point with the end support;

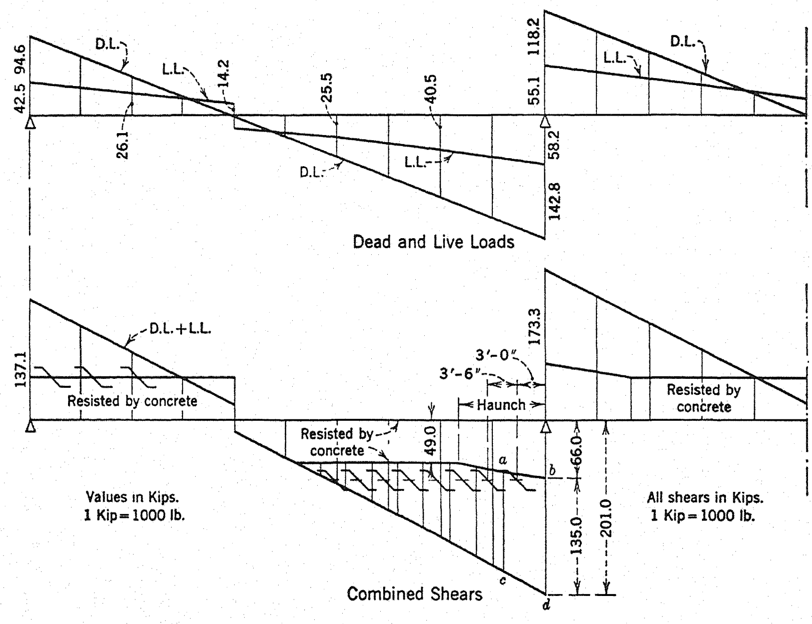


FIG. 82.—External Shear Diagrams. *(See p. 179.)*

and either the work of constructing the parabolas graphically is repeated, or ordinates from the parabola already constructed in the center span are scaled at several points and plotted at the corresponding points in the end span, starting from the closing line.

Largest negative bending moments for live load in this case are obtained by plotting the maximum value at the support, and the values in the end span at points $x = 0.8l$, and $0.9l$, and in the second span at $x = 0.1l$ and $0.2l$, all taken from the summary on p. 176. They are somewhat larger than would have been obtained by drawing parabolas for the most unfavorable full span loadings, because they are produced by partial loadings of the spans.

Shear Diagrams. — Using the values from the summary on p. 176, shear diagrams are drawn separately for dead load and for live load; and the resulting curves are combined as shown in Fig. 82, p. 178. The combined shear diagrams are used to determine the amount and spacing of web reinforcement.

Determination of Dimensions of Girders

Unit Stresses. — In determining the concrete dimensions and the required amount of reinforcement, the following unit stresses are used, all in pounds per square inch: $f_c = 800$ (900 at support of continuous girder); $f_s = 16\,000$; $v = 40$ for concrete alone, and 120 for beam with web reinforcement; $u = 100$; $n = 15$.

Concrete Dimensions at Support Determined by Shear. — The largest end shear acts at the right support of the first span, and it is $V_{2l} = 200\,100$ lb. Assume that $b' = 26$ in. and $j = 0.87$ for girder with steel in top and bottom; then the depth of girder required by shear is

$$d = \frac{V_{2l}}{b'jv} = \frac{200\,100}{26 \times 0.87 \times 120} = 74.0 \text{ in.}$$

Economical Depth of T-beam in Center of Span. — The economical depth is found from the formula[4] $d = \sqrt{\frac{rM}{f_s b'}} + \frac{t}{2}$. Assume ratio of cost of steel per cubic foot to cost of concrete in place, including cost of formwork reduced per cubic foot of concrete, $r = 50$. Then in the center of the first span

$$d = \sqrt{\frac{50 \times 20\,500\,000}{16\,000 \times 26}} + \frac{10}{2} = 49.5 + 5 = 54.5 \text{ in.}$$

Accepted Depths of Girder. — In the center of the span accept the economical depth[5] making

$$h = 60 \text{ in.; and } d = 60 - 4.5 = 55.5 \text{ in.}$$

At the interior support the depth required by shear is used

$$h = 79 \text{ in.; and } d = 79 - 4.5 = 74.5 \text{ in.}$$

Short haunch is used at each interior support. The length of haunch is determined by shear from the shear diagram.

Required Reinforcement at Interior Support. — The combined negative bending moment at support is $M_2 = -2\,081\,000$ ft-lb. $= -24\,970\,000$ in-lb. For specified stresses $f_c = 900, f_s = 16\,000, k = 0.458, j = 0.847$, and $p = 0.0129$.

$$A_s = \frac{24\,970\,000}{0.847 \times 74.5 \times 16\,000} = 24.8 \text{ sq. in.}$$

Use twenty-five 1-in. square bars, giving $A_s = 25.0$ sq. in.

The required steel area is substantially equal to the area from formula $A_s = pbd = 0.0129 \times 26 \times 74.5 = 25.0$ sq. in., which shows that the compression stresses are

[4] See "Concrete, Plain and Reinforced," Vol. I, p. 220.

[5] Smaller depth in the center may be more economical, when the effect of the reduced dead load on bending moments in girder is considered. Also in actual design it may be more economical to use a girder with variable moments of inertia.

within working limits even without any compression steel. When necessary to use compression steel, formulas in "Concrete, Plain and Reinforced," Vol. I, p. 232, apply.

Required Positive Reinforcement in First Span. — In resisting positive bending moments the girder acts as a T-beam with the following dimensions:

$$d = 55.5 \text{ in.}, b' = 26.0 \text{ in.}, t = 10 \text{ in.}$$

The total available width of slab acts as a flange of T-beam, so that

$$b = 53.0 + 26.0 + 64.0 = 144.0 \text{ in.}$$

Assume $j = 0.91$ (see "Concrete, Plain and Reinforced," Vol. I, p. 221). For $M_{1\text{max.}} = 1\,708\,000$ ft-lb. $= 20\,500\,000$ in-lb.

$$A_s = \frac{20\,500\,000}{0.91 \times 55.5 \times 16\,000} = 25.4 \text{ sq. in.}$$

Use eighteen 1-in. square bars plus six $1\frac{1}{8}$-in. square bars in three layers. This gives $A_s = 18 \times 1.0 + 6 \times 1.27 = 25.6$ sq. in.

The compression stresses in flange are almost always lower than the allowable working stresses. They are here checked by the method given in "Concrete, Plain and Reinforced," Vol. I. For $\frac{t}{d} = \frac{10}{55.5} = 0.18$ and for unit stresses $f_c = 800$, $f_s = 16\,000$, and $n = 15$, the largest ratio of tension steel that can be used without exceeding the allowable compression is found from the diagram, Vol. I, p. 874, $p_m = 0.007\,1$. In this case $p = \frac{25.4}{144 \times 55.5} = 0.003\,2$, which is much smaller than the determined value of p_m, indicating that the corresponding compression stresses are much lower than the specified working stresses. Therefore, as far as compression stresses are concerned, appreciably smaller depth of girder could be used. Large depth at support, however, would have to be retained on account of shear, and this would change the girder into one with variable moments of inertia for which the method of design in Chapter X would have to be followed.

Required Positive Reinforcement in Second Span. — For maximum positive bending moment $M_{2\text{max.}} = 780\,000 \times 12 = 9\,360\,000$ in-lb.

$$A_s = \frac{9\,360\,000}{0.91 \times 55.5 \times 16\,000} = 11.6 \text{ sq. in.}$$

Use twelve 1-in. square bars in two layers. $A = 12 \times 1.0 = 12.0$ sq. in.

It is evident from this amount of reinforcement that the depth of girder, as far as positive bending moments are concerned, is too large. A design with variable moments of inertia, and with a small depth in the center span, may be more economical than the design here employed.

Length of Bars and Points of Bending of Reinforcement. — The bending-moment diagram, in conjunction with the shear diagram, is used to determine the points of bending of reinforcement. (See Figs. 81 (*a*) and (*b*), p. 177.) To avoid the use of very long bars, no bottom reinforcement is bent up and carried into the adjoining spans. Instead, the positive and the negative reinforcement are made up of separate sets of bars, which by overlapping give the same effect as if the bars were continuous.

In Fig. 81 (*b*), in the upper part, the reinforcement is shown as assembled. Bend-

ing sketches of the bars are also shown. Three layers of bars are used in the end spans and at the supports, and two layers in the center span. At the support, the fourth lowest layer of bent bars is used only as diagonal tension reinforcement. See p. 397 about the proper manner of arranging bars, supporting them, and keeping them in place.

The bars placed at the bottom of each haunch are not needed there by compression stresses, but they are added to protect this vital part of construction from injury.

In the lower part of Fig. 81 (*b*) is reproduced the combined bending-moment diagram, and upon it is superimposed the diagram of moments of resistance determined by the reinforcement. Where the depth of the girder is constant, the curve for the moment of resistance for a definite number of bars is a straight line parallel to the axis. A break occurs in the diagram where bars are bent and thereby stop being effective as tension reinforcement. At the supports, the lines representing moments of resistance are slanting owing to the variation in the depth of the haunch.

The reinforcement is satisfactory, because the outline representing moments of resistance is outside of the bending-moment diagram. Sufficient leeway is allowed between the points where bars could be bent and the points where they actually are bent, to take care of any misplacement of bars and other contingencies. In some places, bars have been extended farther than necessary by the bending moments, because in this way they become more useful as diagonal tension reinforcement.

Diagonal Tension Reinforcement. — The points of bending of the longitudinal reinforcement having been selected with the aid of the bending-moment diagram and of the shear diagram, it is necessary to investigate whether the bent bars are sufficient to take care of the diagonal tension allotted to them. If not, stirrups must be added, or the points of bending of the bent bars rearranged.

As customary in American practice, concrete is assumed to resist diagonal tension corresponding to a shearing stress of $v = 40$ lb. per sq. in. The amount of shear thus resisted is marked off on the diagram. Next the locations of bars are marked; distances within which each set of bars is effective are determined; shear to be resisted is computed and is compared with the value of bent bars in resisting diagonal tension. This work is not given here because it is substantially the same as shown on p. 109.

In the central portions of the spans, no diagonal tension reinforcement is required, concrete being sufficient to resist all web stresses. On account of the importance of the member, however, stirrups spaced as shown in the figure are introduced. Also stirrups with wide spacing are used in the parts reinforced by bent bars, even when they are not needed by the stresses.

Where regulations require that all diagonal tension should be resisted by web reinforcement, the design here given should be properly adjusted.

CONTINUOUS GIRDERS OF EQUAL SPANS WITH CANTILEVERS

Use of Continuous Girders with Cantilevers. — Slabs, beams, and girders with cantilevers are used very often in bridge design. The use and design of slabs with cantilevers are discussed in Chapter V on p. 53.

Cantilevers of main girders serve several purposes. They often replace costly abutments, in which cases they may be provided at their ends with rigidly attached vertical aprons and wing walls to retain the embankment. Such designs are shown in Figs. 51, p. 124, and 69, p. 158.

Cantilevers also may be used to reduce positive bending moments in the end spans and negative bending moments at the first interior support, thereby equalizing bending moments throughout the girder. Obviously only bending moments produced in the main spans of the girder by the dead loads on the cantilevers can be used for this purpose. Where large reduction of positive bending moments is desired, cantilevers are provided with counterweights, as shown in Fig. 52, p. 125. These are particularly advantageous for girders of two spans. In girders of three spans the beneficial effect in the end spans is offset to some extent by the increase in positive bending moments in the center span.

Bending Moments in Main Spans Due to Loads on Cantilevers. — The loads on the cantilevers produce in the cantilevers bending moments and shears described on p. 119 in connection with cantilevers of one-span girders. The bending moments and shears reach their maximum at the supports of the cantilevers.

At the support of the cantilever, the negative bending moments in the cantilever are transferred in full to the girder. In the end span next to the loaded cantilever, bending moments produced by the cantilever vary according to a straight line from the maximum negative value at the cantilever support to a minimum at the opposite support, where the bending moment is positive and much smaller numerically. In the other spans of the girder, bending moments also vary according to a straight line.

TABLE II

CONTINUOUS GIRDERS OF EQUAL SPANS WITH CANTILEVERS
BENDING MOMENTS IN MAIN SPANS OF CONTINUOUS GIRDERS FOR CANTILEVER LOADS

M_c = bending moment at support of cantilever
(M_c is negative and $-M_c$ is positive)

Number of Equal Spans	First Support	Second Support	Third Support	Fourth Support	Fifth Support
			Left Cantilever Loaded		
2	M_c	$-0.25M_c$	0		
3	M_c	$-0.267M_c$	$0.067M_c$	0	
4	M_c	$-0.268M_c$	$0.072M_c$	$-0.018M_c$	0
			Both Cantilevers Loaded. Symmetrical Loads		
2	M_c	$-0.5M_c$	M_c		
3	M_c	$-0.2M_c$	$-0.2M_c$	M_c	
4	M_c	$-0.286M_c$	$0.143M_c$	$-0.286M_c$	M_c

SHEARS IN MAIN SPANS OF CONTINUOUS GIRDERS FOR CANTILEVER LOADS
M_c = bending moment at support of cantilever; l = span length

Number of Equal Spans	First Span	Second Span	Third Span	Fourth Span
		Left Cantilever Loaded		
2	$-1.25\frac{M_c}{l}$	$0.25\frac{M_c}{l}$		
3	$-1.267\frac{M_c}{l}$	$0.333\frac{M_c}{l}$	$-0.067\frac{M_c}{l}$	
4	$-1.268\frac{M_c}{l}$	$0.34\frac{M_c}{l}$	$-0.09\frac{M_c}{l}$	$0.018\frac{M_c}{l}$
		Both Cantilevers Loaded		
2	$-1.5\frac{M_c}{l}$	$1.5\frac{M_c}{l}$		
3	$-1.2\frac{M_c}{l}$	0	$1.2\frac{M_c}{l}$	
4	$-1.286\frac{M_c}{l}$	$0.429\frac{M_c}{l}$	$-0.429\frac{M_c}{l}$	$1.286\frac{M_c}{l}$

The magnitude of bending moments at all the supports of the main span due to a cantilever bending moment M_c are given in the table on p. 182 for girders with cantilevers of two, three, and four equal spans. Two conditions of loading are considered, in one of which the left cantilever is loaded, and in the other cantilevers on both ends are symmetrically loaded. The bending moments at the supports being known, the bending moments at intermediate points may be obtained by plotting the values at the supports and connecting the points by straight lines, as shown in Figs. 83 to 85, pp. 184 to 186.

Shears in Main Spans Due to Cantilever Loads. — Loads on cantilevers produce shears in all spans of a continuous girder. The shears are uniform throughout each span. For one-sided loading, the largest shears act next to the loaded cantilever, and they diminish gradually in the other spans to a minimum in the opposite end span. For symmetrical loading of cantilevers at both ends, the shear is largest in end spans, and decreases toward the center span of the girder.

The values of shears are given in the table above; and shear diagrams are shown in Figs. 83 to 85.

Bending-Moment and Shear Diagrams for Girders with Cantilevers. — Bending-moment and shear diagrams in a girder with cantilevers due to the loads on the main spans are constructed the same as for a girder

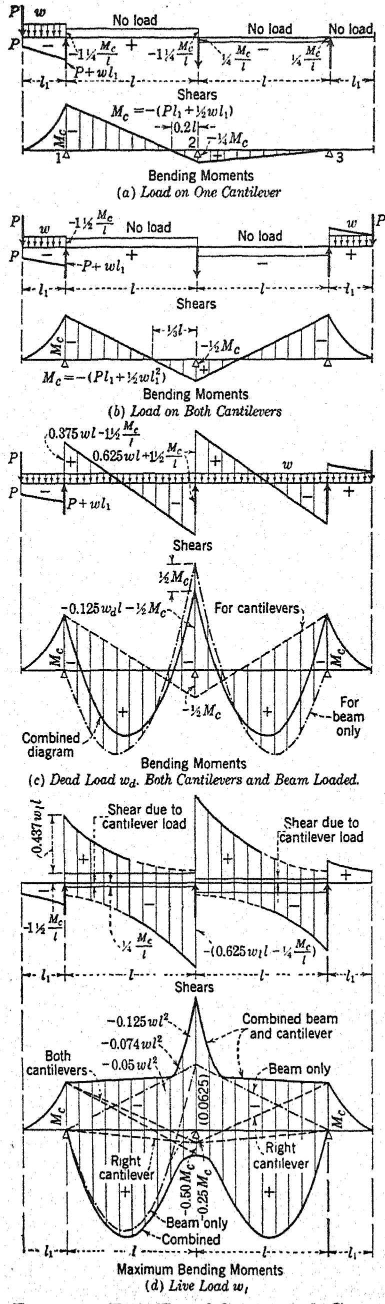


FIG. 83.—Two Equal Spans with Cantilevers. (See p. 184.)

with the same number of spans without any cantilevers. In determining bending moments and shears, it is therefore necessary to compute separately bending moments due to the loads on the main spans and the bending moments and shears due to the cantilever loads. The values are then combined so as to get the most unfavorable results.

The effect of cantilever loads upon continuous girders is shown in Figs. 83 to 85, pp. 184 to 186, for girders of two, three and four spans.

Figures (*a*) in each case give bending moments and shears for loads on left cantilever only. Figures (*b*) give bending moments and shears when both cantilevers are symmetrically loaded.

Figures (*c*) in two cases show bending moments and shears for combined dead load on cantilevers and on the main spans. It will be observed that the loads on cantilevers produce in many parts bending moments of opposite signs to those produced by the main span loading, thereby reducing the bending moments in the main spans. This reduction can be taken advantage of only in case of dead loads.

The figures (*d*) show bending moments and shears for live loads on the cantilevers and on the main spans. Here the cantilever bending moments are used only when the resulting bending moments are larger than the bending moments without cantilevers. For live loads, it is not permissible to utilize bending moments due to cantilever loads to

reduce bending moments in the main span because the two may or may not be loaded simultaneously.

After the bending-moment and shear diagrams are drawn, separately for live load and for dead load, they should be combined in the same manner as explained on p. 168 in connection with girders without cantilevers. Special attention is called to rule 4 given there.

Diagrams for combined bending moments for girders of four spans with cantilevers are not given.

Equivalent Uniformly Distributed Live Load for Cantilevers.—When the main spans of a girder with cantilevers are designed for equivalent uniformly distributed live load in place of the concentrated wheel loads, it is important to remember that it is not permissible to use the same unit loading for the cantilevers as for the main spans for the reasons explained on p. 28. **It is recommended that concentrated wheel loads be used to determine bending moments and shears in the cantilevers even when equivalent loading is used elsewhere.**

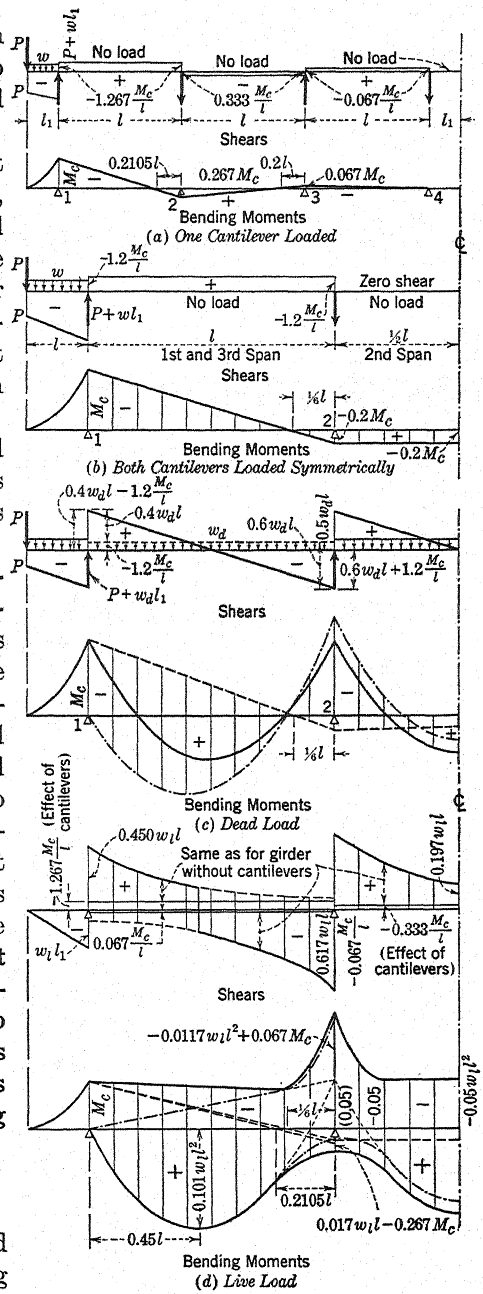


FIG. 84.—Three Equal Spans with Cantilevers. (*See p.* 185.)

NUMERICAL EXAMPLE

The use of formulas and suggestions for determining bending moments and shears in a continuous girder with

cantilevers is illustrated by the following numerical example. The span lengths of the main girder and the number of spans are made the same as in the example of the design of continuous girder on p. 173.

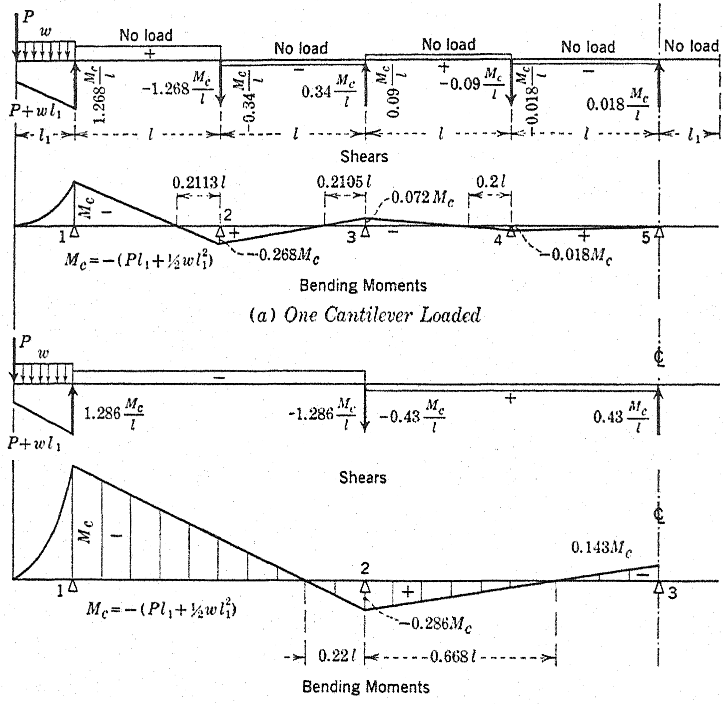


FIG. 85.—Four Equal Spans with Cantilevers. (*See p.* 184.)

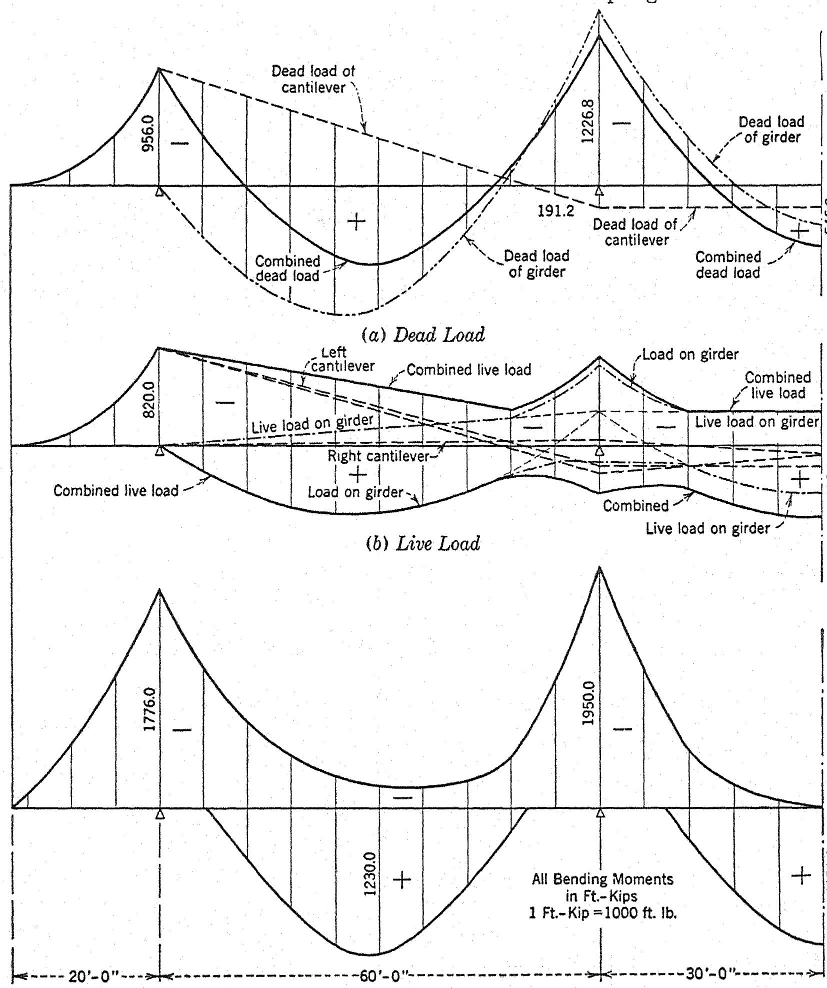
By comparing the final bending-moment diagrams in the two cases, it is possible to gauge the effect of the cantilevers upon the girder.

Example. — Find bending moments in the girders of a continuous girder bridge consisting of three equal spans with two cantilevers. The span lengths of the girders are 60 ft., center to center of supports; the length of each cantilever is 20 ft. Each cantilever carries at the end an apron, 1 ft. thick and 4 ft. deep below the slab, with wing walls to replace the abutments. Width of bridge 21 ft. Arrangement of girders in cross section is the same as in Fig. 80, p. 175. Live loads for main spans are the same as in the example on p. 173. For cantilevers, concentrated wheel loads of a 20-ton truck to be used.

Solution. — Since the bending moments in the girders due to the loads on the main span have already been determined on p. 176, it is only necessary to find bending moments due to cantilever loads, and to combine them with the bending moments for the main span loads.

Dead Load on Cantilever. — The unit dead load on the cantilever is assumed to be the same as for the main spans, namely w_d = 3 940 lb. per lin. ft. of cantilever. The concentrated load on the end of each cantilever equals

Apron	7 200
Wing	1 400
Total	P = 8 600 lb. per girder



(c) *Combined Dead and Live Load*

FIG. 83.—Example of Design. Continuous Girder with Cantilevers. (*See p.* 188.)

Maximum bending moments on the cantilever

For uniformly distributed dead load, $-\frac{1}{2} \times 3\,940 \times 20.0^2 = -788\,000$

For concentrated dead load at the end, $-8\,600 \times 19.5 = -168\,000$

$M_{c(d)} = -956\,000$ ft-lb.

The bending moment at the second support (see the table on p. 182) is

$$M_2 = -0.2M_c = 0.2 \times 956\,000 = 191\,200 \text{ ft-lb.}$$

These two values are sufficient for drawing the bending-moment diagram for dead load. (See Fig. 86 (*a*), p. 187.)

Live Loads. — As explained on p. 185, the equivalent uniformly distributed live load used for the main spans of the girder must not be used for the cantilever. Concentrated wheel loads are used, and it is accepted that each cantilever carries one truck line. The rear axle of a truck is placed 6 in. from the end of the cantilever, making the moment arm for this load 19 ft. 6 in. For the front axle, the moment arm then is $19.5 - 14.0 = 5.5$ ft.

For this loading, the cantilever bending moment for live load, including impact, is

$$M_{c(l)} = -820\,000 \text{ ft-lb.}$$

The bending moments at the other supports of the girder from formulas in the table on p. 182 are as follows.

Both cantilevers loaded

$$M_{2(l)} = M_{3(l)} = -0.2M_{c(l)} = -164\,000 \text{ ft-lb.}$$

Left cantilever loaded

$$M_{2(l)} = -0.267M_{c(l)} = \quad 219\,000 \text{ ft-lb.}$$

$$M_{3(l)} = \quad 0.067M_{c(l)} = -\ 55\,000 \text{ ft-lb.}$$

Combined Bending Moments. — Using the bending moments for the main spans from p. 176, and the just determined bending moments for the cantilever loads, bending-moment diagrams are drawn for the girders with cantilevers as shown in Fig. 86, p. 187.

In Fig. 86 (*a*), bending moments for dead load are combined. In Fig. 86 (*b*), bending moments are plotted for live load on the main spans and for two conditions of loading of the cantilevers. By combining, two curves are obtained, one for positive and the other for negative bending moments. Finally, the bending moments for dead load are combined with the bending moments for live load, as shown in Fig. 86 (*c*). These bending moments should be used in the design of the girder.

Comparison of Bending Moments in Girder with and without Cantilevers. — It is of interest to compare the final bending moments for the girders with cantilevers, as shown in Fig. 86 (*c*), with the final bending moments for the girders without cantilevers, Fig. 81 (*a*), p. 177. To make the comparison easier, the following summary of bending moments at critical sections is prepared.

COMPARISON OF BENDING MOMENTS FOR GIRDERS WITH AND WITHOUT CANTILEVERS. (SEE PP. 176 AND 187.)

	Positive Bending Moments		Negative Bending Moments	
	End Spans, ft-lb.	Center Span, ft-lb.	First Support, ft-lb.	Second Support, ft-lb.
Without cantilevers	1 708 000	780 000	0	−2 081 000
With cantilevers	1 230 000	1 151 300	−1 776 000	−1 950 000

From the summary, it is evident that the bending moments in the girder with cantilevers are better distributed. The positive bending moments in the first and last spans are appreciably smaller than for the girders without cantilevers, so that a much smaller depth of girder could be used economically than would be possible in the girder without cantilevers. On the other hand, the positive bending moments in the center span are increased to about the same magnitude as the positive bending moments in the end spans. This is of advantage, as it permits the use of the same bottom reinforcement in all spans.

The negative bending moments at the interior supports are reduced by about 7 per cent, but large bending moments are developed at the end supports. Haunches will be needed at the end supports and also negative bending moment reinforcement of practically the same amount as at the interior supports.

In this example, the same unit dead load was used in both cases. Actually the depth of the girders could here be made smaller, which would reduce the bending moments for dead load in the girder with cantilevers.

CHAPTER X

CONTINUOUS GIRDERS. FIXED-POINT METHOD

In the design of multi-span reinforced-concrete bridges, it is often necessary to use continuous slabs, beams, and girders of unequal spans. Also, it is often economical, or advisable on account of the headroom, to make the depths of girders in the centers of the spans smaller than at the supports, with the consequence that the moments of inertia of the cross sections become variable. In such cases, formulas in Chapter IX for beams and girders of equal spans and with constant moments of inertia cannot be used and the simplest solution of the problem is by means of the " fixed-point method " described in this chapter.

This fixed-point method may be used to determine bending moments and shears in continuous slabs, beams, and girders of any number of equal or unequal spans; with any condition of restraint at the ends; with or without cantilevers; and with constant or variable moments of inertia. As given in this chapter, the method applies to continuous members not rigidly connected with the supports. When the girders are connected with the supports so as to form rigid frames, the fixed-point method for rigid frames in Chapter XI should be used.

In this chapter, the fixed points are defined; their use in loaded as well as in unloaded spans is explained; and the procedure is outlined for the use of the fixed-point method in design of continuous girders.

Separate formulas for the location of fixed points are given for the following three conditions: (*a*) when moments of inertia in each span are constant, but are different in the various spans; (*b*) when moments of inertia are constant throughout the entire girder; (*c*) when moments of inertia are variable.

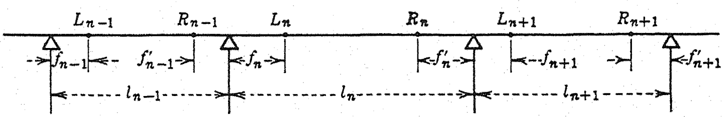
The effect of variable moments of inertia upon bending moments in a girder is discussed in this chapter. For the sake of comparison, in the numerical example here given bending moments are determined for a continuous girder bridge separately for two assumptions as to moments of inertia: (1) moments of inertia of cross sections are constant throughout; (2) moments of inertia are variable. In both cases the same live and dead loads are used.

In this chapter also is discussed the use of fixed points for continuous

girders with cantilevers, and for continuous girders in which the end spans are rigidly connected with the end columns. The use of fixed points for constructing influence lines is briefly outlined.

Finally, formulas are given for determining external shears for known bending moments.

Definition of Fixed Points. — Each span of a continuous slab, beam, or girder has two fixed points, namely, a right fixed point and a left fixed point, as shown in Fig. 87, below. Here points L_{n-1}, L_n and L_{n+1} represent left fixed points, and points R_{n-1}, R_n and R_{n+1} right fixed points.



f_{n-1}, f_n, f_{n+1} *Left Fixed Points.*
f'_{n-1}, f'_n, f'_{n+1} *Right Fixed Points.*

FIG. 87.—Fixed Points in Continuous Girders. (*See p.* 191.)

The left fixed point in any span is the point of intersection with the axis of the straight line representing in that span the bending moments for a condition of loading in which only one span of the girder is loaded, or generates bending moments, and all other spans are not loaded; and the loaded span is to the right of the span under consideration. (See Fig. 88, below.)

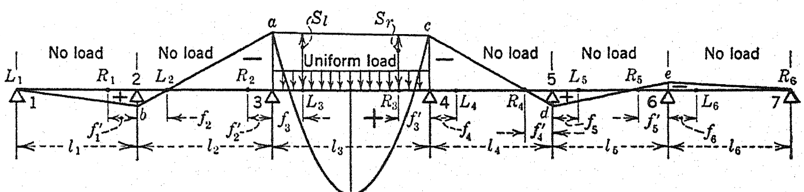


FIG. 88.—Use of Fixed Points. Third Span Only Loaded Span. (*See p.* 191.)

The right fixed point is a similar point of intersection, when the only loaded span is located to the left of the span under consideration.

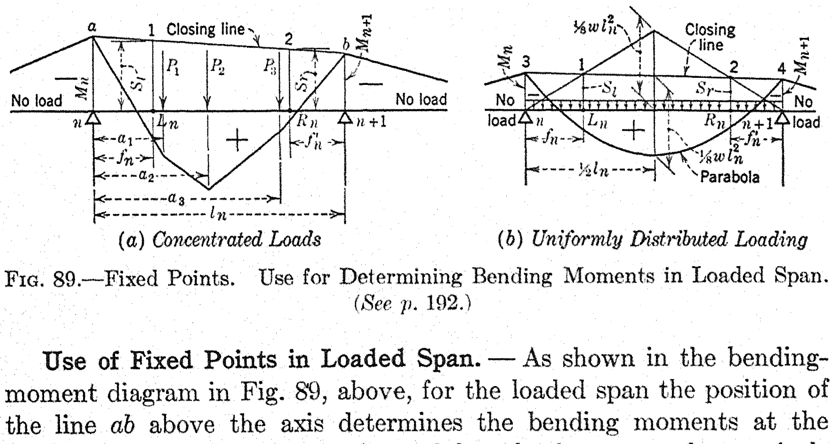
From this definition of fixed points, it follows that, in any span, the location of fixed points being known, and the bending moment at the support nearest to the loaded span having been found, the position of the straight line forming the bending-moment diagram in that span is definitely determined.

Fixed points, also, can be used to determine bending moments in the loaded span when all other spans are not loaded.

Use of Fixed Points When Several Spans Are Loaded. — When several spans of a girder are loaded simultaneously, the effect of loads in each loaded span upon the girder is determined separately, and then the results are added.

Use of Fixed Points in Unloaded Spans. — The location of left fixed points in Fig. 88, p. 191, is indicated by letters L with designating sub-numbers; the location of right fixed points is marked by letters R with sub-numbers. The ends of the girders being free, the left fixed point in the left end span and the right fixed point in the right end span coincide with the respective end supports.

In Fig. 88 the third span is the only loaded span, and its bending-moment diagram is represented by a curve. Since at support 3 the negative bending moment is equal to $3a$, the bending moment diagram to the left of the loaded span is obtained by connecting point a with the left fixed point of the second span, L_2, by extending the line to intersection at b with a vertical at support 2, and finally by connecting point b with the fixed point L_1. Similarly is obtained the broken line $cdeR_6$ representing the bending-moment diagram to the right of the loaded span.



(*a*) *Concentrated Loads* (*b*) *Uniformly Distributed Loading*

Fig. 89.—Fixed Points. Use for Determining Bending Moments in Loaded Span. (*See p.* 192.)

Use of Fixed Points in Loaded Span. — As shown in the bending-moment diagram in Fig. 89, above, for the loaded span the position of the line ab above the axis determines the bending moments at the supports. This line may be located by plotting upon the verticals erected at the fixed points the respective values S_l and S_r, found from the following formulas:

Let S_l, S_r = distances at fixed points to closing line.
f, f' = distances of left and right fixed points from respective supports.
w = uniformly distributed loading.
l = span length.

P, P_1, P_2, P_3 = concentrated loads.
a, a_1, a_2, a_3 = distances of concentrated loads from left support.
ϵ = constant for girder span with unsymmetrical haunches.

Uniformly Distributed Loading:

For spans with constant moments of inertia, and also spans with symmetrical haunches:

$$S_l = -\tfrac{1}{4}wfl \quad \text{at left fixed point} \tag{1}$$

$$S_r = -\tfrac{1}{4}wf'l \quad \text{at right fixed point} \tag{2}$$

For spans with unsymmetrical haunches:

$$S_l = -(\tfrac{1}{4}wfl)(2-\epsilon) \quad (3) \qquad S_r = -(\tfrac{1}{4}wf'l)\epsilon \quad (4)$$

For values of ϵ see pp. 202, 204 and 207.

Concentrated Loads, Constant Moments of Inertia:

One load:

$$S_l = -Pf\frac{a}{l}\left[1-\left(\frac{a}{l}\right)^2\right] \quad (5) \qquad S_r = -Pf'\frac{a}{l}\left(1-\frac{a}{l}\right)\left(2-\frac{a}{l}\right) \quad (6)$$

Several loads:[1]

$$S_l = -f\Sigma P\frac{a}{l}\left[1-\left(\frac{a}{l}\right)^2\right] \quad (7) \qquad S_r = -f'\Sigma P\frac{a}{l}\left(1-\frac{a}{l}\right)\left(2-\frac{a}{l}\right) \quad (8)$$

Concentrated Loads, Variable Moments of Inertia. — Formulas (5) to (8) for S_l and S_r for concentrated loads give only approximate results when used for girders with variable moments of inertia. The difference between the results from the formulas and the correct values is largest for single loads on the span, when it may be as large as ± 10 per cent. The larger the number of loads on the span, however, the smaller becomes the difference; and for three or four loads it is usually negligible. For accurate results use S_l and S_r from diagrams in "Concrete, Plain and Reinforced," Vol. II, pp. 144 and 145.

[1] $f\Sigma P\frac{a}{l}\left[1-\left(\frac{a}{l}\right)^2\right] = f\left\{P_1\frac{a_1}{l}\left[1-\left(\frac{a_1}{l}\right)^2\right] + P_2\frac{a_2}{l}\left[1-\left(\frac{a_2}{l}\right)^2\right] + P_3\frac{a_3}{l}\left[1-\left(\frac{a_3}{l}\right)^2\right] + \cdots\right\}.$

S_l and S_r Determined Graphically for Uniformly Distributed Loading. — For uniformly distributed loading extending over the whole span, S_l and S_r may be determined graphically as shown in Fig. 89 (*b*), p. 192. Plot on the vertical in the center of the span, above the axis, the maximum static bending moment; and connect the apex with the supports. The inclined lines intersect the verticals erected at the fixed points at points 1 and 2, respectively; and $1L$ and $2R$ represent the values S_l and S_r, respectively. The line connecting points 1 and 2, extended on both sides, is the closing line for the bending-moment diagram of the loaded span. The parabola completes the diagram.

This method may be used for spans with constant moments of inertia and for symmetrical spans with variable moments of inertia. For spans with unsymmetrical haunches the value S_r obtained graphically should be multiplied by a constant ϵ, and the value S_l by $(2 - \epsilon)$. Formula for constant ϵ is given on p. 202, and its use is demonstrated in the numerical example on p. 214.

USE OF FIXED-POINT METHOD FOR STRUCTURAL STEEL GIRDERS

The method given in this chapter is applicable not only to continuous concrete girders but also to continuous structural steel girders. The moments of inertia then should be computed for the cross section of the structural steel girder.

DESIGN OF CONTINUOUS GIRDER. FIXED-POINT METHOD

Before using the fixed-point method for determining bending moments in continuous girder of a bridge, it is necessary to perform the following preliminary work.

(*a*) The lengths of spans must be definitely determined.

(*b*) The arrangement of girders in a cross section of the bridge must be selected, as explained on p. 151, and the floor system designed to get the dead load.

(*c*) Decide whether to use girders with constant or with variable moments of inertia. If constant moments of inertia are decided upon, select preliminary dimensions of the girder in each span. If the same depth is used in all spans, it is not necessary to compute moments of inertia, because they disappear in the formulas. When the depth in each span is different, the ratios of moments of inertia must be computed.

(*d*) If girders of variable moments of inertia are selected, decide whether to use symmetrical girders in all spans, or whether to use symmetrical haunches in interior spans and unsymmetrical haunches in

the end spans. Also decide upon the ratio of the maximum depth of girder at the supports to the minimum depth. Finally select the shape and length of haunches.

(*e*) To the dead load of the floor system, add the assumed dead load of the girder to get the total unit dead load per lineal foot of girder.

(*f*) Find the equivalent unit live load, also per lineal foot of girder.

Procedure for Using Fixed Points for Finding Bending Moments. — The procedure here outlined may be used for continuous slabs, beams, and girders, with constant as well as with variable moments of inertia.

1. Lay out the spans of the continuous member to a convenient scale. Where facilities for drafting are not available, a numerical solution of the problem by means of fixed points may be made as outlined on p. 196.

2. For each span, find the position of the left fixed point using appropriate formulas from the table on p. 200. When the arrangement of spans is symmetrical, the right fixed points are symmetrical with the left fixed points. For unsymmetrical arrangement of spans the right fixed points must be computed separately. Plot the fixed points on the spans.

3. Live load and dead load should be considered separately. Assume that each span in turn is the only loaded span, while all other spans are not loaded. In each case, first find the bending moments at the supports in the loaded span as explained on p. 192 under "Use of Fixed Points in Loaded Span," and then complete the bending-moment diagram in the unloaded spans as explained on p. 192 under "Use of Fixed Points in Unloaded Spans."

Where the arrangement of spans of the girder is not symmetrical, a bending-moment diagram must be drawn for every span, so that the total number of diagrams for dead load and for live load is equal to twice the number of spans. Where the spans of the girder are arranged symmetrically, diagrams need to be drawn only for one set of the spans, because the diagrams for the other load conditions may be obtained from symmetry. Thus, in the example on p. 209, where the end spans are symmetrical, bending-moment diagrams were prepared only for the assumptions that, in turn, the first and the second span was the only loaded span. The bending moments in the girder for a condition where the third span is loaded are symmetrical with those when the loading is in the first span.

4. To find bending moments for dead load, consider all spans as loaded simultaneously. Therefore the dead load bending moments for all the individual span loadings must be added. The simplest way of combining bending moments for all the individual span loadings is to add the individual bending moments at each support; to plot at each support the combined negative bending moment; and by connecting

the points thus obtained to get the closing lines for the static bending-moment diagram in all spans. Starting from these closing lines in each span ordinates of the appropriate parabolas are plotted, thereby completing the bending-moment diagram for the dead load.

5. For live-load, bending moment diagrams should be found only for such combinations of loaded spans which give, at the different sections of the girder, the largest positive and negative bending moments, respectively. For each combination of loaded spans a bending-moment diagram is drawn in the same manner as explained under 4 for dead load.

6. Combine bending moments for dead load with the bending moments for the most unfavorable conditions of live load. Separate combinations are required for positive bending moments and for negative bending moments. See remarks on p. 168 about combining bending moments. Diagrams for combined bending moments are used for determining dimensions of the girder, the amounts of reinforcement, and the points of bending reinforcement.

The procedure just outlined was used in the numerical example on p. 209.

Use of Fixed Points for Determination of Bending Moments by Computations. — When it is not convenient to use the graphical method of determining bending moments described in the preceding pages, the problem may be solved by simple computations which do not require much more time than the graphic solution and give more accurate results.

In the loaded span, after the values of S_l and S_r are computed by means of formulas on p. 193, the corresponding bending moments at the supports of the loaded span may be found from the following formulas:

Using notation on p. 192:

Bending Moments at Supports of Loaded Span:

$$M_l = S_l + (S_l - S_r)\frac{f}{l - (f + f')} \qquad \text{at left support} \qquad (9)$$

$$M_r = S_r - (S_l - S_r)\frac{f'}{l - (f + f')} \qquad \text{at right support} \qquad (10)$$

Bending Moments at Supports of Loaded Span, Uniformly Distributed Load:

$$M_l = -2\left[1 + \frac{f - f'}{l - (f + f')}\right]\frac{f}{l}\,M_{\max.} \qquad \text{at left support} \qquad (11)$$

$$M_r = -2\left[1 - \frac{f - f'}{l - (f + f')}\right]\frac{f'}{l}\,M_{\max.} \qquad \text{at right support} \qquad (12)$$

In both formulas, $M_{\text{max.}} = \frac{1}{8}wl^2$.

In the unloaded spans, when the determining bending moment at one support is known, the bending moments at the other support may be found from one of the following formulas:

Bending Moments at Supports of Unloaded Span:

$$M_l = -\frac{f}{l - f}M_r \qquad \text{at left support} \tag{13}$$

$$M_r = -\frac{f'}{l - f'}M_l \qquad \text{at right support} \tag{14}$$

Formula (13) is used when the bending moments at the right support is known, and formula (14) is used for known bending moment at the left support.

Bending moments found from computations may be indicated on freehand sketches or the results may be tabulated.

FORMULAS FOR FIXED POINTS

In formulas for fixed points given on p. 198 and in table on p. 200, let

$l, l_1, l_2 \ldots l_{n-1}, l_n$ = span length of girder.

$I, I_1, I_2 \ldots I_{n-1}, I_n$ = constant moments of inertia; also minimum moments of inertia in members with variable moments of inertia.

I_x = variable moment of inertia at point x from support of any span.

$f, f_1, f_2, \ldots f_{n-1}, f_n$ = distance of left fixed point from left support of span.

$\alpha, \alpha_1, \alpha_2, \ldots \alpha_{n-1}, \alpha_n$ = constants depending upon variation of moments of inertia.

$\alpha', \alpha'_1, \alpha'_2, \alpha'_{n-1}, \alpha'_n$ = constants depending upon variation of moments of inertia.

$\beta, \beta_1, \beta_2, \beta_{n-1}, \beta_n$ = constants depending upon variation of moments of inertia.

ϵ = constant depending upon variation of moments of inertia.

Each subnumber indicates the span for which the particular item is intended. Thus l_3, α_3, α'_3, β_3 and f_3 are values for the third span.

General Formulas for Left Fixed Points. — The following formulas are for a left fixed point in the nth span of a continuous girder. They are general. To use these formulas for any particular span, proper values

in that span and the span to the left should be substituted for the corresponding values in the nth and $(n-1)$th spans.

Moments of Inertia Constant:

$$f_n = \frac{1}{3 + \dfrac{I_n}{I_{n-1}} \dfrac{l_{n-1}}{l_n} q_{n-1}} l_n \tag{15}$$

where

$$q_{n-1} = 2 - \frac{1}{\dfrac{l_{n-1}}{f_{n-1}} - 1} \tag{16}$$

Moments of Inertia Variable:

$$f_n = \frac{\beta_n}{2\alpha_n + \beta_n + \dfrac{I_n}{I_{n-1}} \dfrac{l_{n-1}}{l_n} q_{n-1}} l_n \tag{17}$$

where

$$q_{n-1} = 2\alpha'_{n-1} - \frac{1}{\dfrac{l_{n-1}}{f_{n-1}} - 1} \beta_{n-1} \tag{18}$$

Special Formulas for Left Fixed Points. — For convenience in use separate formulas for fixed points are given in Table I, pp. 200 and 201 for the first, second, third and n-th span of a continuous girder. They were obtained from the general formulas by proper substitutions.

In Table I the following conditions as to moments of inertia are considered: (1) moments of inertia are constant in each span, but are different in different spans (see Fig. 90 (*b*), p. 199); (2) moments of inertia are constant throughout the girder (see Fig. 90 (*a*)); (3) moments of inertia are variable (see Fig. 91, p. 199). In each case three conditions of restraint at the left end are considered.

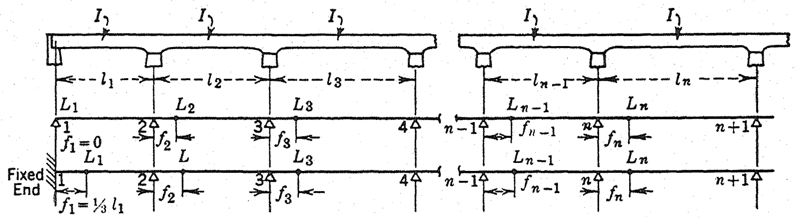
In Figs. 90 and 91, for the sake of clearness, only the left fixed points are shown. For designing purposes, both right and left fixed points are needed.

The computations for fixed points must always start in the end span, where their location depends only upon the condition of restraint at the end. The work then must proceed consecutively from span to span until all spans are taken care of.

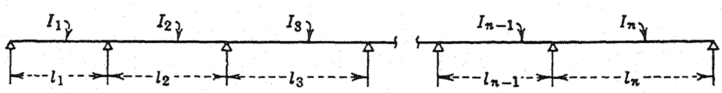
Right Fixed Points. — For symmetrical members, the positions of the right fixed points are symmetrical with those of the left fixed points.

In unsymmetrical members, the right fixed points may be found by renumbering the spans, starting from the right end, and then using formulas in Table I. The work must then proceed from right to left.

Constants α, α' and β. For girders with variable moments of inertia, the value of the fixed-point distance for any span, as determined from



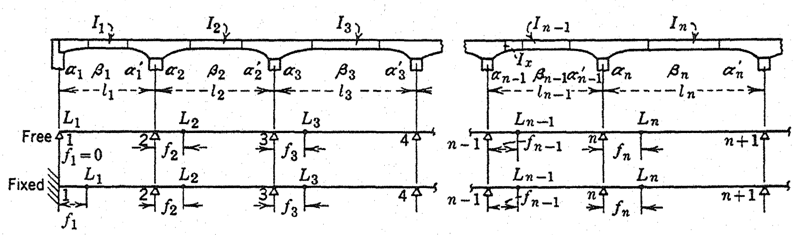
(*a*) *Beam with Constant Moments of Inertia*



(*b*) *Moments of Inertia Constant in each Span*

FIG. 90.—Left Fixed Points. Continuous Girder, Constant Moments of Inertia. (*See pp.* 198 *and* 200.)

formulas in Table I (*c*), p. 200, is governed by constants α and β for that span and by α' and β for the preceding span, which depend upon the variation of moments of inertia.



In spans with symmetrical haunches $\alpha = \alpha'$

FIG. 91.—Left Fixed Points. Continuous Girder, Variable Moments of Inertia. (*See pp.* 198 *and* 200.)

For any span in which the haunches at both ends are symmetrical, constant α is equal to constant α'.

The constants may be different for each span, and then they are designated by α, α', and β with sub-numbers indicating the span; thus in the first span α_1, α'_1, and β_1; in the second, α_2, α'_2, and β_2; and so on.

The constants may be the same for all spans, so that $\alpha_1 = \alpha_2 = \alpha_3 \ldots = \alpha$ and $\beta_1 = \beta_2 = \beta_3 \ldots = \beta$. This happens when the following requirements are fulfilled: the haunches are of the same shape, i.e., they are all straight, or all parabolic; the ratio m of the length of haunch to the span length is the same in all spans; finally, the ratio of minimum to maximum moment of inertia is the same for all spans.

TABLE I

Formulas for Left Fixed Points for Continuous Members Not Rigidly Connected with Supports

Item	Moments of Inertia		Condition at End of Member	1st Span, l_1	2nd Span, l_2
				f_1	f_2
1	Constant, Fig. 90	(a) Different in each span, I_1, I_2, I_3, I_n	Free	0	$\dfrac{1}{3 + 2\dfrac{I_2 l_1}{I_1 l_2}} l_2$
2			Fixed	$\frac{1}{3} l_1$	$\dfrac{1}{3 + 1.5\dfrac{I_2 l_1}{I_1 l_2}} l_2$
3			Restrained	$n l_1$*	$\dfrac{1}{3 + \dfrac{2 - 3n}{1 - n}\dfrac{I_2 l_1}{I_1 l_2}} l_2$*
4		(b) Same in all spans, $I_1 = I_2 = I_3, I_n$	Free	0	$\dfrac{1}{3 + 2\dfrac{l_1}{l_2}} l_2$
5			Fixed	$\frac{1}{3} l_1$	$\dfrac{1}{3 + 1.5\dfrac{l_1}{l_2}} l_2$
6			Restrained	$n l_1$*	$\dfrac{1}{3 + \dfrac{2 - 3n}{1 - n}\dfrac{l_1}{l_2}} l_2$*
7	Variable, Fig. 91	(c) Different each span, I_1, I_2, I_3, I_n	Free	0	$\dfrac{\beta_2}{2\alpha_2 + \beta_2 + 2\dfrac{I_2 l_1}{I_1 l_2}\alpha'_1} l_2$
8			Fixed	$\dfrac{\beta_1}{2\alpha_1 + \beta_1} l_1$	$\dfrac{\beta_2}{2\alpha_2 + \beta_2 + \dfrac{I_2 l_1}{I_1 l_2}\left(2\alpha'_1 - \dfrac{\beta_1}{2\alpha'_1}\beta_1\right)} l_2$
9			Restrained	$n l_1$*	$\dfrac{\beta_2}{2\alpha_2 + \beta_2 + \dfrac{I_2 l_1}{I_1 l_2}\left(2\alpha_1 - \dfrac{n}{1 - n}\beta_1\right)} l_2$*

* For restrained ends, values of n are fractions intermediate between zero and the value for fixed ends.
In members with constant moments of inertia, $I_1, I_2, I_3 \ldots I_n$ are the constant values in each span.
In members with variable moments of inertia $I_1, I_2, I_3 \ldots I_n$ are the smallest values in each span.
Constants α, α', and β, with sub-numbers, in each span depend upon variation of moments of inertia in that span.

TABLE I (*continued*)

FORMULAS FOR LEFT FIXED POINTS FOR CONTINUOUS MEMBERS NOT RIGIDLY CONNECTED WITH SUPPORTS

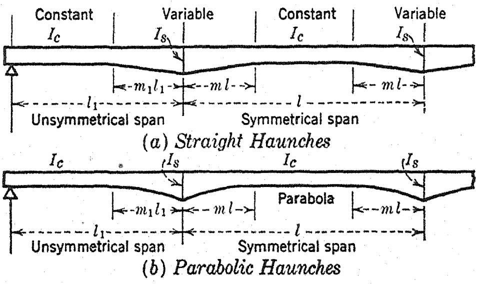
3rd Span, l_3		nth Span, l_n		Item
q_2	f_3	q_{n-1}	f_n	
				1
$2-\dfrac{1}{\dfrac{l_2}{f_2}-1}$	$\dfrac{1}{3+\dfrac{I_3 l_2}{I_2 l_3} q_2} l_3$	$2-\dfrac{1}{\dfrac{l_{n-1}}{f_{n-1}}-1}$	$\dfrac{1}{3+\dfrac{I_n l_{n-1}}{I_{n-1} l_n} q_{n-1}} l_n$	2
				3
				4
$2-\dfrac{1}{\dfrac{l_2}{f_2}-1}$	$\dfrac{1}{3+\dfrac{l_2}{l_3} q_2} l_3$	$2-\dfrac{1}{\dfrac{l_{n-1}}{f_{n-1}}-1}$	$\dfrac{1}{3+\dfrac{l_{n-1}}{l_n} q_{n-1}} l_n$	5
				6
				7
$2\alpha'_2-\dfrac{1}{\dfrac{l_2}{f_2}-1}\beta_2$	$\dfrac{\beta_3}{2\alpha_3+\beta_3+\dfrac{I_3 l_2}{I_2 l_3} q_2} l_3$	$2\alpha'_{n-1}-\dfrac{1}{\dfrac{l_{n-1}}{f_{n-1}}-1}\beta_{n-1}$	$\dfrac{\beta_n}{2\alpha_n+\beta_n+\dfrac{I_n l_{n-1}}{I_{n-1} l_n} q_{n-1}} l_n$	8
				9

See Table II (*a*) and (*b*), p. 203, for values of constants α and β for symmetrical spans with straight and parabolic haunches.

See Table III, p. 204, for values of constants α, α', and β for unsymmetrical haunches. For summation method, see p. 205.

Tables for Constants α and β; and α, α', β, and ϵ. — On p. 203, table is given for α and β for symmetrical spans; and on p. 204 for α, α', β, and ϵ for unsymmetrical spans.

In all cases, the constants depend upon the shape of haunches, whether straight or parabolic; upon ratio of length of haunch to span, m; and upon ratio of minimum to maximum moment of inertia in the span.



For Tables II and III, pp. 203 *and* 204.

FIG. 92.—Girders with Straight and Parabolic Haunches. (*See p.* 202.)

Table II gives constants for α and β for symmetrical haunches. Under (a) values are given for straight-line haunches (see Fig. 92 (a), p. 202); under (b), for parabolic haunches (see Fig. 92 (b)).

Table III gives constants α, α', and β for spans provided with a haunch at one end only. Here also straight and parabolic haunches are treated separately. Values for ϵ to be used in formulas (3) and (4) are also given for the same conditions.

General Formulas for Constants α, α', β, and ϵ. — The constants α, α', and β in formulas for fixed points depend upon the variation of moments of inertia in the span for which they are computed. They are represented by the following formulas.

$$\alpha = 3\int_0^l \frac{I_c}{I_x}\left(1 - \frac{x}{l}\right)^2 d\left(\frac{x}{l}\right) \tag{19}$$

$$\alpha' = 3\int_0^l \frac{I_c}{I_x}\left(\frac{x}{l}\right)^2 d\left(\frac{x}{l}\right) \tag{20}$$

$$\beta = 6\int_0^l \frac{I_c}{I_x}\frac{x}{l}\left(1 - \frac{x}{l}\right) d\left(\frac{x}{l}\right) \tag{21}$$

The constant ϵ in formulas (3) and (4), p. 193 also depends upon $\dfrac{I_c}{I_x}$

$$\epsilon = \frac{2\int_0^l \frac{I_c}{I_x}\left(\frac{x}{l}\right)^2\left(1 - \frac{x}{l}\right) d\left(\frac{x}{l}\right)}{\int_0^l \frac{I_c}{I_x}\left(\frac{x}{l}\right)\left(1 - \frac{x}{l}\right) d\left(\frac{x}{l}\right)} \tag{22}$$

TABLE II (*a*) and (*b*)

CONSTANTS α AND β FOR SPANS WITH SYMMETRICAL HAUNCHES

(See Fig. 92 (*a*) and (*b*), p. 202.)

Haunches	Constant	Ratio m	Values of $\frac{I_c}{I_s}$									
			0.02	0.04	0.06	0.08	0.10	0.15	0.20	0.30	0.60	1.00
(*a*) Straight haunches (See Fig. 92 (*a*).)	$\alpha = \alpha'$	0.15	0.67	0.69	0.71	0.72	0.74	0.76	0.78	0.82	0.91	1.00
		0.2	0.58	0.61	0.63	0.64	0.66	0.69	0.72	0.77	0.88	1.00
		0.3	0.42	0.45	0.48	0.50	0.53	0.57	0.61	0.68	0.83	1.00
		0.4	0.27	0.32	0.35	0.38	0.41	0.46	0.51	0.59	0.79	1.00
		0.5	0.14	0.19	0.23	0.27	0.30	0.36	0.42	0.52	0.75	1.00
	β	0.15	0.91	0.92	0.92	0.93	0.93	0.94	0.95	0.96	0.98	1.00
		0.2	0.85	0.86	0.87	0.88	0.89	0.90	0.91	0.93	0.97	1.00
		0.3	0.68	0.71	0.73	0.74	0.76	0.79	0.81	0.85	0.93	1.00
		0.4	0.47	0.52	0.55	0.58	0.60	0.65	0.69	0.75	0.88	1.00
		0.5	0.23	0.30	0.35	0.39	0.42	0.49	0.55	0.64	0.84	1.00
(*b*) Parabolic haunches (See Fig. 92 (*b*).)	$\alpha = \alpha'$	0.15	0.74	0.76	0.78	0.79	0.80	0.82	0.84	0.87	0.94	1.00
		0.2	0.66	0.69	0.71	0.73	0.74	0.77	0.79	0.83	0.92	1.00
		0.3	0.52	0.56	0.59	0.62	0.63	0.68	0.71	0.76	0.88	1.00
		0.4	0.40	0.45	0.49	0.51	0.54	0.59	0.63	0.70	0.85	1.00
		0.5	0.29	0.35	0.39	0.42	0.45	0.51	0.56	0.64	0.82	1.00
	β	0.15	0.94	0.95	0.95	0.96	0.96	0.97	0.97	0.98	0.99	1.00
		0.2	0.90	0.91	0.92	0.93	0.93	0.94	0.95	0.96	0.98	1.00
		0.3	0.79	0.81	0.83	0.84	0.86	0.88	0.89	0.92	0.96	1.00
		0.4	0.64	0.69	0.72	0.74	0.76	0.79	0.82	0.86	0.93	1.00
		0.5	0.48	0.54	0.58	0.62	0.64	0.69	0.73	0.79	0.90	1.00

TABLE III (*a*) and (*b*)

CONSTANTS α, α', β, AND ϵ FOR SPANS WITH UNSYMMETRICAL HAUNCHES

(Right side only provided with haunch; see Fig. 92 (*a*) and (*b*) p. 202.)

Right Haunch	Constant	$m = 0.2$ Ratio $I_c \div I_s$					$m = 0.3$ Ratio $I_c \div I_s$					$m = 0.4$ Ratio $I_c \div I_s$					$m = 0.5$ Ratio $I_c \div I_s$				
		0.3	0.2	0.1	0.06	0.02	0.3	0.2	0.1	0.06	0.02	0.3	0.2	0.1	0.06	0.02	0.3	0.2	0.1	0.06	0.02
(*a*) Straight	α	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.97	0.97	0.96	0.97	0.96	0.94	0.93	0.92
	α'	0.77	0.72	0.66	0.63	0.59	0.68	0.62	0.54	0.49	0.43	0.61	0.53	0.44	0.38	0.31	0.53	0.46	0.35	0.30	0.22
	β	0.96	0.96	0.94	0.93	0.92	0.92	0.90	0.88	0.86	0.84	0.87	0.84	0.80	0.77	0.73	0.82	0.77	0.71	0.68	0.62
	ϵ	0.97	0.96	0.95	0.95	0.94	0.94	0.93	0.91	0.90	0.87	0.92	0.89	0.86	0.84	0.80	0.89	0.86	0.81	0.78	0.73
(*b*) Parabolic	α	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.98	0.97	0.97	0.96
	α'	0.83	0.80	0.74	0.71	0.66	0.77	0.71	0.64	0.60	0.53	0.71	0.64	0.55	0.50	0.42	0.65	0.58	0.48	0.42	0.34
	β	0.98	0.97	0.97	0.96	0.95	0.96	0.95	0.93	0.92	0.89	0.93	0.91	0.88	0.86	0.82	0.89	0.87	0.82	0.79	0.74
	ϵ	0.98	0.98	0.97	0.97	0.96	0.97	0.96	0.94	0.93	0.92	0.95	0.93	0.91	0.90	0.87	0.93	0.91	0.88	0.86	0.82

ϵ to be used for values of S next to the haunch, in this case S_r. The other S multiply by $(2 - \epsilon)$. See formulas (3) and (4) p. 193. For haunch on left side of span interchange values of α with values of α'.

When the variation of the ratios $\frac{I_c}{I_x}$ can be expressed by a mathematical function which can be easily integrated, the integrals in the formulas for α, α', β, and ϵ may be solved analytically.

When integrations are not feasible, either the summation method, or one of the several known graphical or semi-graphical methods, may be used for solving the integrals. The following summation method is easiest to understand and to apply.

Solving Integrals for α, α', β, and ϵ by Summation. — To solve integrals in formulas (15) to (18) by summation, divide the span into a convenient number of equal divisions, Δx. In most cases ten divisions per span are sufficient. Find at each division point the moment of inertia of the cross section of the girder, and the ratio of the minimum moment of inertia I_c to the moment of inertia I_x at the division point. Tabulate the values as shown in Table IV for symmetrical spans of the girder, and in Table V for unsymmetrical spans.

For spans in which the haunches are symmetrical, the work is appreciably simplified: first, constant α is equal to constant α'; second, in columns for $\frac{x}{l}\left(1 - \frac{x}{l}\right)$ and $\left(1 - \frac{x}{l}\right)^2$ the values for symmetrical points can be added, because they have the same values of $\frac{I_c}{I_x}$. In the table, for ten divisions, the values for points 1 and 9, 2 and 8, 3 and 7, and 4 and 6 are thus combined.

In both tables $\left(1 - \frac{x}{l}\right)^2$ for the zero point, and $\left(\frac{x}{l}\right)^2$ for the last point are entered at one-half of their actual values for the reason explained in the footnote on p. 207. The constants determined in Table V are the values for the end girder used in the numerical example on p. 209.

Effect of Variation of Moments of Inertia upon Bending Moments. — In a continuous slab, beam, or girder, the increase in moments of inertia usually occurs near the supports, and is caused by an increase in depth, by an increase in width of stem of beam or girder, by the introduction of a bottom slab to resist compression stresses, or by any combination of these means.

When moments of inertia of a girder are larger at and near the supports than in the center of the span, larger negative bending moments at the supports are produced than if the moments of inertia were constant; and the larger the increase in moments of inertia, the larger is the increase in the negative bending moments. Not only the magnitude of

TABLE IV

SUMMATION METHOD FOR CONSTANTS α AND β SPAN WITH SYMMETRICAL HAUNCHES

Ten equal divisions of span. $\Delta x = 0.1l$

Point	$\frac{x}{l}$	$1 - \frac{x}{l}$	h_x	$\frac{I_c}{I_x} = \left(\frac{h_c}{h_x}\right)^3$	$\frac{x}{l}\left(1 - \frac{x}{l}\right)$	$\frac{I_c}{I_x}\frac{x}{l}\left(1 - \frac{x}{l}\right)$ (5) × (6)	$\left(1 - \frac{x}{l}\right)^2$	$\frac{I_c}{I_x}\left(1 - \frac{x}{l}\right)^2$ (5) × (8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0 + 10	0 and 1.0	1.0 and 0			0		0.50*	
1 + 9	0.1 and 0.9	0.9 and 0.1			0.18		0.88	
4 + 6	0.4 and 0.6	0.6 and 0.4			0.48		0.52	
5	0.5	0.5			0.25		0.25	
					Sum =		Sum =	

$\alpha = 3\Sigma \frac{x}{c}\left(1 - \frac{x}{l}\right)^2 \frac{\Delta x}{l} = 3 \times$ sum of column (9) $\times \frac{\Delta x}{l}$; $\beta = 6\Sigma \frac{Ic}{Ix}\frac{x}{l}\left(1 - \frac{x}{l}\right)\frac{\Delta x}{l} = 6 \times$ sum of column (6) $\times \frac{\Delta x}{l}$. In this case $\frac{\Delta x}{l} = 0.1$

Note: If ratios of moments of inertia $\frac{Ic}{Ix}$ are determined by any other method, compute the ratios at the various points and insert in column (5) the appropriate values. The same applies to Table V.

TABLE V

SUMMATION METHOD FOR α, α', β AND ϵ FOR UNSYMMETRICAL SPAN OF GIRDER. HAUNCH ON RIGHT END ONLY

$m = 0.5 \quad I_c \div I_s = 0.12 \quad \Delta x = 0.1l \quad h_c = 4.25$ ft.

Point	$\frac{x}{l}$	$1 - \frac{x}{l}$	h_x ft.	$\frac{I_c}{I_x} = \left(\frac{h_c}{h_x}\right)^3$	$\frac{x}{l}\left(1 - \frac{x}{l}\right)$	$\frac{I_c}{I_x}\frac{x}{l}\left(1 - \frac{x}{l}\right)$	$\left(1 - \frac{x}{l}\right)^2$	$\frac{I_c}{I_x}\left(1 - \frac{x}{l}\right)^2$	$\left(\frac{x}{l}\right)^2$	$\frac{I_c}{I_x}\left(\frac{x}{l}\right)^2$	$\frac{I_c}{I_x}\left(\frac{x}{l}\right)^2\left(1 - \frac{x}{l}\right)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
0	0.0	1.0	4.25	1.0	0.0	0.0	0.50*	0.50*	0.0	0.0	0.00
1	0.1	0.9	4.25	1.0	0.09	0.09	0.81	0.81	0.01	0.01	0.009
2	0.2	0.8	4.25	1.0	0.16	0.16	0.64	0.64	0.04	0.04	0.032
3	0.3	0.7	4.25	1.0	0.21	0.21	0.49	0.49	0.09	0.09	0.063
4	0.4	0.6	4.25	1.0	0.24	0.24	0.36	0.36	0.16	0.16	0.096
5	0.5	0.5	4.25	1.0	0.25	0.25	0.25	0.25	0.25	0.25	0.125
6	0.6	0.4	4.43	0.89	0.24	0.214	0.16	0.143	0.36	0.320	0.128
7	0.7	0.3	4.96	0.63	0.21	0.132	0.09	0.057	0.49	0.309	0.093
8	0.8	0.2	5.84	0.39	0.16	0.062	0.04	0.016	0.64	0.250	0.050
9	0.9	0.1	7.08	0.22	0.09	0.020	0.01	0.002	0.81	0.178	0.018
10	1.0	0.0	8.67	0.12	0.0	0.00	0.0	0.00	0.50*	0.060*	0.00
					Sum	1.378	Sum	3.268	Sum	1.667	0.614

$\alpha = 3 \times \text{column (9)} \times \frac{\Delta x}{l} = 3 \times 3.268 \times 0.1 = 0.98$; $\alpha' = 3 \times \text{column (11)} \times 0.1 = 3 \times 1.667 \times 0.1 = 0.5$; $\beta = 6 \times \text{column (7)} \times 0.1 = 6 \times 1.378 \times 0.1 = 0.827$

$$\epsilon = \frac{2 \times \text{col. (12)}}{\text{col. (7)}} = \frac{2 \times 0.614}{1.378} = 0.891; \quad (2 - \epsilon) = 2 - 0.891 = 1.109$$

The girder span here used is the left end span in the example on p. 209, Case 2.

* For points at the ends of span only one-half of the actual values of ordinates is used in the table, because the area represented by the integral equals $\frac{1}{2}\Delta x[a + 2b + 2c + \ldots 2(n-1) + n] = \Delta x[\frac{1}{2}a + b + c + \ldots (n-1) + \frac{1}{2}n]$.

the negative bending moments but also the length of the region subjected to them is larger.

On the other hand, the magnitude of positive bending moments and the length of the region of positive bending moments are smaller for a girder with variable moments of inertia than for a girder with constant moments of inertia.

From the above, it follows that a girder with variable moments of inertia, if designed according to formulas based on the assumption of constant moment of inertia, would be deficient in the amount of negative bending-moment reinforcement; and also the top bars would not extend far enough into the span to take care fully of the regions subjected to negative bending moments. On the other hand, a large part of the positive bending-moment reinforcement would be superfluous. The resulting design, therefore, would be not only unsafe, but also uneconomical.

The numerical example on p. 209 in which bending moments were computed for an assumption of constant moments of inertia, and also for an assumption of variable moments of inertia, gives a good idea of the effect of the variation of moments of inertia.

Method of Computing Moments of Inertia. — How to find moments of inertia of reinforced-concrete continuous T-beams is a moot question. A distinction must be made between the moments of inertia of any section to be used for computing stresses at that section, and the moments of inertia necessary for finding the variation of moments of inertia of the girder.

In the first instance, concrete areas in the compression zone, as well as areas of the tension and compression steel, must be used in computations. The concrete area in the tensile zone, however, must be omitted because the rules assume that it does not contribute anything to the strength of the section.

In the second case, the object is not to find the numerical value of the moment of inertia at any particular section of the girder, but to determine how much larger is the moment of inertia I_x, at a point, x, than the minimum moment of inertia, I_c; and to find at all sections the ratios $\frac{I_c}{I_x}$, i.e., the ratios of the minimum moments of inertia to the moments of inertia at points x.

It is obvious that the rule used in formulas for determining dimensions, which requires that the effect of concrete in tension should be disregarded, cannot be used when finding the variation of the moments of inertia. Possible cracks in concrete in a properly designed structure are restricted to the sections of the member immediately adjacent to the

critical sections. In the bulk of the structure concrete in the tensile zone is always intact, and is active in affecting the moments of inertia of the cross sections.

An exact determination of moments of inertia of reinforced-concrete sections is not possible, but the preponderance of opinion of authorities is in favor of basing computations of moments of inertia, for the purpose of finding the variation, upon concrete sections only, and of considering them as homogeneous sections. In case of T-beams the same width of the flange should be used as is recommended for computations of compression stresses. At the supports, T-sections should be used even though the flange is in the tensile zone. When bottom slabs are used at supports, both flanges should be included in computing moments of inertia. For T-beams having equal flanges and equal widths of stems, but varying depths, it is considered accurate enough to assume that their moments of inertia vary in the same ratio as the third powers of their depths, or $\frac{I_c}{I_x} = \left(\frac{h_c}{h_x}\right)^3$.

NUMERICAL EXAMPLE FOR GIRDERS WITH CONSTANT AND VARIABLE MOMENTS OF INERTIA

The numerical example here given serves the following two purposes: (*a*) to illustrate the use of the fixed-point method for finding bending moments in continuous girders with unequal spans, (1) having constant moments of inertia, and also (2) having variable moments of inertia; (*b*) to demonstrate the effect of the variation in moments of inertia upon bending moments.

Example. — Find bending moments for a continuous girder bridge of three spans with freely supported ends. The spans are $l_1 = 64.0$ ft.; $l_2 = 80.0$ ft., and $l_3 = 64.0$ ft.

Case 1. Moments of inertia are constant throughout the girder.

Case 2. The underside of the girder in the center span is parabolic. In the end spans the underside is parabolic in the half next to the interior support, and straight in the other half. The moments of inertia of the girder are, therefore, variable. The center span is symmetrical and the end spans are unsymmetrical.

Loadings: Dead load, $w_d = 2\,600$ lb. per lin. ft.;

Equivalent live load plus impact, $w_l = 1\,500$ lb. per lin. ft.

Solution. — The fixed-point method is used for solving this problem. For both cases, ratios of spans are: $\frac{l_1}{l_2} = \frac{64.0}{80.0} = 0.8$, $\frac{l_2}{l_3} = \frac{80.0}{64.0} = 1.25$.

Case 1. *Constant Moments of Inertia.* — See Fig. 93, p. 210. Use formulas from the table on p. 200, item 4.

Left fixed points:

First span $\quad f_1 = 0$

Second span $f_2 = \dfrac{1}{3 + 2 \times 0.8} \times 80.0 = \dfrac{1}{4.16} \times 80.0 = 17.4$ ft.; $\dfrac{l_2}{f_2} = 4.60$

Third span $q_2 = 2 - \dfrac{1}{4.60 - 1} = 1.72$

$$f_3 = \frac{1}{3 + 1.25 \times 1.72} \times 64.0 = \frac{1}{5.15} \times 64.0 = 12.40 \text{ ft.}$$

Right fixed points symmetrical with left fixed points.

$$f'_3 = f_1 = 0;\ f'_2 = f_2 = 17.4 \text{ ft.};\ f'_1 = f_3 = 12.40 \text{ ft.}$$

Case 2. *Parabolic Variation of Depths of Girder.* — The dimensions of the girder are shown in Fig. 93, p. 210. It is accepted that moments of inertia vary as the third powers of the depths of cross sections.

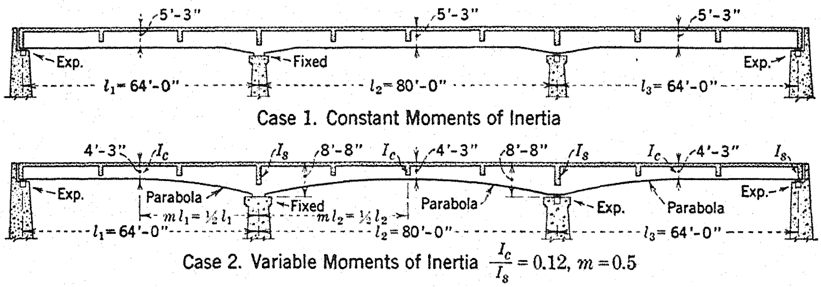


FIG. 93.—General Dimensions of Girders. (*See p.* 210.)

For the unsymmetrical end span constants, found by the summation method shown in the table on p. 207, are: $\alpha_1 = 0.98$, $\alpha'_1 = 0.5$, $\beta_1 = 0.827$, $\epsilon_1 = 0.891$.

For the symmetrical center span, with the ratio of moments of inertia $\dfrac{I_c}{I_s} = \left(\dfrac{4.25}{8.67}\right)^3 = 0.12$, and the ratio of length of haunch to span length $m = 0.5$, constants taken from Table II (*b*), p. 203, by interpolation, are: $\alpha_2 = \alpha'_2 = 0.47$, $\beta_2 = 0.66$.

In the third span the conditions are the same as in the first span, only reversed. The constants, therefore, are: $\alpha_3 = 0.5$, $\alpha'_3 = 0.98$, $\beta_3 = 0.827$, $\epsilon_3 = (2 - 0.891)$.

Since the minimum depths in all spans are the same,

$$I_1 = I_2 = I_3 \quad \text{and} \quad \frac{I_2}{I_1} = \frac{I_3}{I_2} = 1.0.$$

Using formulas for fixed points from the table on p. 207, item 7:
Left fixed points:

First span $f_1 = 0$

$$\text{Second span } f_2 = \frac{0.66}{2 \times 0.47 + 0.66 + 2 \times 1.0 \times 0.8 \times 0.5} \times 80.0$$

$$= \frac{1}{3.64} \times 80.0 = 22.0 \text{ ft.};\ \frac{l_2}{f_2} = 3.64$$

$$\text{Third span } q_2 = 2 \times 0.47 - \frac{1}{3.64 - 1} \times 0.66 = 0.69$$

$$f_3 = \frac{0.827}{2 \times 0.5 + 0.827 + 1.0 \times 1.25 \times 0.69} \times 64.0$$

$$= 0.31 \times 64.0 = 19.84 \text{ ft.}$$

Right fixed points: Owing to symmetry of spans, right fixed points are symmetrical with left fixed points. Thus

$$f'_3 = f_1 = 0;\ f'_2 = f_2 = 22.0 \text{ ft.};\ f'_1 = f_3 = 19.84 \text{ ft.}$$

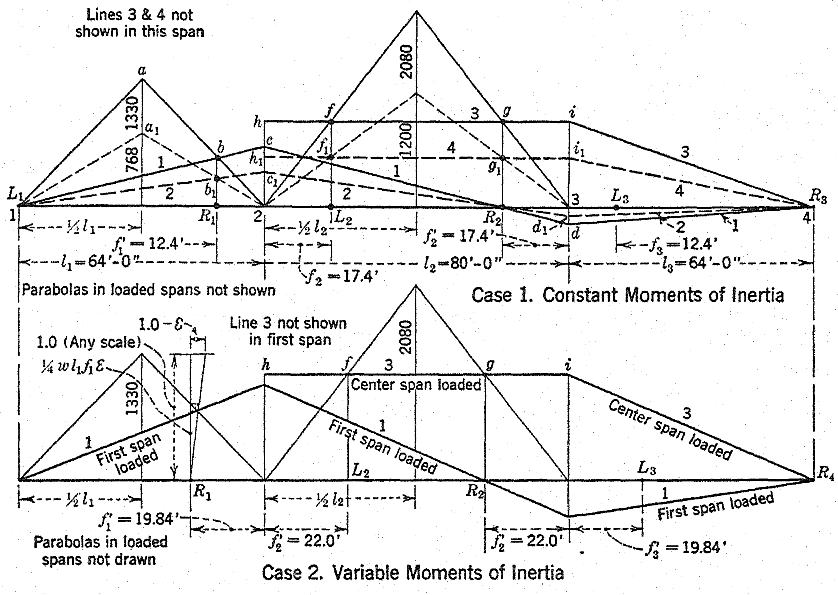


FIG. 94.—Example of Use of Fixed Points for Determining Bending Moments. (*See p.* 211.)

Static Bending Moments. — For simply supported spans the maximum static bending moments in the centers of spans are:

Dead load: End spans: $M_{s(d)} = \frac{1}{8} \times 2\,600 \times 64^2 = 1\,330\,000$ ft-lb.
Center span: $M_{s(d)} = 2\,080\,000$ ft-lb.

Live load and impact: End spans: $M_{s(l)} = \frac{1}{8} \times 1\,500 \times 64^2 = 768\,000$ ft-lb.
Center span: $M_{s(l)} = 1\,200\,000$ ft-lb.

Parabolas representing bending-moment diagrams are shown in Fig. 95, p. 213.

Determination of Bending Moments Using Fixed Points. — The procedure outlined on p. 195 is here followed.

1 and 2. The spans are laid out to scale in Fig. 94, Case 1, and the fixed points, previously computed, are plotted.

3. The first span is assumed to be the only loaded span; and on a vertical in the center of the span are plotted above the axis separately the maximum static bending

moment for dead load and for live load, which gives points a and a_1. These are connected with supports 1 and 2. At the fixed points verticals are erected. In this case the left fixed point L_1 coincides with the support 1. The diagonals intersect the vertical at R_1 at point b and b_1. Connect point 1 with points b and b_1 and extend the lines to intersection at points c and c_1. This determines the negative bending moments for live and dead load at support 2 for a condition when the first span only is loaded. The parabolas in the loaded span do not need to be drawn now. Starting with bending moments at the support 2, the bending-moment diagram is completed by connecting points c and c_1 with R_2; extending the lines to intersection at points d and d_1 at third support; and finally connecting these points with R_3 in the third span, which coincides with support 4. The lines L_1cdR_3 and $L_1c_1d_1R_3$ represent diagrams for conditions when the first span is the only loaded span for dead load and for live load, respectively.

Perform the same work for the assumption that the second span is the only loaded span, and get L_1hiR_3 for dead load and $L_1h_1i_1R_3$ for live load. Parabolas in the loaded center span do not need to be drawn.

Since the girder is symmetrical about the center, no diagram is drawn for the assumption that the third span is the only loaded span. The diagram for this condition is symmetrical with the diagram for the first span.

4. For dead load acting simultaneously on all spans, bending moment at the support 2 is found by adding $2c$ and $2h$ and subtracting $3d$, because, from symmetry, $3d$ is equal to the bending moment at support 2 when the third span is loaded. In Fig. 95, plot the total bending moment for dead load at support 2 and get point j. Only one-half of the diagram is drawn because the other half is symmetrical. Connect j with 1 and draw in the center span a line parallel to the axis. Starting from these closing lines plot the ordinates of parabolas for dead load, using values in Fig. 94. This gives the final bending-moment diagram for dead load.

5. For live load, consider the following three conditions of loading:

(*a*) First and second span loaded, gives maximum bending moments at supports. At support 2 add $2c_1$ to $2h_1$; and at support 3 from $3i_1$ subtract $3d_1$. Plot at the supports 2 and 3 the corresponding sum, and in Fig. 95 get points k and l. Draw parabolas for live load in the loaded spans.

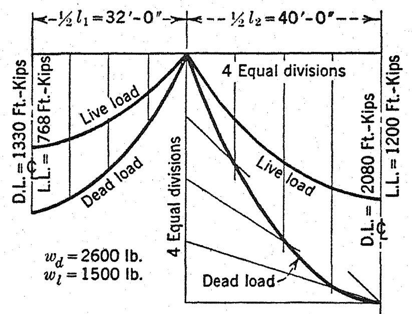
(*b*) Center span loaded, gives maximum positive bending moments in the center span, and largest negative bending moments in the central part of the end spans. Negative bending moment at each support equal to $2h_1$. Plot this in Fig. 95, and plot parabola for live load in the center span.

(*c*) Both end spans loaded, gives maximum positive bending moments in the end spans, and largest negative bending moments in the central part of the center span. The negative bending moment at each interior support equals $2c_1$ minus $3d_1$. Plot the value at support 2 and draw parabola in end span.

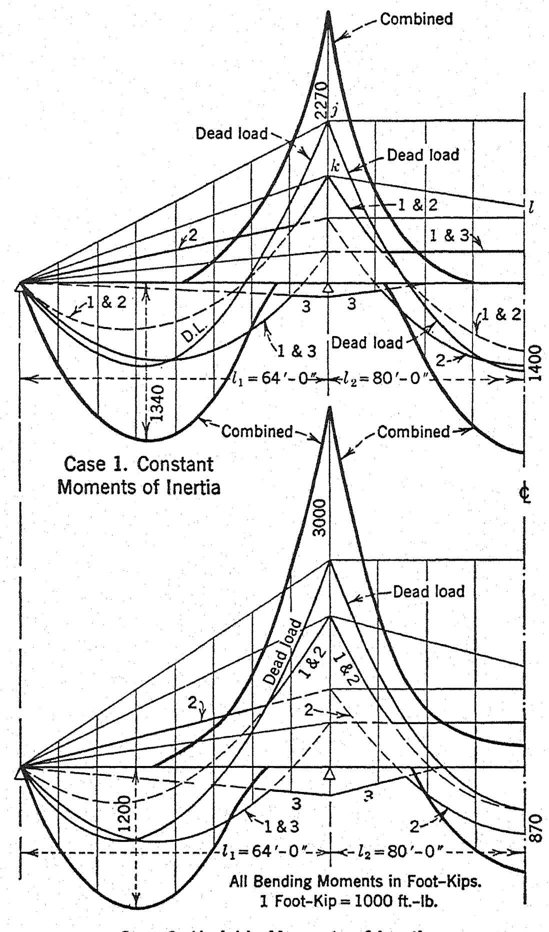
(*d*) To get positive bending moments at support 2, the third span is considered as loaded.

6. Combined Bending Moments: Bending moments for dead load are combined with bending moments for live load in the manner suggested on p. 196. In Fig. 95 the combined bending moments are indicated by heavy lines. These should be used for determining the dimensions of girders, the amount of reinforcement, and finally the points of bending of reinforcement.

Bending Moments for Girders with Variable Moments of Inertia. — The work performed for the girder with constant moments of inertia is duplicated for the girder with variable moments of inertia. The results are shown in Figs. 94 and 95, p. 213.



(*a*) *Static Bending Moment Curves*



Case 2. Variable Moments of Inertia

FIG. 95.—Example. Combined Bending-Moment Diagrams. (*See p.* 212.)

In the unsymmetrical end span the value of S_r is found graphically as for a symmetrical span, and then multiplied by the constant $\epsilon_1 = 0.891$ found by summation on p. 207.

Comparison of Bending Moments for Girders with Constant and Variable Moments of Inertia. — The effect of the variation of moments of inertia is evident from the comparison of the bending-moment diagrams in Fig. 95, Case 1, p. 213, with the diagrams in Fig. 95, Case 2, p. 213. Bending moments at critical sections are given in the following summary:

Moments of Inertia	Positive Bending Moments		Negative Bending Moments
	First Span $M_{1\max.}$, ft-lb.	Second Span $M_{2\max.}$, ft-lb.	Second Support M_2, ft-lb.
Constant	1 340 000.0	1 400 000.0	−2 270 000.0
Variable	1 200 000.0	870 000.0	−3 000 000.0

As evident from the summary, the maximum negative bending moment at the support for the girder with variable moments of inertia is 32 per cent larger than for the girder with constant moments of inertia. The maximum positive bending moments, on the other hand, are 11 per cent smaller in the first span, and 38 per cent smaller in the center span.

In a girder with variable moments of inertia, it is possible to adapt the dimensions at the various sections to the bending moments and to the shears there expected. This permits a better distribution of materials. Also it is possible to get a much larger headroom, because the depth of the girders in the central portion of each span, and particularly of the center span, may be made appreciably smaller than for a girder with constant moments of inertia.

In the above comparison, the advantage of the reduced dead load of the girder with variable moments of inertia is not taken into account. The same dead load was used in both cases in order to avoid the introduction of another variable.

Comparison of Continuous Girder with Multi-Span Rigid Frame. — To get a direct comparison of bending moments in a continuous girder with the corresponding bending moments in a rigid frame, a numerical example is given on p. 235 in which the spans, the dimensions of members, and the loadings are the same as in this example, the only variable being the fact that there the girder is rigidly connected with the supports to form a rigid frame. The effect of this connection may be evaluated by comparing the bending-moment diagram in Fig. 95 (case 1), p. 213, with that in Fig. 105 (case 1), p. 240, for constant moments of inertia. For variable moments of inertia compare Fig. 95 (case 2), p. 213, with Fig. 105 (case 2), p. 240.

USE OF FIXED POINTS FOR CONTINUOUS GIRDERS WITH CANTILEVERS

The fixed-point method may also be used advantageously for continuous beams and girders having cantilevers at one or at both ends.

The cantilevers do not affect the position of the fixed points. The bending moments in the main spans for the loads on the main spans are found in the same manner as for girders without cantilevers. The loads on the cantilevers produce bending moments in all spans of the continuous girder.

To get bending moments in the main spans produced by the cantilever loads, proceed as follows:

For cantilever loads, find the maximum bending moment at the support of the cantilever separately for dead load and for live load, and plot each at the support. Connect each point with the opposite fixed point of the span next to the loaded cantilever, and extend the line to intersection with a vertical at the opposite support of this span. Connect this new point with the opposite fixed point in the second span, and continue the work until all spans are taken care of, as shown in Fig. 96, below.

Bending-moment diagrams should be drawn separately for dead load and for live load. When in a symmetrical girder the cantilevers at both ends are of equal lengths, and carry the same loads, the bending-moment diagrams for one cantilever are sufficient, because for the other cantilever they are obtained from symmetry. When cantilevers are unsymmetri-

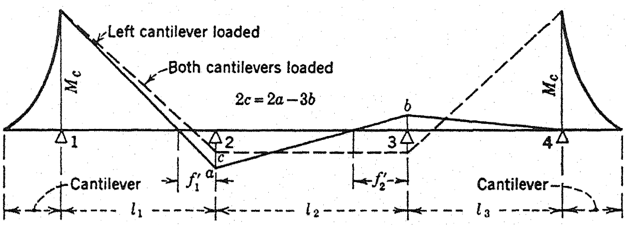


FIG. 96.—Bending Moments in Continuous Girder for Cantilever Loads. (See p. 215.)

cal, separate bending-moment diagrams must be drawn for each cantilever. For dead load, bending moments for both cantilevers are combined, as shown in Fig. 96, p. 215, by dash lines.

After the bending moments for the cantilever loads are plotted, they are combined with the bending moments due to the loads on the main spans in the same manner as explained on p. 184 in connection with equal-span girders with cantilevers.

Shears for Continuous Girders with Cantilevers. — The cantilever loads produce shears in all spans of a continuous girder. These are constant throughout each span, and their magnitude may be found from formula (19), p. 217, after the bending moments at the supports are computed.

USE OF FIXED POINTS WHEN END OF GIRDER IS RIGIDLY CONNECTED WITH SUPPORTING COLUMNS

Often, it is of advantage to connect rigidly the ends of a continuous girder with the supporting end columns, even when the girder is not connected with the other supports. In such case the supporting end column may be considered as an additional span of the girder; and the girder may be treated in the same manner as if this additional span were horizontal. The length of the end span should be taken as equal to the theoretical height of the end column, and the condition of restraint at the end should be accepted the same as the condition of restraint of the column at the bottom.

When the end column supports a wall resisting earth pressures, the bending moment in the column and in the girder due to this pressure should be found as explained on p. 247, using the fixed points. No correction is necessary here for unsymmetrical loading, such as is required in a rigid frame, because the girder is fixed to the support at the fixed bearing, and therefore the column heads cannot move horizontally. (See also p. 263.)

USE OF FIXED POINTS FOR CONSTRUCTING INFLUENCE LINES

Fixed points may be used to construct influence lines for bending moments for continuous girder of any number of equal or unequal spans.

The method of using fixed points for this purpose is fully explained in "Concrete, Plain and Reinforced," Vol. II, p. 162, and illustrated there by a numerical example.

EXTERNAL SHEARS IN CONTINUOUS GIRDERS

External shears in any span of a continuous beam or girder for any loading may be expressed by the external shear in that span, considering it as simply supported at both ends, and in terms of the bending moments at both supports of that span for the same loading. Thus, bending moments in the girder having been found by means of fixed points, the external shears may be easily computed from the following formulas.

Let V_n = end shear for continuous girder.

V_s = corresponding end shear for span considered as simply supported.

V_x = external shear at any point x for continuous girder.

V_{sx} = corresponding static shear at point x for simply supported span.

M_n = bending moment at left support of continuous span.

M_{n+1} = bending moment at right support of continuous span.

l_n = span length.

End Shear at Left Support of Continuous Span:

$$V_n = V_s - \frac{M_n - M_{n+1}}{l_n} \tag{19}$$

External Shear at any Point x from Left Support:

$$V_x = V_{sx} - \frac{M_n - M_{n+1}}{l_n} \tag{20}$$

In the above formulas, the values of M_n and M_{n+1} must be taken with their signs. Both values may be negative, or one value may be negative and the other positive. The second term in formulas (19) and (20) may be either positive or negative, so that the resulting shear may be either larger or smaller than for a simply supported span.

When the bending moments are in foot-pounds, the length of the span in the formulas should be in feet; and for inch-pounds in inches. In both cases the result will be in pounds.

External Shears in Unloaded Spans of Continuous Girder. — Unloaded spans of a continuous girder, when any other span is loaded, are subjected to a shear constant throughout the span. Its magnitude may be found from formula (19), p. 217, by substituting V_n equal to zero.

Shears in Span of Continuous Girder for Moving Live Loads. — The maximum positive end shear in a span due to moving live loads is produced when all spans producing positive shears in the span under consideration are fully loaded, and also the span under consideration is fully loaded. For this condition of loading find bending moments at both supports of the span and the static shear as required by Fig. 97, p. 218.

The construction of the shear diagram for moving loads is shown clearly in Fig. 97, p. 218. This diagram is not quite exact, as it does not take into account the changes in bending moments at the supports when the load in the span is gradually reduced; however, it is accurate enough for practical purposes.

The external negative shears in the span for moving live loads may be found similarly. Negative bending moments at supports in Fig. 97 for this case are designated by M'_n and M'_{n+1} to distinguish them from bending moments M_n and M_{n+1} for the loading used for the positive shears in the span.

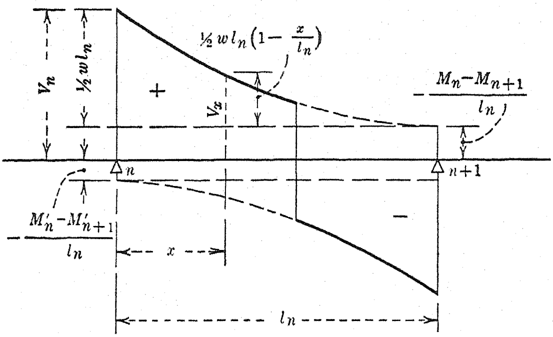


FIG. 97.—Maximum Shears for Moving Loads. (*See p.* 217.)

Loadings of Continuous Girder which Produce Largest External Shears. — In determining largest external shears, the following loadings should be considered:

For dead loads, all spans should be considered as loaded simultaneously.

For uniformly distributed live loads, at the end supports maximum end shear is produced by the same loading which produces maximum positive bending moment in the end span.

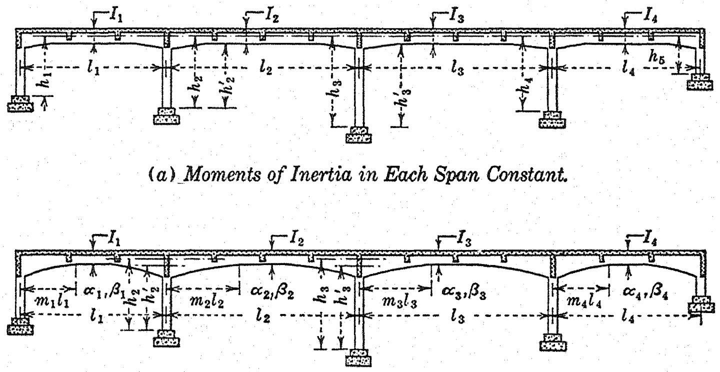
At any other support, maximum end shears are produced by the loadings which produce maximum negative bending moments at that support.

CHAPTER XI

MULTI-SPAN RIGID FRAMES. FIXED-POINT METHOD

Multi-span rigid frame bridges, which are so economical because they take advantage of the rigidity of joints readily attained in reinforced concrete, are described and illustrated in Chapter VIII. In this chapter are given means for determining bending moments and shears in multi-span frames, which may be used for frames of any number of equal or unequal spans supported by, and rigidly connected with, vertical members of equal or unequal heights, and with any conditions of restraint at the ends of the girders and of the columns. No higher mathematics is involved in the solution of problems connected with the design of rigid frames. The treatment is relatively simple and easy to understand.

General arrangements of members of frames here treated are shown in Fig. 98, below.



(a) Moments of Inertia in Each Span Constant.

Note: Since girder in each span is symmetrical $\alpha_1 = \alpha'_1, \alpha_2 = \alpha'_2, \alpha_3 = \alpha'_3$

(b) Moments of Inertia in Horizontal Members Variable.

FIG. 98.—Multi-span Rigid Frames. (*See p.* 219.)

The fixed-point method is used here to solve the problems. Separate formulas are given for frames in which moments of inertia in each member are constant, and for frames in which moments of inertia in the horizontal members are variable and in vertical members constant.

The method of procedure for determining bending moments is outlined and illustrated by a numerical example.

To illustrate the effect upon the bending moments in a frame, of the variation of moments of inertia in the horizontal members, bending moments in the numerical example on p. 235 are computed for two conditions: (1) when moments of inertia are constant throughout the whole length of the girder; (2) when moments of inertia of horizontal members are variable.

To illustrate the effect upon bending moments in the horizontal members of the rigid connection with the columns, the number and the lengths of spans in the numerical example in this chapter, as well as the loading, are made the same as in the numerical example for continuous girders given on p. 209. The results are compared and discussed.

The method here given may be used without corrections for all conditions of vertical loadings of frames consisting of three or more spans, and for all symmetrical loadings of all symmetrical frames. For frames of one or two spans, bending moments for unsymmetrical loadings obtained by this method should be corrected in the manner outlined on pp. 245, 306 and 317, respectively.

Bending moments in all frames due to unsymmetrical cantilever loadings also should be corrected as explained on p. 246.

The use of the fixed-point method treated in this chapter for determining the effect of horizontal pressures such as earth pressures, acting upon the vertical members of rigid frames, is fully discussed in Chapter XII. In the same chapter are given means for finding the effects upon the frames of temperature changes, shrinkage, and settlements of foundations. These analyses also are based on the fixed-point method given in this chapter. Effect of traction forces are considered on p. 249.

Definition of Multi-Span Rigid Frames. — By the term "multi-span rigid frame" is meant a unit consisting of a longitudinal continuous member which is rigidly connected with all the elastic vertical members upon which it rests. By "rigid connection" is meant a connection in which each joint between the members is able to resist all bending moments and shears to which it may be subjected.

Each vertical member of a frame must be connected with the foundation so as to be able to resist horizontal thrusts acting at its bottom end. The connections at the end may be hinged, partly restrained, or fixed.

Fixed-Point Method. — The fixed-point method for rigid frames is based upon the same general principles as those for continuous girders in Chapter X. The location of fixed points is shown in Fig. 99, p. 221. Here, also, each member has two fixed points. But in addition to these fixed points, at each column there are indicated ratios r_n, r_{n+1},

r_{n+2}, etc., the significance of which is explained later. The definition, in this case, of the fixed points is the same as for continuous girders, on p. 191. Here also they can be used directly only where one span or one vertical member is loaded or originates bending moments, while all other spans and members are not loaded.

To find bending moments in a frame for a condition of loading where several spans or vertical members are loaded simultaneously, it is necessary to treat the loading of each member separately, and then to add the results.

Degree of Exactness of Fixed-Point Method for Rigid Frames. — The method given in this chapter may be used without corrections only for vertical loadings of frames consisting of three or more spans. For unsymmetrical loadings of one- and two-span frames the results should be corrected as explained on pp. 245, 306 and 317.

When used without corrections, the fixed-point method does not take into account the horizontal movements of the column heads which take

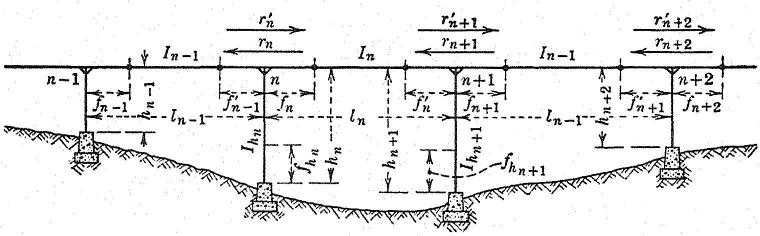


FIG. 99.—Fixed Points for Rigid Frame. (*See p.* 220.)

place under some conditions of loadings, the effect of which is fully discussed on p. 252. For symmetrical loadings of symmetrical frames, no such horizontal movements of the column heads take place. For unsymmetrical loadings, however, all column heads move either to the right or to the left, which means that the frame as a whole tilts slightly, and the girder as a whole moves horizontally. The effect upon bending moments of such movements for frames of three or more spans is negligible, and no corrections are necessary. For frames of one or two spans, the results should be corrected.

For unsymmetrical horizontal pressures and for unsymmetrical cantilever loadings, the effect of horizontal movements of column heads is always appreciable. See p. 246.

Difference between Multi-Span Continuous Girders and Multi-Span Frames. — The fixed-point method as applied to frames may be better understood by considering the difference between the action of a continuous girder and that of a multi-span rigid frame.

In a continuous girder which is not connected with the supports, the bending moments in the girder on one side of any support are transmitted in full to the girder on the other side of that support. For a one-span loading when the bending moments at the supports of the loaded span are known, the bending moments in the balance of the girder can be easily obtained by using the fixed points.

In a multi-span frame, on the other hand, three members meet at each support, namely, two spans of the horizontal member and one vertical member. The bending moments acting at the end of one horizontal member are not transmitted in full to the horizontal member in the next span; but a part of the bending moments is transferred to the horizontal member and the balance is resisted by the vertical member. Consequently, to solve the problem of bending moments in rigid frames it is not sufficient to know the positions of the fixed points, but also it is necessary to know the ratio by which the original bending moment must be multiplied to get the transferred bending moment. This ratio is called the ratio of transference, and is indicated by the letter r. Its practical use is illustrated in examples which follow.

Fixed Points and Ratios of Transference in Rigid Frame. — Fixed points and ratios of transference in a rigid frame are shown in Fig. 99, p. 221. Each horizontal member has a left and a right fixed point in each span. Each vertical member also has two fixed points, the upper and the lower fixed point; but for vertical loadings only the lower fixed point is needed.

For vertical loadings, there are two ratios of transference at each joint. The left ratio is used when bending moments are transferred from the right span to the left span; and the right ratio is used when they are transferred in the opposite direction. The left ratios are designated by the letter r with sub-numbers indicating the column to which they apply. The right ratios are indicated by r' with similar sub-numbers.

Ratios of transference may be indicated by diagonals drawn at the joints at such angles that they may be used for graphical determinations of the transferred bending moments. The construction and the use of these diagonals are clearly shown in Figs. 104, p. 235, and 112, p. 262.

When the bending moments at both supports of the loaded span are known, the bending-moment diagram in the balance of the frame may be easily drawn using the ratios of transference and the fixed points.

Use of Fixed Points in Loaded Span. — The location of the fixed points being known, the bending moments at the supports in the loaded span, for a condition of loading when only one span is loaded, may be found by plotting at the fixed points of that span the values S_l and

S_r, which are computed and used in the same manner as for continuous girders. Formulas for S_l and S_r are given on p. 193; the graphical method for uniformly distributed loads is discussed on p. 194. Formulas for bending moments at supports are also given on p. 196.

In Fig. 100, p. 223, the third span is loaded, and the bending moments at supports determined as explained above are M_{3r} and M_{4l}.

Use of Fixed Points in Unloaded Spans. — To be able to complete a bending-moment diagram when bending moments at the supports of the loaded span are known, it is necessary to know the location of fixed points and the ratios of transference. When the bending moment at the support in one span is known, the bending moment in the adjoining span at the same support is obtained by multiplying the original bending moment by the proper ratio of transference. The left ratio of transference is used when the span under consideration is to the left of the loaded span; and the right fixed point, marked r', is used when the span is to the right of the loaded span.

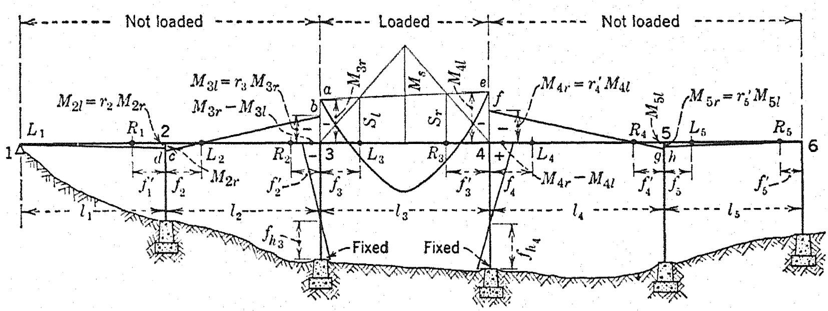


FIG. 100.—Use of Fixed Points in Rigid Frame. (*See p.* 223.)

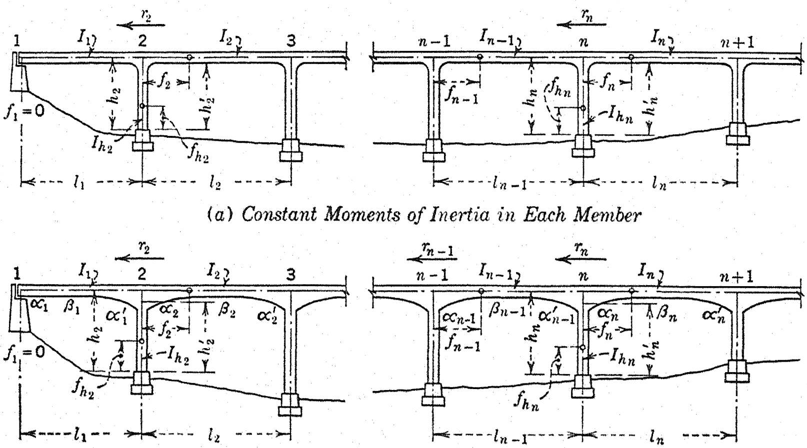
The use of fixed points and ratios of transference is illustrated in Fig. 100, p. 223, in which the third span is the loaded span, and all other spans are not loaded. At the left support of the loaded span, i.e., at support 3, the bending moment in the loaded span is M_{3r} and is indicated by $3a$. By multiplying this value by the left ratio of transference r_3, M_{3l} is obtained at support 3 in the second span; this plotted gives the point b. By connecting point b with the left fixed point in the second span, and extending it to point c, M_{2r} is obtained, which is the bending moment to the right of support 2. This multiplied by r_2 gives the bending moment to the left of support 2, which plotted gives point d. Line dL_1 completes the diagram.

To the right of the loaded span, M_{4r} equals M_{4l} multiplied by r'_4. The rest of the diagram is completed in the same manner as for the other side.

Bending Moments in Columns. — For any condition of loading, the bending moment at the top of any column is equal to the difference between the bending moment acting in the horizontal member to the right of that column and the bending moment to its left.

Thus, in Fig. 100, p. 223, the bending moment at the top of column 3 is equal to $M_{3r}-M_{3l}$, and at the top of column 4 to $M_{4r}-M_{4l}$. In each case bending moments should be used with their signs.

To find bending moments in columns at intermediate points, plot for each column the bending moment on a horizontal erected at the top of the diagram, and connect the point with the lower fixed point as shown in Fig. 100, p. 223, for columns 3 and 4. The straight line is the bending-moment diagram.



(*a*) *Constant Moments of Inertia in Each Member*

(*b*) *Variable Moments of Inertia*

Note: Only left fixed points and left ratios of transference are indicated.

FIG. 101.—Left Fixed Points. Rigid Frame, No End Columns. (*See p.* 224.)

FORMULAS FOR FIXED POINTS AND RATIOS OF TRANSFERENCE

Assumption as to Moments of Inertia. — Formulas for fixed points and ratios of transference are here given for two assumptions as to moments of inertia.

1. Moments of inertia are different in different members, but constant throughout the length of each member.

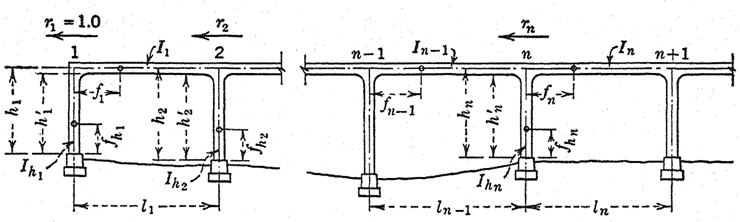
2. Moments of inertia of horizontal members are variable, and in vertical members constant.

These conditions are shown in Figs. 101 and 102, p. 225, where for the sake of clearness only those left fixed points and ratios of transference are indicated for which formulas are given on pp. 228 to 230.

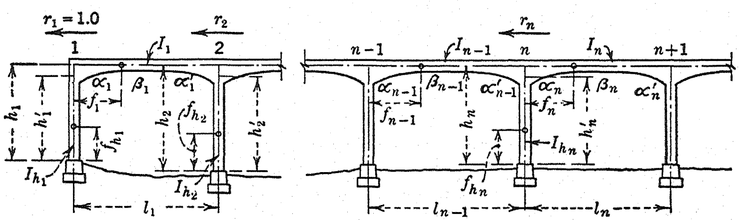
In both cases, moments of inertia are affected by the solid blocks of concrete at the intersections of the horizontal members with the vertical members. Since the cross section of each member within these intersection blocks is appreciably larger than outside of the blocks, the moments of inertia are proportionally increased.

So far as horizontal members are concerned, the effect of the intersection blocks upon their moments of inertia may be disregarded because the lengths of the blocks are usually small in comparison with the lengths of the members.

The conditions are different with the vertical members. In bridge design, the depth of the blocks often forms a considerable part of the height of each vertical member, and therefore, these blocks affect to



(*a*) *Constant Moments of Inertia in Each Member*



(*b*) *Variable Moments of Inertia*

Note: Only left fixed points and left ratios of transference are indicated.

FIG. 102.—Left Fixed Points. Rigid Frame with End Columns. (*See p.* 224.)

some extent the moments of inertia. To take care of this in formulas here given, vertical members are considered as having constant moments of inertia only throughout their clear heights, h', while in the portions within the intersection blocks the moments of inertia are assumed to be infinitely large. The actual restraint exerted by vertical members thus stiffened, upon the horizontal members is larger than it would be if their moments of inertia were constant throughout. The ratio of transference from the girder on one side to the girder on the opposite side of the vertical member is then somewhat smaller, because a larger proportion of the bending moment is transferred to the vertical member stiffened by the intersection block.

The effect of the intersection blocks upon the fixed points and ratios of transference is taken care of by the introduction in the formulas of the ratios $\frac{h'}{h}$, where h and h' are as defined under the next heading.

Frame Axes and Theoretical Lengths of Members. — In frames with straight members, all axes coincide with the straight lines drawn through the centers of gravity of concrete cross sections of the members. Short haunches may be disregarded.

When horizontal members of a frame are provided with haunches of appreciable length, the lines connecting the centers of gravity of concrete sections are composite lines the shapes of which depend upon the lengths and the shapes of the haunches. These composite lines should be replaced by a straight axis so located that the areas below the axis comprised between the composite lines and the axis are about equal to the areas above the axis.

The theoretical span length of a horizontal member, l, is measured between the center lines of the supporting vertical members.

The theoretical height of a vertical member, h, is measured from the horizontal axis to the top of the footing, if the dimensions of the footing are substantially larger than the dimensions of the vertical member. Where the dimensions of the footing do not exceed appreciably the dimensions of the vertical member, the height of the column should be measured from the bottom of the footing. For intermediate conditions the theoretical location of the bottom of the vertical member must be selected by judgment.

The clear height of the vertical member, h', as used in the formulas, is the distance from the bottom of the member to the underside of the horizontal member. Where haunches are used h' is the distance from the bottom of the member not to the underside of the haunch, but to a point within the haunch where the cross section of the vertical member has been very appreciably increased by the intersection block. (See assumption under preceding heading.) Short haunches may be disregarded, and the clear height measured to the underside of the girder. For haunches of appreciable lengths, a part, but not more than one-third, of their depth may be deducted from the clear height as determined for girders without haunches.

Notation. — In the formulas that follow, in addition to the notation on p. 197, let:

$h, h_1, h_2, h_{(n-1)}, h_n$ = theoretical height of vertical member in frame.

$h', h'_1, h'_2, h'_{(n-1)}, h'_n$ = clear height of vertical frame member as defined above.

$I_h, I_{h1}, I_{h2}, I_{h(n-1)}, I_{hn}$ = constant moments of inertia of vertical members.

$f_h, f_{h1}, f_{h2}, f_{h(n-1)}, f_{hn}$ = distances of lower fixed points in vertical members from bottom.

$r, r_1, r_2, r_{(n-1)}, r_n$ = left ratios of transference from girder to girder.

Lower Fixed Points in Vertical Members. — The position of the lower fixed point in a vertical member depends upon the degree of restraint at the bottom of the member, and upon the ratio of the clear height to the theoretical height, $\frac{h'}{h}$. It may be obtained from the following formulas:

Lower Fixed Point in Vertical Member:

$$f_h = 0 \quad \text{member hinged at bottom} \tag{1}$$

$$f_h = \frac{1}{9}\left(2 + \frac{h'}{h}\right)h \quad \text{member fixed at bottom}$$

$$\left(\text{for } \frac{h'}{h} = 1, \quad f_h = \tfrac{1}{3}h\right) \tag{2}$$

For members partially restrained at the bottom, intermediate values for the fixed points should be used.

Upper Fixed Point in Vertical Members. — For solving problems treated in this chapter, upper fixed points are not needed. They are needed, however, for determining the effects of horizontal pressures and of changes of temperature, and may be found from formulas on p. 255.

Moments of Inertia of Vertical Members. — Formulas above are based on the assumption of constant moments of inertia in vertical members, except as modified by the intersection block.

For tapering columns such as shown in Fig. 124, p. 287, it is accurate enough to use formulas for fixed points and ratios of transference based on the assumption of constant moments of inertia in vertical members, provided that the value of the moment of inertia I_h used in the formulas is made equal to the moment of inertia of a section of the vertical member located above the footing a distance equal to $0.65h$, where h is the theoretical height.

General Formulas for Left Fixed Points and Left Ratios of Transference in Horizontal Members. — Left fixed points and left ratios of transference from girder to girder may be found from the following formulas, using the previous notation.

Moments of Inertia in Each Member Constant. — (See Figs. 101 (*a*) and 102 (*a*), pp. 224 and 225.)

Left Ratio of Transference at nth Column:

$$r_n = \frac{1}{1 + \dfrac{I_{hn} l_{(n-1)}}{I_{(n-1)} h_n} \dfrac{q_{(n-1)}}{q_{hn}}} \tag{3}$$

Distance of Left Fixed Point in nth Span from nth Column:

$$f_n = \frac{1}{3 + r_n \dfrac{I_n l_{(n-1)}}{I_{(n-1)} l_n} q_{(n-1)}} l_n \tag{4}$$

where

$$q_{(n-1)} = 2 - \frac{1}{\dfrac{l_{(n-1)}}{f_{(n-1)}} - 1} \tag{5}$$

$$q_{hn} = \left(\frac{h'_n}{h_n}\right)^2 \left[2\frac{h'_n}{h_n} - \frac{1}{\dfrac{h_n}{f_{hn}} - 1}\left(3 - 2\frac{h'_n}{h_n}\right)\right] \tag{6}$$

For $\dfrac{h'_n}{h_n} = 1.0$, formula (6) changes to $q_{hn} = 2.0$ for hinged end of vertical member; and to $q_{hn} = 1.5$ for fixed end.

Moments of Inertia in Horizontal Members Variable. — (See Figs. 101 (*b*) and 102 (*b*), pp. 224 and 225.)

Left Ratio of Transference at nth Column:

$$r_n = \frac{1}{1 + \dfrac{I_{hn} l_{(n-1)}}{I_{(n-1)} h_n} \dfrac{q_{(n-1)}}{q_{hn}}} \tag{7}$$

Distance of Left Fixed Point in nth Span from nth Column:

$$f_n = \frac{\beta_n}{2\alpha_n + \beta_n + r_n \dfrac{I_n l_{(n-1)}}{I_{(n-1)} l_n} q_{(n-1)}} l_n \tag{8}$$

where

$$q_{(n-1)} = 2\alpha'_{(n-1)} - \frac{1}{\dfrac{l_{(n-1)}}{f_{(n-1)}} - 1} \beta_{(n-1)} \tag{9}$$

$$q_{hn} = \left(\frac{h'_n}{h_n}\right)^2 \left[2\frac{h'_n}{h_n} - \frac{1}{\dfrac{h_n}{f_{hn}} - 1}\left(3 - 2\frac{h'_n}{h_n}\right)\right] \tag{10}$$

For $\dfrac{h'_n}{h_n} = 1.0$, formula (10) changes to $q_{hn} = 2.0$ for hinged end of vertical member; and to $q_{hn} = 1.5$ for fixed end.

Constants $\alpha'_{(n-1)}$, $\beta_{(n-1)}$, α_n, and β_n depend upon the variation of moments of inertia in the $(n-1)$th and nth spans of the horizontal members, respectively. Formulas for these constants are given on p. 202; tables of constants for straight and parabolic haunches are given on p. 203. Summation method of finding constants for any shape of haunches is given on p. 205.

HOW TO SOLVE FORMULAS FOR FIXED POINTS AND RATIOS OF TRANSFERENCE

In determining the left fixed points and the left ratios of transference, it is first necessary to find the lower fixed points for all vertical members, using formulas (1) and (2), p. 227. Next, values are determined for the horizontal member starting at the left end of the frame, and proceeding in an orderly sequence from span to span until all spans are taken care of.

Frames Without End Columns at Left End. — For frames without end columns at the left end, the fixed point in the first span depends only upon the degree of restraint at its end. When the end is free, the distance f_1 equals zero for constant as well as for variable moments of inertia. See Fig. 101, p. 224.

Left Fixed Point in First Span, End of Girder Free:

$$f_1 = 0 \tag{11}$$

When the first span is fixed at the end,

Left Fixed Point in First Span, Girder Fixed at End:

$$f_1 = \tfrac{1}{3}l_1 \quad \text{for constant moments of inertia} \tag{12}$$

$$f_1 = \frac{\beta_1}{2\alpha_1 + \beta_1}\, l_1 \text{ for variable moments of inertia} \tag{13}$$

For the second span use the general formulas by making the following substitutions:

$$\frac{l_{(n-1)}}{f_{(n-1)}} = \frac{l_1}{f_1};\ \frac{I_{hn}l_{(n-1)}}{I_{(n-1)}h_n} = \frac{I_{h2}l_1}{I_1h_2};\ \frac{I_nl_{(n-1)}}{I_{(n-1)}l_n} = \frac{I_2l_1}{I_1l_2};\ \left(\frac{h'_n}{h_n}\right)^2 = \left(\frac{h'_2}{h_2}\right)^2$$

When moments of inertia are variable, substitute also $\alpha'_{(n-1)} = \alpha'_1$; $\alpha_n = \alpha_2$; $\beta_{(n-1)} = \beta_1$; and $\beta_n = \beta_2$.

Frames With End Column at Left End.— For a frame provided at the left end with an end column, consider the end column as the first span of a continuous girder and the first span of the frame as the second span

of the continuous girder. (See Fig. 102, p. 225.) The formulas then become:

Left Fixed Points in First Span, Frame with End Columns. Constant Moments of Inertia:

$$f_1 = \frac{1}{3 + \dfrac{I_1 h_1}{I_{h1} l_1} q_{h1}} l_1 \tag{14}$$

where[1]

$$q_{h1} = \left(\frac{h'_1}{h_1}\right)^2 \left[2\,\frac{h'_1}{h_1} - \frac{1}{\dfrac{h_1}{f_{h1}} - 1}\left(3 - 2\frac{h'_1}{h_1}\right)\right] \tag{15}$$

Left Fixed Point in First Span. Frame with End Columns. Variable Moments of Inertia:

$$f_1 = \frac{\beta_1}{2\alpha_1 + \beta_1 + \dfrac{I_1 h_1}{I_{h1} l_1} q_{h1}} l_1 \tag{16}$$

where[1]

$$q_{h1} = \left(\frac{h'_1}{h_1}\right)^2 \left[2\,\frac{h'_1}{h_1} - \frac{1}{\dfrac{h_1}{f_{h1}} - 1}\left(3 - 2\frac{h'_1}{h_1}\right)\right] \tag{17}$$

For the second span, use the general formulas on p. 228, making the same substitutions as are indicated on p. 229 for the frame without end columns.

RIGHT FIXED POINTS AND RIGHT RATIOS OF TRANSFERENCE

For symmetrical frames, after the left values are computed, values for right fixed points and right ratios of transference are obtained from symmetry.

For unsymmetrical frames, the right values must be computed separately. For this purpose renumber the spans, starting from the right end, and use the general formulas in the same manner as for left fixed points but progressing from right to left. It should be noted that

[1] For $\dfrac{h'_1}{h_1} = 1$, $q_{h1} = 2.0$ for hinged end of vertical member; and $q_{h1} = 1.5$ for fixed end.

for members with variable moments of inertia, after the spans are renumbered, the designating marks of the constants α and α' in each span also should be transposed.

Right fixed points are indicated by $f'_1, f'_2 \ldots f'_n$.

Right ratios of transference are indicated by $r'_1, r'_2 \ldots r'_n$.

DESIGN OF MULTI-SPAN RIGID FRAMES

Before it is possible to use the fixed-point method for determining bending moments and shears in the frame of a bridge, it is necessary to perform a considerable amount of preliminary work in the same manner as explained on p. 194 in connection with continuous girder bridges. In addition to the work there outlined under a to f, for a rigid frame it is necessary to determine the theoretical heights of the vertical members, $h_1, h_2 \ldots$, and their clear heights, $h'_1, h'_2 \ldots$, as defined on p. 226. Also, it is necessary to decide whether to make the ends of the columns fixed or hinged. The manner of ending up the bridge should be decided upon, namely: whether to use separate abutments, and rest the end spans upon them; or whether to use end columns rigidly connected with the end spans. The advisability of using cantilevers at the ends should be considered also.

Preliminary dimensions of the horizontal members should be accepted in order to determine the clear height of the columns. Definite ratios of moments of inertia $\frac{I_n}{I_{hn}}$ should be accepted. In the final design, columns of such cross sections should be used that the ratios of the actual moments of inertia agree closely with the ratios assumed in computations.

It is obvious that the preliminary work outlined above is of great importance, because it affects to a great extent the cost of the structure. Examples of structures actually built may serve as a guide in selecting the preliminary dimensions and the arrangement of frames in the structure to be designed.

After the preliminary work is performed, the bending moments in the frame may be determined by the following procedure. After the bending moments and shears are computed, final concrete dimensions are determined, as well as the amount and length of the reinforcement.

Procedure for Finding Bending Moments in Frames. — This procedure may be followed for frames with constant as well as with variable moments of inertia.

1. Lay out to a conveniently large scale the span lengths and also the theoretical heights of the columns. When the numerical solution of bending moments described on p. 196 is used, freehand sketches of the frame are sufficient.

2. Find the locations of the left fixed points, and the left ratios of transference, using the formulas on pp. 228 to 230, and proceeding as explained on p. 229. Also find the lower fixed points for columns. For symmetrical frames the right fixed points and the right ratios of transference are obtained from symmetry; for unsymmetrical frames they are computed separately as outlined on p. 230. Plot all fixed points, and indicate the ratios of transference.

3 to 6. Find bending moments for all the individual loadings of the spans, then final bending moments for dead loads, and finally bending moments for the most unfavorable positions of live load. Find the combined bending moments in the same manner as outlined under 3 to 6 on p. 195 in connection with continuous girders. In this case, bending moments in the unloaded spans should be determined as given on p. 223, and bending moments in the columns as explained on p. 224.

External Shears in Multi-Span Frames. — The shears in multi-span frames depend upon the loading of the member under consideration, and also upon the bending moments at the supports of the same member. Formulas for external shears in this case are the same as those given on p. 217 for continuous girders.

For live loads, the conditions of loading for maximum shears are the same as given on p. 218 for continuous girders.

Horizontal Thrusts and Shears in Vertical Members. — In a rigid frame, all vertical members are subjected to bending moments and to horizontal thrusts acting at the bottom of each vertical member. There is a definite relation between the bending moments in the columns and the thrusts. The horizontal thrusts must be resisted at the bottom of each column by the foundation. The shears in the columns produced by the thrusts must be resisted in the same manner as external shears in beams.

When the frame is subjected only to vertical loading, for known bending moments at top and bottom of column, the horizontal thrust may be found from the following formula.

Let M_T = bending moment in vertical member at top.
M_B = bending moment in vertical member at bottom.
h = theoretical height of column.

Then:

Horizontal Thrust:

$$H = \frac{1}{h}\left(M_T - M_B\right) \qquad (18)$$

In the formula just given, use the bending moments M_T and M_B with their signs. The resulting sign of the thrust determines its direction.

Horizontal Thrusts, Vertical Member Subjected to Earth Pressures. — When the vertical member is subjected to earth pressures, the horizontal thrust at the bottom of the member is found as given on p. 248 as part of Fig. 107.

Sign of Thrusts. — The sign of thrusts is positive, when they act from right to left; and negative when they act from left to right.

Sum of Thrusts in Rigid Frame. — For a rigid frame in equilibrium subjected to vertical loads, the sum of the thrusts at all vertical members must be equal to zero. This means that the sum of all thrusts acting from left to right must be equal to the sum of all thrusts acting in the opposite direction.

External Shears Due to Thrust in Vertical Members. — Horizontal thrusts produce in vertical members shears which are constant throughout the length of each member.

Direct Pressure upon Horizontal Members Due to Thrusts. — Horizontal members are subjected to direct pressures due to the horizontal thrusts. These may be either direct tension or direct compression, depending upon the direction of the thrust. For vertical loadings, the effect of these direct pressures is negligible.

Effect of Temperature Changes and Shrinkage upon Rigid Frames. — In a rigid frame, temperature changes and shrinkage produce bending moments in all members of the frame. These can be determined by the method given in Chapter XII on p. 250.

The effect of temperature changes upon a frame in which the ends are not connected with the end supports, but are there provided with expansion bearings, is comparatively small and in many cases may be disregarded.

The effect is appreciably larger for frames in which end columns rigidly connected with the horizontal member are used. The bending moments are particularly large in each end column and in the span of the girder adjoining the end column. The larger the relative rigidity of the end columns, the larger are the bending moments in the frame produced by changes of temperature.

The character and the signs of bending moments caused by changes of temperature are evident from Fig. 103 (*a*) and (*b*), p. 234, which represent bending-moment diagrams for a three-span frame. In the end column and in the end span, bending moments for a rise of temperature are of the same sign as the bending moments due to vertical loads. Therefore they increase the combined bending moments, and require

additional reinforcement unless larger unit stresses are allowed for a combination of the effect of vertical loadings with the effects of temperature.

For a fall of temperature, the bending moments in the end column and the end span are of opposite sign to those for vertical loadings. Their effect upon the stresses is therefore small, and only a small extension of the positive bending-moment reinforcement in the end span may be needed. The horizontal thrust produces in the end span a pull or tension, which should be provided for by additional horizontal bars.

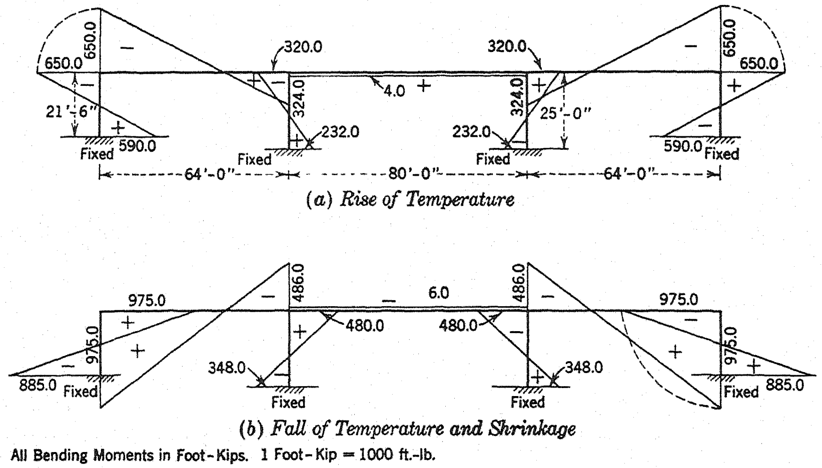


FIG. 103.—Bending Moments in Rigid Frame due to Temperature Changes. *(See p. 233.)*

The effect of temperature changes upon the second column and the second span from the end is much smaller than upon the end column, and in many cases it may be disregarded.

In Fig. 103 the full effect of the changes in length due to temperature changes is shown. Possible reductions of these bending moments before using them in design are discussed on p. 251.

NUMERICAL EXAMPLE OF FIXED-POINT METHOD FOR RIGID FRAMES

The purpose of this example is twofold: to illustrate the use of the fixed-point method for determining bending moments and shears in a rigid frame; and to show the effect upon bending moments in a frame of

the variation in moment of inertia of the horizontal member. To obtain the second object, separate solutions are here given for an assumption of constant moments of inertia in horizontal member, and an assumption of variable moments of inertia.

Example. — Find bending moments in a symmetrical frame of three spans provided with end columns at both ends. The theoretical spans are: $l_1 = 64$ ft.; $l_2 = 80$ ft.; and $l_3 = 64$ ft. The theoretical heights of the columns are $h_1 = h_4 =$ 21 ft. 6 in.; $h_2 = h_3 = 25$ ft. All columns are considered fixed at the bottom.

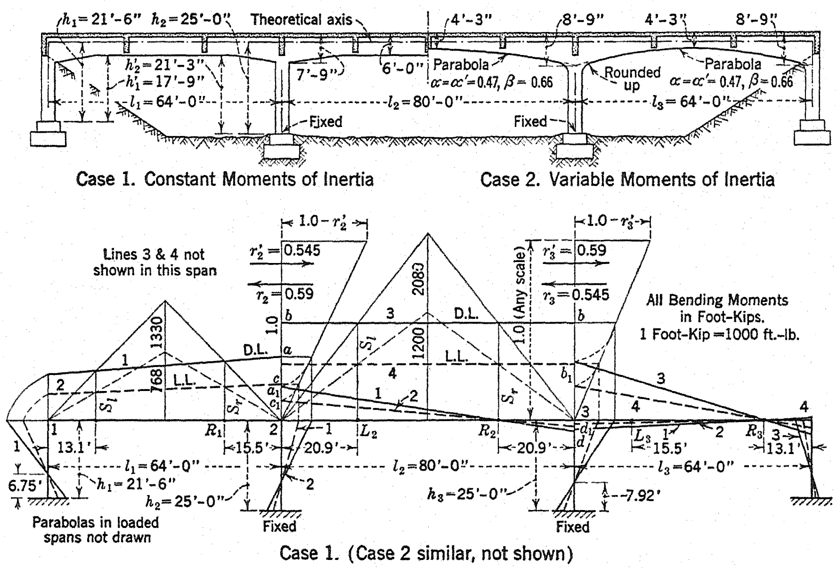


FIG. 104.—Example of Rigid Frame Design. (*See p.* 235.)

The arrangement of spans and the general dimensions are shown in Fig. 104, p. 235, both for constant and for variable moments of inertia.

Case 1. Moments of inertia are constant throughout the whole length of the horizontal member, so that $I_1 = I_2 = I_3 = I_4 = I$. The moments of inertia of all columns are also constant, and the ratios of the moments of inertia of the horizontal members and those of the vertical members are: $\dfrac{I_1}{I_{h1}} = \dfrac{I_2}{I_{h2}} = \dfrac{I_3}{I_{h3}} = \dfrac{I_4}{I_{h4}} = 8.0.$

Case 2. The moments of inertia of horizontal members are variable; the arrangements of haunches in all spans are symmetrical; the underside of the girder in all spans is parabolic; the ratio of the minimum moment of inertia I_c to the maximum I_s is in all spans $\dfrac{I_c}{I_s} = 0.12$; the values of I_c in all spans are the same and are equal to 0.35 of the constant moment of inertia in case 1, or $I_c = 0.35I$. The columns of the same dimensions are used as in case 1; therefore, the ratios for all spans of minimum

moment of inertia in girder to moment of inertia in column is

$$\frac{I_c}{I_{h1}} = \frac{I_c}{I_{h2}} = \frac{I_c}{I_{h3}} = \frac{I_c}{I_{h4}} = 8.0 \times 0.35 = 2.8$$

Unit Loadings. The following unit loadings are used in both cases:

Dead load: $w_d = 2\,600$ lb. per lin. ft.
Live load: $w_l = 1\,500$ lb. per lin. ft.

Maximum static bending moments for these loads are given on p. 211.

Actually, in case 2 the unit dead load is somewhat smaller on account of smaller depth of the girders. This was disregarded in this example so as to avoid the introduction of another variable in the comparison. In actual design, different unit dead load should be used.

Solution. — The fixed-point method is used for solving this problem. The method of procedure outlined on p. 231 is followed.

1. The frame is laid out to scale in Fig. 104, p. 235, for case 1.
2. Fixed points and ratios of transference are computed separately for each case.

Case 1. *Constant Moments of Inertia.*—Use formulas (2), p. 227, (14) and (15), p. 230, and (3) to (6), p. 228.

The clear heights of columns are $h'_1 = h'_4 = 17$ ft. 9 in.; and $h'_2 = h'_3 = 21$ ft. 3 in.; and ratios

$$\frac{h'_1}{h_1} = \frac{h'_4}{h_4} = \frac{17.75}{21.5} = 0.825; \left(\frac{h'_1}{h_1}\right)^2 = 0.681, \frac{h'_2}{h_2} = \frac{h'_3}{h_3} = \frac{21.25}{25.0} = 0.85;$$

$$\left(\frac{h'_2}{h_2}\right)^2 = 0.722; \ \frac{I_1}{I_{h1}}\frac{h_1}{l_1} = 8.0 \times \frac{21.5}{64.0} = 2.69$$

Columns, ends fixed. Formula (2), p. 227.

$$f_{h1} = \frac{1}{9}\left(2 + \frac{h'_1}{h_1}\right)h_1 = \frac{2.825}{9} \times 21.5 = 6.75 \text{ ft.}; \ \frac{h_1}{f_{h1}} = 3.18$$

$$f_{h2} = \frac{2.85}{9} \times 25 = 7.92 \text{ ft.}; \ \frac{h_2}{f_{h2}} = 3.16$$

First span, formulas (14) and (15), p. 230.

$$q_{h1} = 0.681\left[2 \times 0.825 - \frac{1}{3.18 - 1}\left(3 - 2 \times 0.825\right)\right] = 0.702$$

$$f_1 = \frac{1}{3 + 2.69 \times 0.702} \times 64.0 = \frac{1}{4.89} \times 64.0 = 13.1 \text{ ft.}; \ \frac{l_1}{f_1} = 4.89$$

Second span. Make the following substitutions in the general formulas (3) to (6), p. 228.

$$\left(\frac{h'_n}{h_n}\right)^2 = \left(\frac{h'_2}{h_2}\right)^2 = 0.722; \quad \frac{h_n}{f_{hn}} = \frac{h_2}{f_{h2}} = 3.16; \quad \frac{l_{(n-1)}}{f_{(n-1)}} = \frac{l_1}{f_1} = 4.89;$$

$$\frac{I_{hn}}{I_{(n-1)}} \frac{l_{(n-1)}}{h_n} = \frac{I_{h2}}{I_1} \frac{l_1}{h_2} = \frac{1}{8} \times \frac{64.0}{25.0} = 0.32; \quad \frac{I_n}{I_{(n-1)}} \frac{l_{(n-1)}}{l_n} = \frac{64.0}{80.0} = 0.8$$

$$q_1 = 2 - \frac{1}{4.89 - 1} = 1.74;$$

$$q_{h2} = 0.722\left[2 \times 0.85 - \frac{1}{3.16 - 1}\left(3 - 2 \times 0.85\right)\right]$$

$$= 0.79; \quad \frac{q_1}{q_{h2}} = \frac{1.74}{0.79} = 2.21;$$

$$r_2 = \frac{1}{1 + 0.32 \times 2.21} = 0.59$$

$$f_2 = \frac{1}{3 + 0.59 \times 0.8 \times 1.74} \times 80.0 = \frac{1}{3.82} \times 80.0 = 20.9 \text{ ft.}; \quad \frac{l_2}{f_2} = 3.82$$

Third span. After similar substitutions

$$q_2 = 2 - \frac{1}{3.82 - 1} = 1.65; \quad q_{h3} = q_{h2} = 0.79; \quad \frac{q_2}{q_{h3}} = \frac{1.65}{0.79} = 2.09;$$

$$r_3 = \frac{1}{1 + 0.40 \times 2.09} = 0.545$$

$$f_3 = \frac{1}{3 + 0.545 \times 1.25 \times 1.65} \times 64.0 = \frac{1}{4.13} \times 64.0 = 15.5 \text{ ft.}$$

Right fixed points for case 1 are symmetrical with the left fixed points.

$$f'_1 = 15.5 \text{ ft.}, \; f'_2 = 20.9 \text{ ft.}, \; f'_3 = 13.1 \text{ ft.}$$

Case 2. *Variable Moments of Inertia.*—Use formulas (7) to (10), p. 228, and (16) and (17), p. 230. Since in the horizontal member the variation of depths of girder in each span is parabolic, the constants α, α', and β may be taken from the table on p. 203. For symmetrical spans with $\frac{I_c}{I_s} = 0.12$ and $m = 0.5$, the constants are $\alpha = \alpha' = 0.47$ and $\beta = 0.66$. These are used for all spans.

The theoretical and the clear heights of the columns are assumed to be the same as in case 1, in order to avoid the introduction of additional variables. Actually the clear heights are somewhat smaller. The ratios of clear heights to theoretical heights are the same as for case 1. The ratio $\frac{I_1}{I_{h1}} \frac{h_1}{l_1} = 2.80 \times \frac{21.5}{64.0} = 0.94$.

Column, ends fixed. (Same as for case 1.)

$$f_{h1} = 6.75 \text{ ft.}, \frac{h_1}{f_{h1}} = 3.18; \; f_{h2} = 7.92 \text{ ft.}; \frac{h_2}{f_{h2}} = 3.16$$

First span formulas (16) and (17), p. 230

$q_{h1} = 0.702$ (same as case 1)

$$f_1 = \frac{0.66}{2 \times 0.47 + 0.66 + 0.94 \times 0.702} \times 64.0 = \frac{0.66}{2.26} \times 64.0 = 18.8 \text{ ft.} \quad \frac{l_1}{f_1} = 3.42$$

Second span. Make following substitutions in the general formulas (7) to (10), p. 228.

$$\alpha_n = \alpha'_{(n-1)} = \alpha = 0.47; \; \beta_n = \beta_{(n-1)} = \beta = 0.66, \left(\frac{h'_n}{h_n}\right)^2 = \left(\frac{h'_2}{h_2}\right)^2 = 0.722$$

$$\frac{I_{hn}}{I_{(n-1)}} \frac{l_{(n-1)}}{h_n} = \frac{1}{2.80} \times \frac{64.0}{25.0} = 0.91; \; \frac{I_n}{I_{(n-1)}} \frac{l_{(n-1)}}{l_n} = \frac{64.0}{80.0} = 0.8$$

$$q_1 = 2 \times 0.47 - \frac{1}{3.42 - 1} \times 0.66 = 0.67, \quad q_{h2} = 0.79 \text{ (see case 1)};$$

$$\frac{q_1}{q_{h2}} = \frac{0.67}{0.79} = 0.85$$

$$r_2 = \frac{1}{1 + 0.91 \times 0.85} = \frac{1}{1.77} = 0.56$$

$$f_2 = \frac{0.66}{2 \times 0.47 + 0.66 + 0.56 \times 0.8 \times 0.67} \times 80.0 = \frac{0.66}{1.90} \times 80.0 = 27.8 \text{ ft.}$$

$$\frac{l_2}{f_2} = 2.88$$

Third span. After similar substitutions

$$q_2 = 2 \times 0.47 - \frac{1}{2.88 - 1} \times 0.66 = 0.59, \; q_{h3} = 0.79, \frac{q_2}{q_{h3}} = \frac{0.59}{0.79} = 0.75;$$

$$r_3 = \frac{1}{1 + 1.14 \times 0.75} = 0.54$$

$$f_3 = \frac{0.66}{2 \times 0.47 + 0.66 + 0.54 \times 1.25 \times 0.59} \times 64.0 = \frac{0.66}{2.00} \times 64.0 = 21.1 \text{ ft.}$$

Right fixed points are obtained from symmetry.

3. For case 1, bending moments for individual loadings are obtained as shown in Fig. 104, p. 235. Thus, in the center of the first span a vertical is erected, and upon it is plotted above the axis the maximum static bending moment in that span for dead load, taken from p. 211; the apex is connected with supports 1 and 2; the points of intersection of the diagonals with the verticals at the fixed points are connected and the line extended to point a at support 2. This line is the closing line of the bending-

moment diagram in this loaded span; however, it is not necessary to draw the parabola to finish up the diagram. $2a$ at support 2 represents the negative bending moment at that support for this loading. Multiply this by the right ratio of transference $r'_2 = 0.545$ to get $2a_1$, the bending moment in the second span at support 2. Connect a_1 with the right fixed point R_2, and extend the line to point d at support 3. $3d$ is the bending moment in the second span at support 3. Multiply this by the right ratio of transference $r'_3 = 0.59$ and get $3d_1$ as the bending moment in the third span at support 3. Connect d_1 with the right fixed point R_3 and complete the bending-moment diagram in the horizontal member. At the end, the total bending moments at the supports are transferred from the girder to the column. At interior columns the difference between the bending moments in the horizontal members at both sides of the column are transferred to the column. Thus aa_1, is the bending moment in column 2 for this condition of loading. Plot this on a horizontal drawn at the top and connect the point with the lower fixed point to get the bending-moment diagram in the column.

Similar work is performed for the dead load in the second span. $3b$ is the negative bending moment at support 3 in the second span. The locations of points a_1, b_1, and others are here found by the graphical method clearly shown in the figure. Connect point b_1 with R_3 in the third span to complete the diagram. In the figure, bending-moment diagrams for dead load are shown by solid lines marked by 1 and 3. Parabolas in the loaded spans do not need to be drawn in this figure.

The work is repeated for the live load. In the figure, bending-moment diagrams for live load are shown by dash lines and are marked by 2 and 4.

No diagrams are prepared for the third span, because the frame is symmetrical and bending moments for this condition of loading may be obtained by symmetry from the diagrams for the first span.

4. Bending moments in the frame for dead load are obtained by adding at each support the bending moments for the individual loadings of the three spans. The bending moments at the supports are plotted in Fig. 105, closing lines are drawn, and, starting from these, ordinates of parabolas, scaled from Fig. 104, are plotted. It is necessary to draw a dead load diagram only for one half of the symmetrical frame.

5. For live load, three conditions of loading are considered: (1) first and second span loaded; (2) second span loaded; (3) first and third spans loaded. For each of these loading conditions bending moments at the supports are found and plotted; closing lines are drawn in loaded spans; and parabolas are plotted in loaded spans. Diagrams for these three conditions are shown in Fig. 105, p. 240.

6. Bending moments for dead load are combined with the proper bending moments for live load so as to get the largest negative and also the largest possible positive bending moments. The final combined bending moments, indicated by heavy lines, should be used in determining dimensions of the girder and the amount of reinforcement.

For case 2 the work of determining bending moments for individual loadings is repeated, using the appropriate fixed points and ratios of transference. The figure is similar to Fig. 104, p. 235, and is not here reproduced. Bending-moment diagrams for the dead load and for the three conditions of live load, as well as the combined bending-moment diagram for case 2, are shown in Fig. 105, p. 240.

Comparison of Final Bending Moments. — In the table, p. 241, bending moments are tabulated for the following conditions: final bending moments for continuous girders with constant moments of inertia, and with variable moments of inertia, taken

from the example on p. 209; also final bending moments for frames with constant moments of inertia, and with variable moments of inertia, taken from this example. Since the assumption as to spans and loads are the same in all cases, the effect of the various modifications in design are clearly evident.

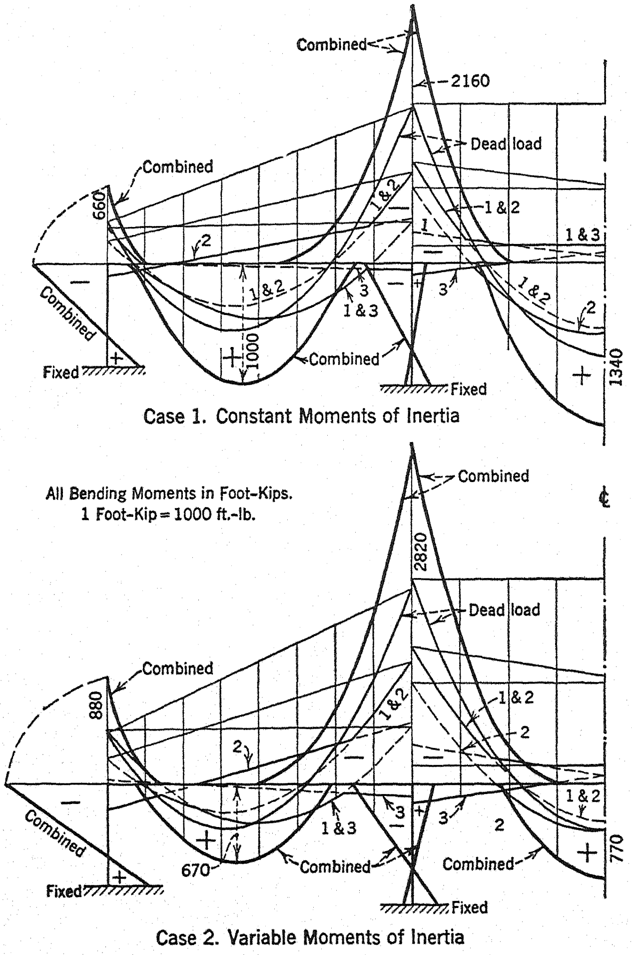


FIG. 105.—Rigid Frame. Combined Bending-Moment Diagrams. (*See p.* 239.)

Type	Moments of Inertia	Maximum Positive Bending Moments		Negative Bending Moments	
		1st Span, ft-lb.	2nd Span, ft-lb.	1st Support ft-lb.	2nd Support, ft-lb.
Continuous girder	Constant	1 340 000	1 400 000	0	2 270 000
	Variable	1 200 000	870 000	0	3 000 000
Rigid frame	Constant	1 000 000	1 340 000	660 000	2 160 000
	Variable	670 000	770 000	880 000	2 820 000

Vertical Members. — Vertical members are subjected to direct pressures and bending moments. The bending moments are particularly large in the end vertical members. For example of design see pp. 297 and 369.

Effect of Temperature Changes. — The bending moments in the above table do not include bending moments due to temperature changes. In continuous girders provided with proper expansion bearings no bending moments are produced by temperature changes. In making comparisons, this should be considered as an item favorable to continuous girders.

In rigid frames with constant moments of inertia the rise of temperature of 30° F. causes bending moments shown in Fig. 103 (*a*), p. 234. For fall of temperature plus shrinkage the bending moments are of opposite signs and are numerically 50 per cent larger as shown in Fig. 103 (*b*). The bending moments for temperature changes were determined as is explained on p. 252 and is shown in the example on p. 265. For frames with variable moments of inertia, temperature bending moments are similar to those shown in Fig. 103.

These bending moments may be reduced for the reasons given on p. 251.

Traction Forces.—In this example the effect of traction forces does not need to be computed. Any traction forces developed in the structure would be resisted directly by the passive earth pressures at the far end of the frame.

If necessary, however, to compute the effect of the traction forces, find the magnitude of the forces acting on one frame and consider them as applied at the level of the horizontal member. Then proceed as explained on p. 249 under the heading "Traction Forces." Traction forces should be assumed as acting in either direction.

The character of the bending-moment diagram for traction forces is the same as shown in Fig. 111 (*e*), p. 260.

CHAPTER XII

SPECIAL PROBLEMS IN RIGID-FRAME DESIGN FIXED-POINT METHOD

Solutions are given in this chapter for special problems in rigid-frame design which cannot be solved directly in the manner described in the previous chapter. The chapter is divided into the following sections:

1. Corrected bending moments for vertical loadings.
   (*a*) One-span frames.
   (*b*) Two-span frames.
   (*c*) Frames with cantilevers.
2. Bending moments in frames for horizontal forces.
3. Bending moments in frames for temperature changes and shrinkage.
4. Movements of column heads, their causes and effects.
5. Formulas for bending moments caused by horizontal and vertical movements of column heads.
6. Numerical example.

1. CORRECTED BENDING MOMENTS FOR VERTICAL LOADINGS

The fixed-point method for rigid frame given in Chapter XI may be used without corrections for symmetrical loadings of all frames, and for any vertical loading of frames consisting of three or more spans. For one and two-span frames loaded by unsymmetrical vertical loadings the results should be corrected as explained in this chapter.

In the fixed-point method without corrections, it is accepted that, after loading, all column heads of the frame remain in the same positions as before loading. This is substantially true only for symmetrical loadings. For unsymmetrical loadings the horizontal member of the frame moves slightly, as a whole, either to the right or to the left; and this movement produces additional bending moments in the frame members. These bending moments are negligible for frames of three or more spans, and have appreciable values only for one- and two-span frames. (See also pp. 221 and 245.)

To get correct bending moments, the effect of the horizontal movement of the column heads must be found and added to the bending moments due to vertical loading previously determined.

Procedure for Finding Corrected Bending Moments. — For unsymmetrical loadings of frames, corrected bending moments are found in the following manner.

1. Using the fixed-point method in the manner explained in Chapter XI, bending-moment diagrams are prepared for such unsymmetrical

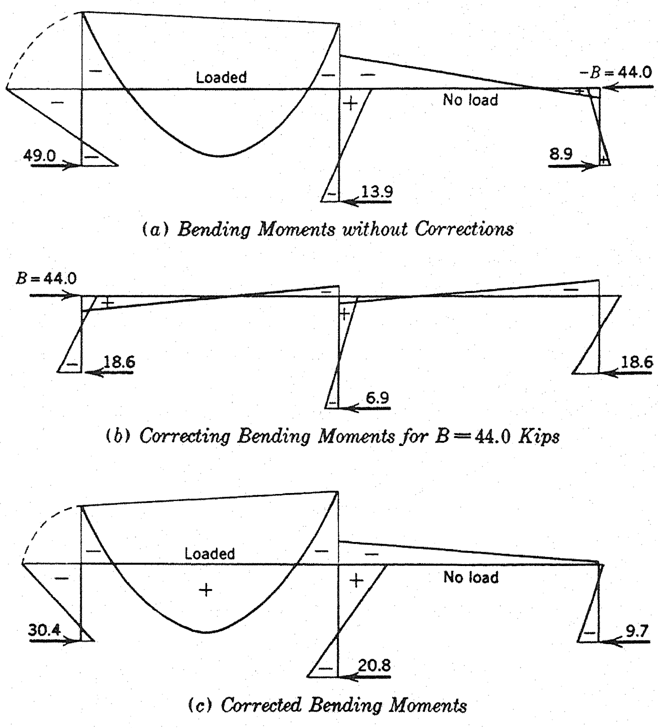


FIG. 106.—Two-span Frame. Bending Moments for Unsymmetrical Vertical Loading. (*See p.* 243.)

loadings as are required for designing the frame. For an example of such diagrams, see Fig. 106 (*a*), above.

2. For all these loadings, horizontal thrusts acting at the bottom of all columns are computed using formula (18), p. 232.

3. For each loading, all thrusts acting to the left are added, and the sum is compared with the sum of all thrusts acting to the right. If the two sums are substantially equal, no correcting bending moments are necessary.

If these sums are not equal, it is obvious that, to keep the frame in equilibrium, there must be acting upon the frame an additional force ($-B$), equal, but of opposite sign, to the unbalanced thrusts, and applied at the level of the horizontal axis of the frame. This force

restrains free action of the frame by preventing horizontal movements of the column heads; therefore, it must be eliminated.

4. To eliminate the restraining force $(-B)$, and at the same time to balance the horizontal thrusts in the frame, a balancing force B is now introduced, which is equal to the restraining force but acts in the opposite direction, and also is applied at the level of the horizontal axis. This balancing force produces bending moments and horizontal thrusts in the frame. A bending-moment diagram due to force B is now prepared as explained under a separate heading below. An example of such a diagram is shown in Fig. 106 (*b*), p. 243.

5. For each vertical loading, the original bending-moment diagram is combined with the corresponding bending-moment diagram for force B, and the resulting diagram is the corrected bending-moment diagram for that particular loading. Force B and force $(-B)$ are eliminated because they are equal and act in opposite directions; but the horizontal thrusts produced by the balancing force B serve the purpose of balancing all horizontal thrusts in the frame. In Fig. 106 (*c*), p. 243, is shown the corrected bending-moment diagram obtained by combining figure (*a*) with figure (*b*). In this final diagram the sum of all thrusts is zero.

Bending Moment for Balancing Horizontal Force B. — Bending moments for any balancing force B are found in the following manner:

(*a*) Find the final bending-moment diagram for a condition when all column heads of the frame move simultaneously and in the same direction an arbitrarily selected distance Δ_l. For this condition find horizontal thrusts for each column. Add all the thrusts at all columns, with their signs, and the sum gives the magnitude of the force F which, acting at the level of the horizontal axis, produces the assumed horizontal movement Δ_l of the column heads. One such diagram is shown in Fig. 111 (*e*); and the force producing the horizontal movement there is $F = -15.57I_{h1}$.

(*b*) Correcting bending moments, for any balancing force B, are obtained by multiplying the bending moments in the final bending-moment diagram due to the force F, just described, by the ratio $\frac{B}{F}$, where both values are used with their signs.

Each loading has a different balancing force B, and therefore a different correcting bending-moment diagram. However, one diagram for the force F is sufficient for all conditions of loadings, horizontal as well as vertical.

(*c*) To get the final bending-moment diagram described under (*a*), it is first necessary to find for each column head of the frame an individual bending-moment diagram, obtained by assuming that the column head under consideration is the only one to move, all other column

heads remaining stationary. For such condition the bending moments at the top and the bottom of the column whose column head has moved are found from formulas (12) and (13), p. 256; and the bending moments in the other members of the frame are found in the usual manner using the fixed points and the ratios of transference. Thus, as many individual diagrams are obtained as there are column heads. In symmetrical frames, however, it is sufficient to get diagrams for one set of the symmetrical spans, because bending moments for the other set may be obtained from symmetry. (See Fig. 111 (*c*) and (*d*).) Bending moments may be obtained graphically, or if desired, by computations using the formulas on p. 197.

The final bending moment for a simultaneous movement of all column heads is obtained by adding the corresponding values from all individual bending-moment diagrams.

This procedure is clearly illustrated in the numerical example on p. 258.

(*a*) One-Span Rigid Frames

In the majority of one-span frames, the treatment and the formulas given in Chapter XIII provide a simpler solution of the problem than the fixed-point method with the necessary corrections.

However, in some cases it may be necessary to resort to the fixed-point method, as explained on p. 304 for frames with hinged ends, and on p. 316 for frames with fixed ends. The corrections for one-span frames are much simpler than for multi-span frames, and they are treated separately under proper headings in Chapter XIII.

(*b*) Two-Span Frames

For two-span frames the fixed-point method, with corrections where needed, may be used for equal and unequal spans; for equal and unequal column heights; for constant and variable moments of inertia; and for any restraint at the bottoms of the vertical members.

For symmetrical loadings of symmetrical frames, no corrections are necessary. For unsymmetrical loadings and for frames with cantilevers, proceed as outlined on pp. 244 and 246.

In the numerical example on p. 258, correcting bending moments are found for the following conditions: for unsymmetrical live loads; for unsymmetrical earth pressures; and for one-sided cantilever loadings. The effect of temperature changes also is determined in the example.

The auxiliary bending-moment diagram for the horizontal movement of column heads is given in Fig. 111 (*e*), p. 260. It is sufficient to solve all problems just enumerated.

(c) Rigid Frame with Cantilevers

Bending moments in rigid frames with cantilevers may be easily determined by the fixed-point method. Separate bending moments must be found for the loads on the main spans of the frame by disregarding the cantilevers. The effect of the cantilever loads upon the frame is also determined separately, and the results are combined so as to get the most unfavorable bending moments.

Fixed Points for Frame with Cantilevers. — Fixed points and ratios of transference for a frame with cantilevers are found in the same manner as for a frame without cantilevers, except that the ratio of transference from the cantilever to the main span is found by considering that cantilever as a new span. (See p. 266.)

How to Find Bending Moments in Frame for Cantilever Loads. — To find bending moments produced in a frame by cantilever loads, compute the maximum cantilever bending moment at the support, and, by multiplying this value by the ratio of transference, get the bending moment at the first support in the first horizontal span of the frame. Beginning with this bending moment, find bending moments in the remaining members of the frame, using fixed points and ratios of transference in the usual manner. (See Fig. 115, p. 267.) The results are the uncorrected bending moments.

For symmetrical cantilever loadings of a frame with two cantilevers, no corrections are necessary.

For unsymmetrical loadings, find correcting bending moments in substantially the same manner as explained on p. 243 in connection with the unsymmetrical vertical loadings. The correcting bending moments for cantilever loads are usually appreciably larger than for unsymmetrical vertical loadings of the same frame. Final bending moments due to unsymmetrical cantilever loadings are obtained by combining the original bending-moment diagram with the correcting bending-moment diagram. (See the numerical example on p. 258.)

Bending moments due to cantilever loads are combined with those due to the loads on the main spans of the frame as explained on p. 184.

Numerical Example. — A numerical example of the use of fixed points for cantilever loads is given on p. 266. A two-span frame is used in the example.

Formulas and Bending-Moment Diagrams for Frames with Cantilevers. — The bending moments in Table III on p. 347 worked out for flat slab bridges with cantilevers and the corresponding bending-moment diagrams may be used for all types of frames in which the ratios of rigidity of the columns to those of the girders are the same as there used.

2. BENDING MOMENTS IN FRAMES FOR HORIZONTAL FORCES

Horizontal longitudinal forces which may need consideration in frame design are: (1) earth pressures; (2) traction forces.

Earth Pressures. — In many cases, vertical members of a frame are subjected to earth pressures which produce bending moments and shears in all members of the frame. The earth pressures consist of the earth pressures proper, and of the pressures due to the surcharge and to the live loads transmitted to the vertical member by the fill. The earth pressures may be represented in each case by a trapezoid, which in computations may be replaced by a triangle and a rectangle.

When a rigid frame is subjected to earth pressures at both ends, the earth pressures, proper, may be assumed as acting simultaneously at both ends. The horizontal pressure due to the live loads, however, should be considered as live loading, which may act at either end or at both ends, whichever condition gives the most unfavorable results at any section of the frame under consideration.

Although in many instances bending moments in horizontal members due to earth pressure are of opposite signs to the bending moments for dead load, it is not advisable to take advantage of this fact for the purpose of reducing bending moments in the combined bending-moment diagram.

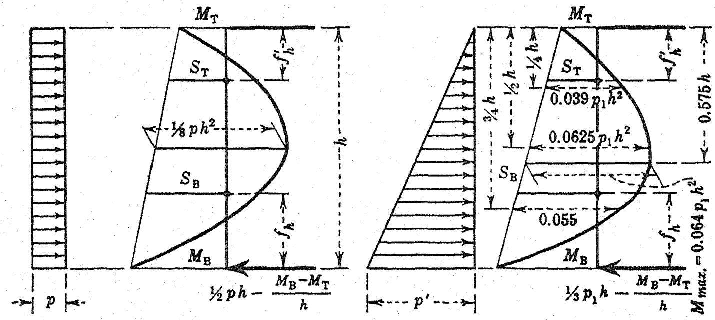
Use of Fixed Points for Earth Pressures. — Bending moments for earth pressures may be found in the following manner:

Consider the earth pressure as acting at one side only, for instance at the left side as in Fig. 107, p. 248. Treat separately the pressures represented by a triangle and those represented by a rectangle.

Lay out the frame to scale, plot the fixed points, and indicate the ratios of transference. The fixed points and the ratios of transference are the same as are used for vertical loadings, except that in addition it is necessary to find the upper fixed points in the vertical members. Compute separately for triangular and for rectangular pressures the values of S_T and S_B from formulas (1) to (4), p. 248. Upon horizontal lines erected at the fixed points of the vertical member subjected to earth pressure plot these values S_T and S_B, and get for each case separately a closing line for the bending-moment diagram in the loaded member. This line extended to the supports determines the negative bending moments at the top and the bottom of the member. The bending moments in the rest of the frame are found in the known manner by using the fixed points and the ratios of transference. In this way two bending-moment diagrams are obtained: one for triangular and the other for rectangular pressures.

For each bending-moment diagram, compute thrusts at the bottom of all vertical members; find the difference between the sum of the earth

pressure and of the thrusts acting in the same direction as the earth pressures, and the sum of the thrusts acting in the opposite direction. This difference is equal to the balancing force B. the effect of which upon the frame it is now necessary to find.



(a) *Rectangular Pressure* (b) *Triangular Pressure*

Horizontal Thrust

$$H = \frac{1}{2}ph - \frac{1}{h}(M_B - M_T) \qquad H = \frac{1}{3}p_1h - \frac{1}{h}(M_B - M_T)$$

FIG. 107.—Fixed Point Method. Horizontal Earth Pressures. (*See p.* 247.)

To find the effect of the force B, it is first necessary to prepare a diagram for the effect of the horizontal movement of all column heads in the manner outlined on p. 244. Bending moments for force B are then obtained as explained on p. 244 under (b). This gives the correcting bending-moment diagram. To get the corrected bending-moment diagram proceed as outlined on p. 244 under 5 for vertical loadings. For numerical example see p. 263.

Values of S_T and S_B for Horizontal Earth Pressures. — The following formulas give values of S_T and S_B for uniformly distributed and triangular earth pressures. The use of these values is shown in Fig. 107, p. 248.

Uniformly Distributed Earth Pressure:

$$S_T = -\frac{1}{4}\left[1 - 2\left(1 - \frac{h'}{h}\right)^2\right]f'_hph \tag{1}$$

$$S_B = -\frac{1}{4}\left[1 + 2\left(1 - \frac{h'}{h}\right)^2\right]f_hph \tag{2}$$

Triangular Earth Pressure, p_1 at bottom, zero at top:

$$S_T = -\frac{1}{8.57}\left[1 - \left(1 - \frac{h'}{h}\right)^2\right]f'_hp_1h \tag{3}$$

$$S_B = -\frac{1}{7.5}\left[1 + 3\left(1 - \frac{h'}{h}\right)^2\right]f_hp_1h \tag{4}$$

These simplified formulas give results accurate enough for practical purposes. For $h = h'$ the values in square brackets are equal 1.

Static Bending-Moment Diagrams for Earth Pressure. — The bending-moment diagram in the loaded vertical member may be completed by plotting the ordinates of the static bending-moment diagram starting from the closing line.

For uniformly distributed earth pressures, p, the static bending-moment diagram is a parabola with a maximum ordinate in the center equal to $\frac{1}{8}ph^2$.

For triangular earth pressures, the reactions, the maximum bending moment, and bending moments at intermediate points are given in Fig. 107 (*b*), p. 248.

Horizontal Thrusts at Bottom of Vertical Members. — The horizontal thrust acting at the bottom of the vertical member of a frame directly subjected to earth pressures may be found from expressions indicated at the bottom of vertical members in Fig. 107 (*a*) for rectangular distribution of earth pressures and in Fig. 107 (*b*) for triangular distribution. In both cases, the horizontal thrust is expressed in terms of the earth pressures and of the bending moments acting at the top and bottom of the vertical member. These should be taken with their signs. The horizontal thrust here always acts in the opposide direction to the earth pressures.

For other vertical members, not directly subjected to earth pressures, horizontal thrusts may be found from formula 18, p. 232.

Numerical Example for Earth Pressures. — A numerical example for determining bending moments for earth pressures is given on p. 263.

Traction Forces. — Bending moments due to traction forces may be obtained by multiplying the values in the diagram for the effect of horizontal movements of column heads by the ratio $\frac{T}{F}$, where F is the force producing the horizontal movements as explained on p. 244, and T is the traction force. (See also p. 241.)

3. BENDING MOMENTS IN FRAMES FOR TEMPERATURE CHANGES AND SHRINKAGE

Expansion and contraction of members forming a rigid frame due to changes of temperature produce bending moments in all members of the frame. Shrinkage may be considered together with the fall of temperature because their effects are similar.

Temperature changes produce bending moments in a frame only when they cause either horizontal or vertical movements of its column

heads while at the same time the bottoms of the vertical members remain stationary. To simplify the matter, the effects of horizontal and vertical movements of column heads are considered separately. The problem of determining the effect of temperature changes may be reduced to: (*a*) finding the magnitude of the movement for each column head; (*b*) determining bending moments in the frame for these movements.

Effect of Horizontal Movements of Column Heads Due to Temperature Changes. — The extent of the horizontal movements of column heads due to temperature changes depends not only upon the number, length, and dispositions of the spans of the frame, but also upon the conditions at both ends of the frame as far as they affect expansion or contraction of the horizontal members.

When the conditions of both ends of the frame are the same, a symmetrical frame expands or contracts symmetrically on both sides of its line of symmetry. The movement Δ_l of any column head is then

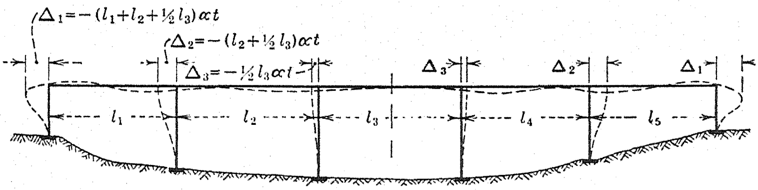


FIG. 108.—Movements of Column Heads Due to Rise of Temperature. (*See p.* 250.)

equal to its distance from the line of symmetry l multiplied by the coefficient of expansion α and by the number of degrees of the change of temperature to be provided for, t. Or, $\Delta_l = \alpha t l$. (See also p. 374.)

In Fig. 108, p. 250, is shown the expansion of a symmetrical frame consisting of five spans. The dash lines show an exaggerated deflection of the frame after expansion. Column heads 1, 2, and 3 move to the left; 4, 5, and 6 move to the right. The movement of column head 3 is due to the expansion of one-half of the center span, and the movement of column head 1 is due to the expansion of two and one-half spans.

For fall of temperature and shrinkage, the column heads 1, 2, and 3 move to the right; and column heads 4, 5, and 6 to the left.

For unsymmetrical frames, the position of the point from which expansion or contraction begins must be estimated from an investigation of the rigidity of both ends of the frame. It is logical to assume that the location of the point of zero movement is such that the work required to move the column heads on one end is equal to the work required at the other end.

The problem is more complicated when the conditions at both ends of the frame are different, as, for instance, when one end of the frame abuts against an embankment and the other end is free to move. Under such circumstances it may be necessary to assume that expansion will take place only in one direction, i.e., toward the free end.

Effect of Vertical Movements of Column Heads Due to Temperature Changes. — When the vertical members of a frame change their lengths as the result of expansion or contraction, the whole frame moves bodily up or down. When the lengths of all vertical members are the same, the vertical movements of all column heads are the same, and no bending moments are produced in the frame members. For unequal column length the vertical movements of the longer columns are larger than those of the shorter columns, and the effect of the difference is similar to the effect produced by an unequal settlement of foundations. Ordinarily such movements are small and do not require any consideration.

Actual vs. Computed Movements of Column heads for Temperature Changes. — The actual movements of column heads due to temperature changes are usually much smaller than the computed movements.

Fall of temperature, in addition to the actual shortening of the horizontal member, and a consequent movement of column heads, causes opening of construction joints and produces a number of incipient cracks distributed along the length of the member. Therefore the final movement of the column head is reduced by the aggregate widths of the cracks, and this in turn reduces bending moments in the frame due to the fall of temperature.

The expansion of the horizontal members of a frame due to temperature changes may be affected by the conditions at the ends of the frame. If the frame is connected at both ends with abutment walls subjected to earth pressure, the active and the passive earth pressures may prevent, either partially or wholly, the expansion of the horizontal member. Rise of temperature in such cases would produce compression stresses in the horizontal member and a consequent shortening of its length; and this shortening would eliminate either partially or wholly the movements of column heads due to expansion. Since bending moments in a frame due to temperature changes are caused by the movements of the column heads, they are reduced in proportion to the reduction in the extent of these movements.

In both cases, to solve the problem it is necessary to assume the fraction by which the theoretical values should be multiplied to get the final results.

Procedure for Finding Effect of Temperature Changes. — To find the effect of temperature changes upon a frame proceed as follows:

1. Decide upon the extent of the variation of temperature to be provided for.

2. For the accepted variation of temperature compute the horizontal movement of each column head in the frame as explained on p. 250 and shown in Fig. 108, p. 250. Thus Δ_1 for column 1; Δ_2 for column 2; Δ_3 for column 3; etc.

3. Find the effect upon the frame of a condition in which only one column head moves horizontally, all other column heads remaining stationary. This assumption should be made consecutively for each column head. Use formulas (12) and (13) for bending moments in the column whose head moves, and for this condition determine bending moments in the other members of the frame by means of the fixed points and the ratios of transference. There will be as many bending-moment diagrams as there are columns in the frame whose heads have moved.

4. Combine bending-moment diagrams for all the individual movements of column heads, and get a combined diagram which represents bending moments in the frame caused by the change of temperature. If the original bending-moment diagram is for a rise of temperature, bending moments for a fall of temperature will be of opposite signs and their magnitude will be proportional to the ratio of the accepted range for the rise to the range of the fall of temperature.

The above procedure is illustrated in the numerical example on p. 265.

4. MOVEMENTS OF COLUMN HEADS, THEIR CAUSES AND EFFECTS

In the first part of this chapter correcting bending moments caused in rigid frames by movements of the column heads were discussed. These movements will now be discussed as well as their causes and effects.

In a rigid frame, column heads do not always remain in fixed positions, but they often move horizontally or vertically either because of unsymmetrical loadings or because of temperature changes and shrinkage.

In discussing the causes and effects of the movements of the column heads, the horizontal and vertical movements are considered separately; and separate formulas are given on pp. 256 and 257 for the effect of each of the two movements.

Horizontal Movements of Column Heads. — Horizontal movements of column heads may be due to one or several of the following causes:

1. Unsymmetrical vertical loading. In such case the whole top of the frame moves bodily either to the right or to the left. For frames of three or more spans such movements are small, and their effects are

negligible. However, for frames of one or two spans the effect of such movements may need consideration.

2. Unsymmetrical cantilever loading. The effect upon the frame of unsymmetrical loading of cantilevers is similar to, but much larger than, the effect of unsymmetrical loading of the main spans.

3. Unsymmetrical horizontal forces acting upon the frame. This is discussed more fully under "Earth Pressures," on p. 247.

4. Changes in length of horizontal members due to direct compression or tension; and also shortening of the span due to the curvature of the deflection curve. Neither of these changes needs to be considered in design. They correspond to the rib shortening in arches, but their effect upon bending moments is appreciably smaller.

5. Changes of temperature and shrinkage. These are discussed under the heading: "Effect of Temperature Changes and Shrinkage," on p. 249.

In cases 1 to 3, for any particular condition of loading, the horizontal movements of all column heads in the frame are the same, and in the same direction, because the whole horizontal member moves bodily either to the right or to the left. In case 4 the change in length of each span may be different from that of any other span, and, therefore, each column head may move a different distance. Finally, in case of temperature changes and shrinkage, one set of column heads at one end may move to the right, while at the same time the set at the other end may move to the left.

Vertical Movements of Column Heads. — Vertical movements of column heads in most cases are so small as to require no consideration. They may be due to one or several of the following causes:

(*a*) Change in length of columns due to compression caused by vertical reactions of the loads, and due to expansion and contraction of columns produced by temperature changes and shrinkage. These changes affect bending moments in the frame only when the movement of one column head is appreciably larger than the movement of the adjoining column heads. Consideration of vertical movements due to these causes is seldom necessary, and that only in frames with columns of different heights, where the differences are large.

(*b*) Unequal settlement of foundations. When the settlement of all foundations in a frame is substantially the same, no bending moments are produced thereby in the frame. Bending moments result only when one or more columns settle more than the adjoining columns. The difference in settlement of two adjoining columns is the vertical movement of the column head of the frame to be used in computations.

The effects of vertical movements of a column head may be found from formulas (14) and (15), p. 257.

5. FORMULAS FOR BENDING MOMENTS CAUSED BY HORIZONTAL AND VERTICAL MOVEMENTS OF COLUMN HEADS

In this section, formulas are given for bending moments at the top and bottom of a column of a frame for a condition when the head of this column moves longitudinally a distance Δ_l, while the heads of all other columns remain stationary.

Another set of formulas gives bending moments at the ends of a girder span for a condition when one of its ends moves vertically up or down a distance Δ_v, while the ends of all other spans remain stationary.

In both cases, bending moments in the remaining members of the frame are found by means of the fixed points and the ratios of transference in the usual manner.

For a condition where several column heads move simultaneously it is necessary first to find the effects of all individual column head movements, and then to add the results.

Notation:

Δ_l = horizontal movement of column head.
Δ_v = vertical movement of column head.
E = modulus of elasticity of concrete.
l_l = length of span to the left of column under consideration.
l_r = length of span to the right of column.
f_l = left fixed point in left span.
f'_r = right fixed point in right span.
h = theoretical height of column.
h' = clear height of column as explained on p. 226.
f_h = lower fixed point of column.
f'_h = upper fixed point of column.
I_l = constant moment of inertia of left span, or smallest moment of inertia for span with variable moments of inertia.
I_r = constant moment of inertia of right span, or smallest moment of inertia for span with variable moments of inertia.
I_h = moment of inertia of column.
α'_l, β_l = constants for left span for girder with variable moments of inertia.
α_r, β_r = constants for right span, variable moments of inertia.

By the expression " column " is meant any vertical support of the frame.

By the expression " column head " is meant the juncture of any vertical support of the frame with the horizontal member of the frame.

Upper Fixed Point in Column and Ratio of Transference from Column to Girder. — In addition to the fixed points in the frame determined by formulas (3) to (10), p. 228, it is necessary to find the upper fixed points for all columns and the ratios of transference from each column to both spans of the horizontal member. Formulas for these are here given. Moments of inertia of the column are assumed to be constant, and in the horizontal members either constant or variable.

Ratio of Transference from Column to Left Span:

$$r_{cl} = \frac{1}{1 + \dfrac{I_r l_l}{I_l l_r} \dfrac{q_l}{q'_r}} \tag{5}$$

Ratio of Transference from Column to Right Span:

$$r_{cr} = 1 - r_{cl} \tag{6}$$

Upper Fixed Point in Column:

$$f'_h = \frac{\left(\dfrac{h'}{h}\right)^2 \left(3 - 2\dfrac{h'}{h}\right)}{3\left(\dfrac{h'}{h}\right)^2 + r_{cl} \dfrac{I_h l_l}{I_l h} q_l} h \tag{7}$$

in which for constant moments of inertia of horizontal members

$$q_l = 2 - \frac{1}{\dfrac{l_l}{f_l} - 1} \quad (8) \qquad q'_r = 2 - \frac{1}{\dfrac{l_r}{f'_r} - 1} \quad (9)$$

and for variable moments of inertia of horizontal members

$$q_l = 2\alpha'_l - \frac{1}{\dfrac{l_l}{f_l} - 1} \beta_l \quad (10) \qquad q'_r = 2\alpha_r - \frac{1}{\dfrac{l_r}{f'_r} - 1} \beta_r \quad (11)$$

The values of constants α'_l and β_l for the left span, and of α_r and β_r for the right span, depend upon the variation of the moments of inertia. See p. 199.

By the expression " column " is meant any vertical support of the frame.

Horizontal Movements of Column Heads. — Bending moments in a frame produced when one column head moves horizontally a distance Δ_l while all other column heads are stationary are shown in Fig. 109, p. 256. The bending moments in the column under consideration are the generating bending moments, and they vary from a maximum positive value at

one end to a maximum negative value at the opposite end. Bending moments in the remaining members of the frame may be found by means of the fixed points and the ratios of transference.

By the expression " column head " is meant the juncture of any vertical support of the frame with the horizontal member of the frame.

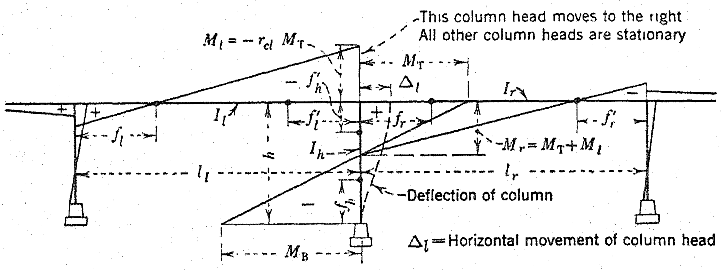


FIG. 109.—Bending Moments Due to Horizontal Movement of One Column Head. (*See p.* 256.)

Values for bending moments at the top and bottom of the column may be found from the following formulas.

Bending Moments in Column due to Horizontal Movement:

$$M_T = \frac{1}{\left(\frac{h'}{h}\right)^2\left(3 - 2\frac{h'}{h}\right)} \frac{6EI_h\Delta_l}{h^2} \frac{f'_h}{h - (f_h + f'_h)} \qquad \text{Top}^1 \quad (12)$$

$$M_B = -\frac{1}{\left(\frac{h'}{h}\right)^2\left(3 - 2\frac{h'}{h}\right)} \frac{6EI_h\Delta_l}{h^2} \frac{f_h}{h - (f_h + f'_h)}$$

$$= -M_T\frac{f_h}{f'_h} \qquad \text{Bottom}^1 \quad (13)$$

Δ_l is positive for a movement from left to right, as in Fig. 109, p. 256; and negative for a movement from right to left.

If E is in pounds per square foot, I_h in feet to fourth power, h and Δ_l in feet, the bending moments are in foot-pounds. If Δ_l is in inches, with the other values remaining as before, the bending moments are in inch-pounds.

[1] For $\frac{h'}{h} = 1, \quad \frac{1}{\left(\frac{h'}{h}\right)^2\left(3 - 2\frac{h'}{h}\right)} = 1.$

Use of Formulas for Horizontal Movements. — Formulas (12) and (13) for horizontal movements are used to find the effect of all the horizontal movements enumerated on p. 252.

The procedure for finding correcting bending moments for unsymmetrical vertical loadings is given on p. 243, and illustrated in the example on p. 262.

The use of the formulas for earth pressures is described on p. 247.

The use of formulas for temperature changes is described on p. 252.

Vertical Movement of One End of Span. — In this case one end of one horizontal span moves down a distance Δ_v. This produces bending

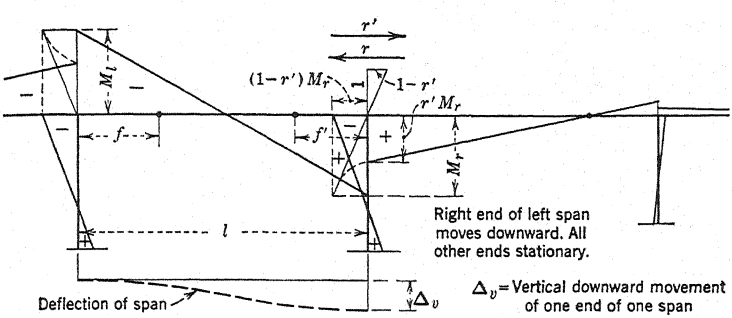


FIG. 110.—Bending Moments Due to Vertical Movement of One End of Span. (*See p.* 257.)

moments at the ends of the span under consideration, as given in formulas (14) and (15), p. 257. These are then transferred in the known manner from span to span to the whole frame.

Bending Moments at Ends of Span, for Movement of Right End:

$$M_r = \frac{6EI\Delta_v}{\beta l^2}\,\frac{f'}{l-(f+f')} \quad \text{Moved end (right)} \tag{14}$$

$$M_l = -\frac{6EI\Delta_v}{\beta l^2}\,\frac{f}{l-(f+f')} \quad \text{Stationary end (left)} \tag{15}$$

Δ_v is positive when the movement is downward, and negative when it is upward.

For constant moments of inertia in horizontal member, $\beta = 1.0$. For variable moments of inertia, values of β may be taken from the tables on p. 203, or found as explained on p. 205.

A bending-moment diagram in a frame due to a downward movement of one end of one span is shown in Fig. 110, above.

Vertical Movement of Column Head. — When a column head moves vertically, it carries with it the right end of the span to the left of the column, and the left end of the span to the right. To get the total effect upon the frame of the vertical movement of one column head, it is necessary to find separately the effect of the vertical movement of the end of the span on one side of the column and of the end of the span on the other side of the column, using formulas given above; and to add the two sets of bending moments.

6. NUMERICAL EXAMPLE

The use of formulas (5) to (13), p. 255, is illustrated by the following numerical example in which bending moments are computed for a rigid frame of two spans. In this example, following problems are solved:

1. Corrected bending moments are found for unsymmetrical vertical loading.
2. Bending moments are found for horizontal earth pressures.
3. Effect of temperature changes and shrinkage is found.
4. Bending moments are found for a frame with cantilevers.

Example. — Find corrected bending moments for unsymmetrical live loads, for earth pressures, and for changes of temperature and shrinkage, for a frame of two spans with the following general dimensions.

Spans: $l_1 = l_2 = 80.0$ ft.; theoretical column heights: $h_1 = h_3 = 24$ ft.; $h_2 = 30.0$ ft.

Clear column heights: $h'_1 = h'_3 = 20$ ft. 3 in.; $h'_2 = 26$ ft. 3 in. (see p. 226 for definition).

Only shallow haunches are used for the girders; therefore moments of inertia may be considered as constant, so that $I_1 = I_2 = I$. Accept the following relations between the moments of inertia of the girders and of the vertical members:

$$I_{h1} = 0.3I;\ I_{h2} = 0.15I;\ \frac{I_{h1}}{I} = 0.3,\ \frac{I_{h2}}{I} = 0.15;\ I_{h1} = 2I_{h2},\ I_{h1} = I_{h3}$$

All vertical members are assumed to be fixed at the bottom. Equivalent uniformly distributed loading is $w_l = 1\,800$ lb. per lin. ft.

Solution. — Find the fixed points for the frame using formulas (3) to (17), pp. 228 to 230.

$$\text{Column ratios: } \frac{h'_1}{h_1} = \frac{20.25}{24.0} = 0.845,\ \left(\frac{h'_1}{h_1}\right)^2 = 0.714$$

$$\frac{h'_2}{h_2} = \frac{26.25}{30} = 0.875,\ \left(\frac{h'_2}{h_2}\right)^2 = 0.766$$

Lower fixed points in columns. (Formula (2), p. 227.)

$$f_{h1} = f_{h3} = \frac{1}{9}(2 + 0.845) \times 24.0 = 7.6 \text{ ft.},\ \frac{h_1}{f_{h1}} = 3.16$$

$$f_{h2} = \frac{1}{9}(2 + 0.875) \times 30.0 = 9.6 \text{ ft.},\ \frac{h_2}{f_{h2}} = 3.12$$

Left fixed points in horizontal members.

First span. Use formula (14), p. 230: $\dfrac{I}{I_{h1}}\dfrac{h_1}{l_1} = \dfrac{1}{0.3} \times \dfrac{24}{80} = 1.0.$

$$q_{h1} = 0.714\left[2 \times 0.845 - \frac{1}{3.16 - 1} \times (3 - 2 \times 0.845)\right] = 0.772$$

$$f_1 = \frac{1}{3 + 1.0 \times 0.772} \times 80.0 = \frac{1}{3.772} \times 80.0 = 21.2 \text{ ft.}, \frac{l_1}{f_1} = 3.772$$

Second span. Use formulas (3) to (6), p. 228, making proper substitutions.

$$\frac{I_{h2}}{I}\frac{l_1}{h_2} = 0.15 \times \frac{80.0}{30.0} = 0.4, \quad q_1 = 2 - \frac{1}{3.772 - 1} = 1.64,$$

$$q_{h2} = 0.766\left[2 \times 0.875 - \frac{1}{3.12 - 1}(3 - 2 \times 0.875)\right] = 0.889,$$

$$\frac{q_1}{q_{h2}} = \frac{1.64}{0.889} = 1.84$$

$$r_2 = \frac{1}{1 + 0.4 \times 1.84} = \frac{1}{1.74} = 0.575$$

$$f_2 = \frac{1}{3 + 0.575 \times 1.0 \times 1.64} \times 80.0 = \frac{1}{3.94} \times 80.0 = 20.4 \text{ ft.}, \frac{l_2}{f_2} = 3.94$$

Upper fixed points in columns. Use formulas (5) to (11), p. 255:

First column. Same as third column determined below.

Second column.

Owing to symmetry $\dfrac{I_r}{I_l}\dfrac{l_l}{l_r} = 1.0, \quad \dfrac{q_l}{q'_r} = 1.0$ and $r_{cl} = \dfrac{1}{1+1} = 0.5,$

$$\frac{I_h}{I_l}\frac{l_l}{h} = 0.15 \times \frac{80}{30} = 0.4 \text{ and } q_l = q_1 \text{ found above.}$$

$$f'_{h2} = \frac{0.766(3 - 2 \times 0.875)}{3 \times 0.766 + 0.5 \times 0.4 \times 1.64} \times 30.0 = \frac{1}{2.74} \times 30 = 10.9 \text{ ft.}$$

Third column. $l_r = 0, \quad \dfrac{I_h}{I_l}\dfrac{l_l}{h} = 0.3 \times \dfrac{80}{24} = 1.0.$

$$r_{cl} = 1.0; \; q_l = 2 - \frac{1}{3.94 - 1} = 1.66 \text{ because } \frac{l_l}{f_l} = \frac{l_2}{f_2} = 3.94$$

$$f'_{h3} = \frac{0.714(3 - 2 \times 0.845)}{3 \times 0.714 + 1.0 \times 1.0 \times 1.66} \times 24.0 = \frac{1}{4.07} \times 24.0 = 5.9 \text{ ft.}$$

All fixed points and ratios of transference are indicated in Fig. 111, p. 260.

Diagrams for Effect of Horizontal Movement of Column Heads. — All correcting bending moments are found by considering the effect of the horizontal movement of the column heads. Assume $E\Delta_l = 10\,000$ ft-lb. Use formulas (12) and (13), p. 256.

1. End column head moves, other column heads stationary.

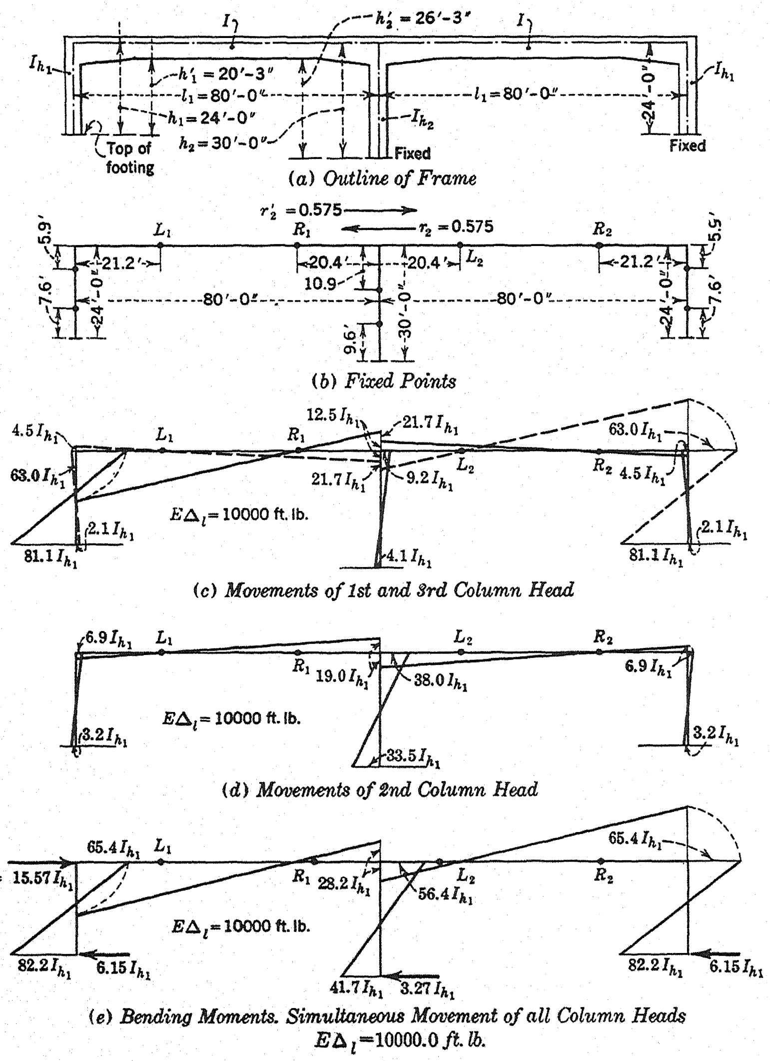


FIG. 111.—Example. Two-Span Rigid Frame. General Data. (See p. 259.)

$$M_T = \frac{1}{0.714 \times (3 - 2 \times 0.845)} \times \frac{6 \times 10\,000 I_{h1}}{24^2} \times \frac{5.9}{24.0 - 13.5} = 63 I_{h1}$$

$$M_B = -M_T \times \frac{7.6}{5.9} = -81.1 I_{h1}$$

Bending moments in the other members of the frame are found using the fixed points and the ratios of transference as shown in Fig. 111 (*c*), p. 260, by solid lines. The dash lines in the same figure indicate bending moments due to a similar movement of the third column head.

2. Center column head moves, other column heads stationary. $I_{h2} = 0.5 I_{h1}$.

$$M_T = \frac{1}{0.766 \times (3 - 2 \times 0.875)} \times \frac{6 \times 10\,000.0 I_{h2}}{30.0^2} \times \frac{10.9}{30.0 - 20.5} = 76.0 I_{h2}$$

$$= 76.0 \times 0.5 I_{h1} = 38.0 I_{h1}$$

$$M_B = -M_T \times \frac{9.6}{10.9} = -33.5 I_{h1}$$

Bending moments in the balance of the frame due to the movement of the center column head are shown in Fig. 111 (*d*), p. 260.

Final Bending-Moment Diagram for Simultaneous Movement of all Column Heads. — By adding the corresponding values from the diagrams in Fig. 111 (*c*) and (*d*), final bending moments are obtained, as shown in Fig. 111 (*e*). Horizontal thrusts are now found using formula (18), p. 232; and finally the sum of thrusts is computed:

First column	$(65.4 + 82.2) I_{h1} \div 24.0 =$	$6.15 I_{h1}$
Second column	$(56.4 + 41.7) I_{h1} \div 30.0 =$	$3.27 I_{h1}$
Third column	Same as first column $=$	$6.15 I_{h1}$
	Sum of thrusts	$15.57 I_{h1}$

As explained on p. 244 the force producing a simultaneous movement of all column heads for which $E\Delta_l = 10\,000$ ft-lb. is equal to the sum of all horizontal thrusts in the frame, and it acts at the level of the horizontal axis in the opposite direction to the thrusts. Therefore, $F = -15.57 I_{h1}$ lb. The horizontal thrusts and the force F are expressed in terms of the moment of inertia of the end column I_{h1}. The actual value may be obtained by substituting a numerical value for I_{h1}. Usually, however, this is not necessary because I_{h1} is cancelled out in computations.

Correcting Bending Moments for Unsymmetrical Live Load. — In the first place, bending moments are found in the frame for the unsymmetrical live load using the fixed points and the ratios of transference in the ordinary manner. In this case, the left span is assumed to be loaded. The work is clearly shown in Fig. 112 (*a*).

For $w_l = 1\,800$ lb., and $l = 80$ ft., $M_{s\max.} = \frac{1}{8} \times 1\,800 \times 80.0^2 = 1\,440\,000.0$ ft-lb.

For the bending moments in the vertical members the horizontal thrusts are found using formula (18), p. 232.

First column	$H_1 = -(787\,000 + 370\,000) \div 24.0 =$	$-48\,200.0$
Second column	$H_2 = (310\,000 + 146\,000) \div 30.0 =$	$15\,200.0$
Third column	$H_3 = -(150\,000 + 70\,000) \div 24.0 =$	$-9\,200.0$
	$\Sigma H =$	$-42\,200$ lb. or -42.2 kips

The sum of the thrusts is not equal to zero; therefore corrections are necessary. Restraining force is $(-B) = -\Sigma H = 42.2$ kips. The effect upon the frame is now found of a balancing force $B = -42.2$ kips, which is equal to the restraining force and acts in the opposite direction.

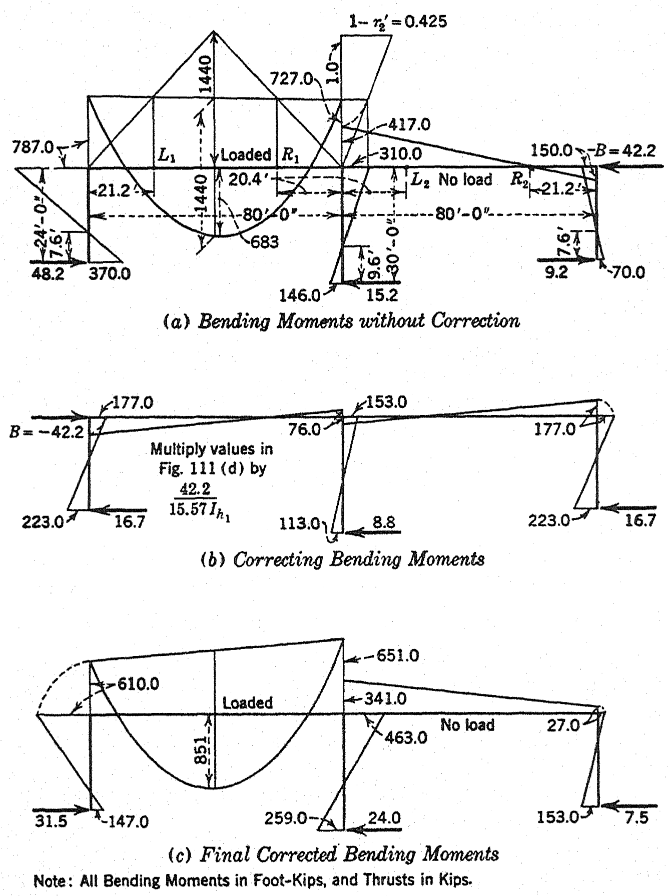


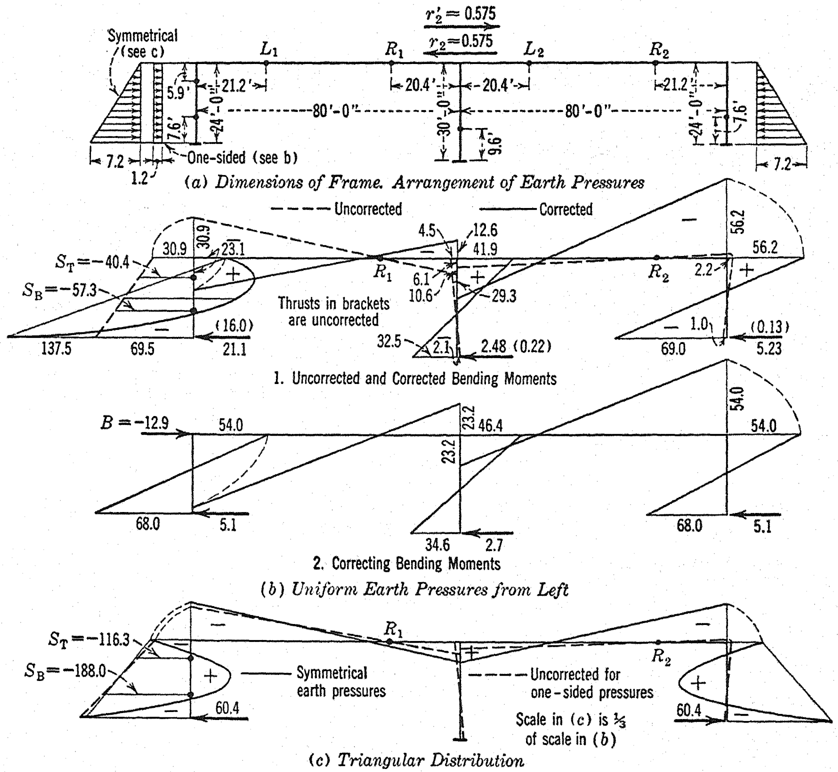
FIG. 112.—Example. Two-Span Rigid Frame. Bending Moments for Unsymmetrical Vertical Loading. (*See p.* 262.)

Correcting Bending Moments for Vertical Loading. — The correcting bending moments produced by the balancing force $B = -42.2$ kips are obtained by multiplying the known bending moments for the horizontal force $F = -15.57 I_{h1}$ shown in Fig. 111 (*e*), p. 260, by the ratio $\dfrac{B}{F} = \dfrac{-42.2}{-15.57 I_{h1}}$. The resulting bending moments and thrusts are shown in Fig. 112 (*b*).

Final Corrected Bending Moments. — By combining the bending moments in Fig. 112 (*a*) with those in Fig. 112 (*b*), corrected bending moments for unsymmetrical live loads are obtained. These are shown in Fig. 112 (*c*).

Earth Pressures

The horizontal earth pressures used in this example are shown in Fig. 113 (*a*), p. 263. They are represented by a trapezoid which is replaced by a rectangle with a uniform pressure $p = 1\ 200$ lb. per lin. ft.; and a triangle with a maximum pressure at the bottom, $p_1 = 7\ 200$ lb. per lin. ft.



Note: All Bending Moments in Foot-Kips. 1 Foot-Kip = 1000 ft.-lb. All Thrust in Kips. 1 Kip = 1000 lb.

FIG. 113.—Example. Bending Moments in Frame due to Earth Pressures. *(See p. 263.)*

For the reason given on p. 247, the rectangular pressures are assumed to act as live loads, while the triangular pressures are taken as acting simultaneously on both sides of the frame.

Uniform Pressures Represented by Rectangle. — $p = 1\ 200$ lb.; $ph = 28\ 800$ lb.

To get the effect of the one-sided pressure, it is necessary to find first the uncorrected values by the fixed-point method, and then to determine the correcting bending moments.

Find first the static reactions and the maximum static bending moment for earth pressures.

$$R_{\text{top}} = R_{\text{bot.}} = 1\,200 \times \frac{24.0}{2} = 14\,400 \text{ lb. or } 14.4 \text{ kips}$$

$$M_{s\max.} = \tfrac{1}{8} \times 1\,200 \times 24.0^2 = 86\,400 \text{ ft-lb. or } 86.4 \text{ ft-kips}$$

Find the values of S_T and S_B at the fixed points from formulas (1) and (4), p. 248.

$$S_T = -\tfrac{1}{4}[1 - 2(1 - 0.845)^2]5.9 \times 28\,800 = -40\,400 \text{ ft-lb. or } 40.4 \text{ ft-kips}$$
$$S_B = -\tfrac{1}{4}[1 + 2(1 - 0.845)^2]7.6 \times 28\,800 = -57\,300 \text{ ft-lb. or } 57.3 \text{ ft-kips}$$

In Fig. 113 (*b*), plot these values of S_T and S_B on horizontals at the fixed points and determine the negative bending moments at the ends of the vertical member. Draw the balance of the diagram for this condition, using the fixed points and the ratios of transference. The uncorrected diagram is shown in the figure by dash lines. Parabola for the loaded member not shown.

Compute all the horizontal thrusts in all vertical members using bending moments from the diagram. See p. 232 for instructions how to determine horizontal thrusts.

First column	$14.4 - \dfrac{-69.5 + 30.9}{24.0}$	$= 16.0$ kips
Second column	$-\dfrac{4.5 + 2.1}{30.0}$	$= -0.22$ kip
Third column	$\dfrac{2.2 + 1.0}{24}$	$= 0.13$ kip

Sum of thrusts and horizontal pressures is $-28.8 + 16.0 - 0.22 + 0.13 = -12.89$ kips. Hence $B = -12.89$ kips.

By multiplying the values in Fig. 111 (*e*) for force $F = -15.57 I_{h1}$ by the ratio $\dfrac{B}{F} = \dfrac{12.89}{15.57 I_{h1}}$, correcting bending-moment diagram is obtained as shown in Fig. 113 (*c*). By combining the uncorrected bending moments in Fig. 113 (*b*) with the bending moments in Fig. 113 (*c*), corrected bending moments are obtained which are shown in Fig. 113 (*b*) by solid lines. Attention is called to the large effect of the horizontal movement of column heads upon bending moments in this case.

Triangular Pressures. — $p_1 = 7\,200$ lb. per lin. ft., $p_1 h = 172\,800$ lb. Since in this case only symmetrical pressures are considered, no correcting bending moments are necessary. Bending moments are found first for pressures acting at the left end of the frame; and to these are added bending moments for the pressures acting at the right end, the latter being obtained from symmetry.

From formulas (3) and (4), p. 248, values of S_T and S_B at fixed points are

$$S_T = -\frac{1}{8.57}[1 - (1 - 0.845)^2]5.9 \times 172\,800 = -116\,300 \text{ ft-lb. or } -116.3 \text{ ft-kips}$$

$$S_B = -\frac{1}{7.5}[1 + 3(1 - 0.845)^2] \times 7.6 \times 172\,800 = -188\,000 \text{ ft-lb. or } -188.0 \text{ ft-kips}$$

In Fig. 113 (*d*), the dash line at the end column shows the closing line for the bending-moment diagram. With the negative bending moments thus determined, the bending-moment diagram for this condition is completed in the usual manner. Dash lines show the values for one-sided earth pressure acting from the left. For earth pressures acting from the right, bending moments are obtained from symmetry.

For triangular pressures acting symmetrically at both ends of the frame, bending moments are obtained by adding bending moments for the left pressures to the corresponding bending moments for the right pressures. The final diagram is shown in Fig. 113 (*d*) by solid lines. No corrections are needed for symmetrical loadings.

Effect of Temperature Changes and Shrinkage

Assume the range of temperature changes to be provided for as ±30° F. Shrinkage will be replaced by a fall of temperature of 15°. Therefore for fall of temperature plus shrinkage $t = 45°$.

Coefficient of expansion is $\alpha = 0.000\,005\,5$ per degree Fahrenheit. Modulus of elasticity of concrete $E = 144 \times 2\,000\,000$ lb. per sq. ft.

Owing to symmetry, the center column head remains stationary during expansion and contraction of the frame members. For the fall of temperature each end column moves toward the center column a distance $\Delta_l = 0.000\,005\,5 \times 45.0 \times 80.0 = 0.02$ ft.

Bending moment in the end column due to the movement of the left end column is found from formulas (12) and (13), p. 256.

$$h_1 = 24.0 \text{ ft.}, \; I_h = 12.0 \text{ ft.}^4; \quad \text{therefore}$$

$$\frac{6EI_h\Delta_l}{h^2} = \frac{6 \times 144 \times 2\,000\,000 \times 12.0 \times 0.02}{24.0^2} = 720\,000 \text{ ft-lb.}$$

$$M_T = \frac{1}{0.714 \times (3 - 2 \times 0.845)} \times 720\,000 \times \frac{5.9}{24.0 - 13.5} = 436\,000 \text{ ft-lb. or } 436.0 \text{ ft-kips}$$

$$M_B = -436.0 \times \frac{7.6}{5.9} = -562.0 \text{ ft-kips}$$

The same values would be obtained by multiplying the bending moments on p. 261 for $E\Delta_l = 10\,000$ ft-lb. by $\dfrac{144 \times 2\,000\,000 \times 12.0 \times 0.02}{10\,000} = 6\,900.0$.

In Fig. 114 the bending moments at the left column are plotted and the bending-moment diagram for the whole frame completed. It is shown by dash lines. Similar diagram should be drawn for the movement of the third column. By combining bending moments for the two individual movements of the end columns, final bending moments are obtained for the effect of the fall of temperature plus shrinkage. It is shown in Fig. 114 by solid lines.

The horizontal thrust at each end column is −43 000 lb. This thrust produces direct tension in the horizontal member and, therefore, should be provided for by additional longitudinal reinforcement.

For rise of temperature, the signs of the bending moments are opposite to those for the fall of temperature. The values in the diagram should be multiplied by $-\frac{30}{45}$. In these computations is determined the full effect of the movements of column heads due to the temperature. See p. 251 for discussion of possible reductions.

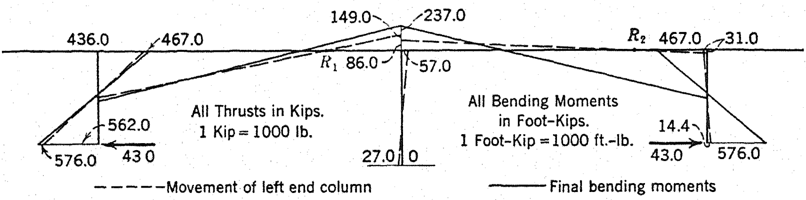


FIG. 114.—Example. Bending Moments in Frame, due to Fall of Temperature. *(See p. 265.)*

TWO-SPAN FRAME WITH CANTILEVERS

The use of the fixed-point method for frames with cantilevers is here illustrated. The general dimensions of the frame are the same as in the previous cases, and the cantilevers are 25 ft. long, each. In this case the same moments of inertia of the columns are used as in the previous cases. In an actual design the moment of inertia of the end columns may be reduced because in a frame with cantilevers end columns are subjected to comparatively small bending moments, there being no earth pressures; and besides the bending moments in the column due to loads on the main spans are balanced to a considerable extent by the bending moments due to the dead load on the cantilevers.

First, bending moments at the supports of the cantilevers are computed as explained on p. 119. For live load, concentrated wheel loads are used instead of the equivalent uniformly distributed loading for the reasons given on p. 28. Compare the bending moments for the wheel loads with the bending moments which would have been obtained by using equivalent uniformly distributed loading determined for the main span.

Dead load	$M_{c(d)} = -$ 880 000 ft-lb. or -880.0 ft-kips
Live load	$M_{c(l)} = -1\,200\,000$ ft-lb. or $-1\,200.0$ ft-kips

The ratio of transference from the cantilever to the main span is found by considering the cantilever as a third span and using formulas (3) and (5), p. 228. This gives the ratio of transference from right cantilever to the end span. Use values from p. 259 and substitute $\frac{I_{h3}}{I_2}\frac{l_2}{h_3} = 1.0$, $\frac{l_2}{f_2} = 3.94$, $q_{h3} = q_{h1} = 0.772$.

Then

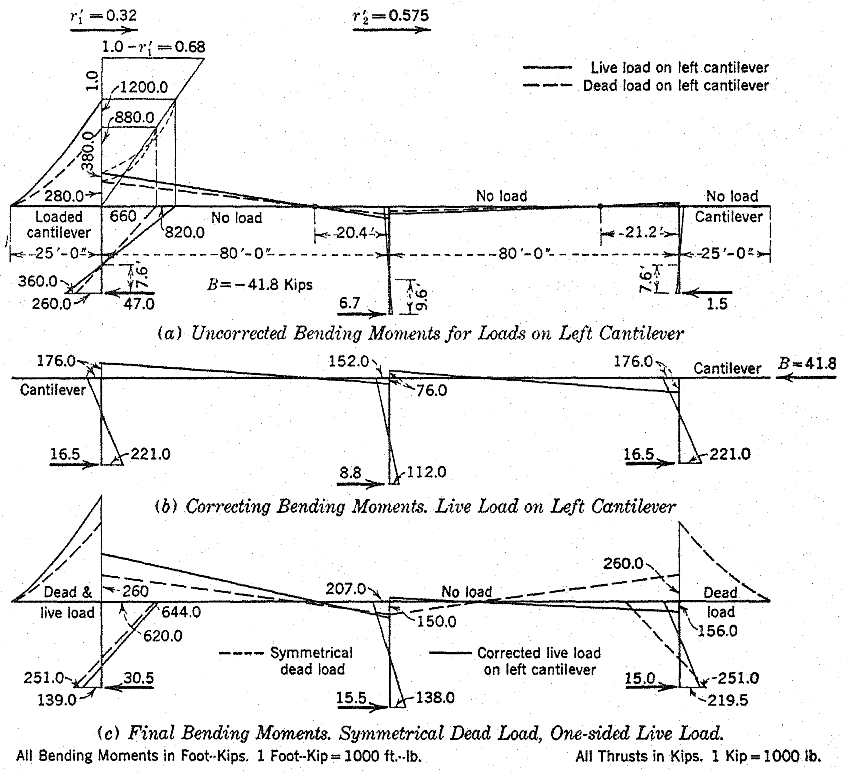
$$q_2 = 2 - \frac{1}{3.94 - 1} = 1.66, \quad \frac{q_2}{q_{h3}} = \frac{1.66}{0.772} = 2.15$$

$$r_3 = \frac{1}{1 + 1.0 \times 2.15} = 0.32$$

The ratio of transference from the left cantilever to the first span is the same as the value just computed for the second span so that $r'_1 = r_3 = 0.32$.

For the cantilever bending moment acting at the left end of the frame, bending moments in the other members of the frame are found in the known manner as shown in Fig. 115 (*a*), where solid lines indicate bending moments for live loads, and dash lines bending moments for dead loads.

For dead loads, no correcting bending moments are required. Final bending moments are obtained by combining bending moments for the left cantilever with the symmetrical values for the right cantilever. The results are shown in Fig. 115 (*c*) by dash lines.



(*a*) *Uncorrected Bending Moments for Loads on Left Cantilever*

(*b*) *Correcting Bending Moments. Live Load on Left Cantilever*

(*c*) *Final Bending Moments. Symmetrical Dead Load, One-sided Live Load.*

All Bending Moments in Foot-Kips. 1 Foot-Kip = 1000 ft.-lb. All Thrusts in Kips. 1 Kip = 1000 lb.

FIG. 115.—Example. Bending Moments in Rigid Frame due to Cantilever Loads. (*See p.* 267.)

For unsymmetrical live load, correcting bending moments must be found. The horizontal thrusts found as in the previous cases are shown in the figure. The sum of the thrusts is 41.8 kips. For this force, the bending-moment diagram is found by multiplying values in Fig. 111 (*e*) by $\dfrac{41.8}{-15.57 I_{h1}}$. The results are shown in Fig. 115 (*b*).

The corrected bending-moment diagram obtained by combining the uncorrected diagram with the correcting diagram is shown in Fig. 115 (*c*) by solid lines. The effect of the corrections should be carefully noted.

CHAPTER XIII

ONE-SPAN RIGID FRAME BRIDGES

Where reasonably unyielding foundations are easily obtainable, the most economical type for one-span bridges is usually the rigid-frame type to which the present chapter is devoted. Although the design of a rigid frame is more complicated than that of the simply supported type, the treatment in this chapter is sufficiently clear and simple and easy to follow.

The text in this chapter contains formulas and information necessary for rational designing of structures the general type of which is shown in Fig. 116, p. 269. Numerical examples make clear the use of the formulas and recommendations; and the presentation of the subject is completed by descriptions and illustrations of completed structures from actual practice.

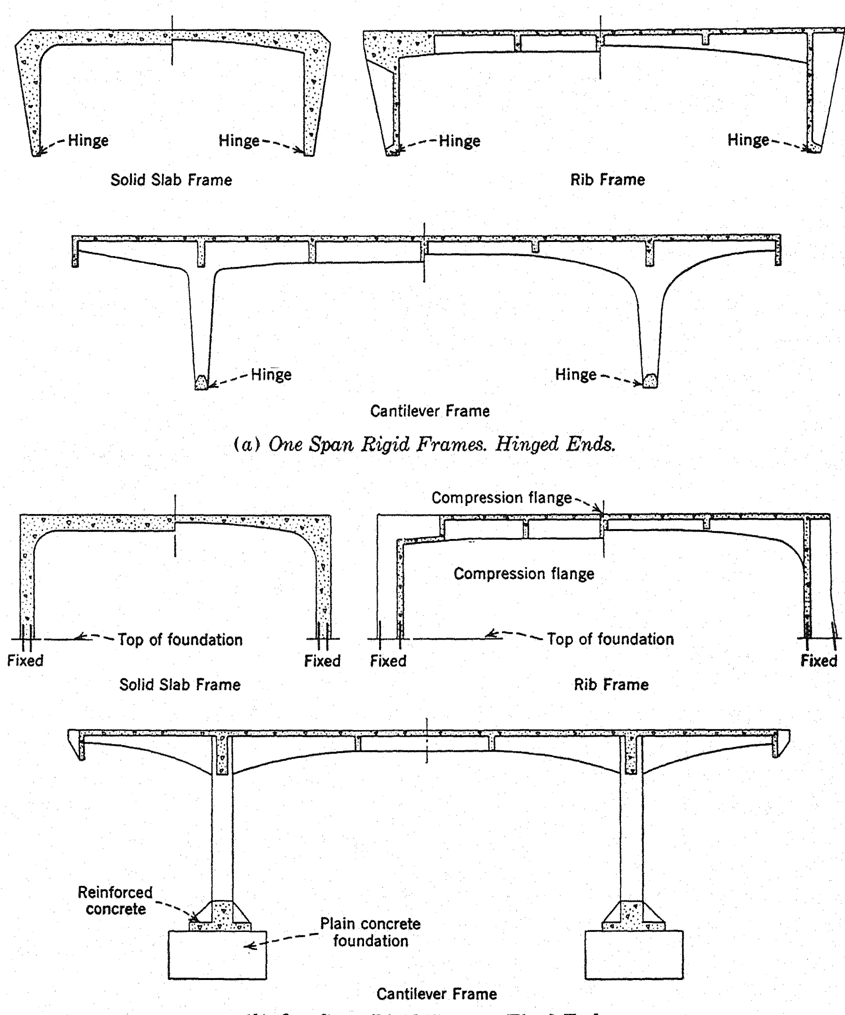
In most cases, it is possible to use the final formulas here given for bending moments and shears. In special cases, however, the fixed-point method may be found useful, as explained on p. 304, for frames with hinged ends, and on p. 316 for frames with fixed ends of vertical members.

Simplicity of Work in Connection with Design of Rigid Frames. — The use of formulas and recommendations given in this chapter is very simple. For an experienced designer, the difference between the length of time required for a design of a rigid frame and that required for an ordinary design of a simply supported girder bridge should not exceed one day.

Use of One-Span Rigid Frames. — Rigid frames of one span are very useful in bridge design for crossings over tracks, intersecting roads, streams and canals, etc. Where solid foundations are easily obtainable, the total cost of a structure using rigid-frame design is appreciably less than that of a structure using simply supported slab or girder designs supported by concrete abutments. Pleasing appearance can be attained by curving the outline to resemble an arch.

In a rigid-frame design, and particularly one with solid slabs, an appreciably smaller depth of construction may be used than is possible in simply supported deck designs. This is of great advantage where the headroom is limited, as in grade separations, because then the height of

the fill approaches, and also their lengths, may be reduced. When the elevation of the top of the bridge is fixed, the rigid-frame design provides the largest effective clearance.



(a) *One Span Rigid Frames. Hinged Ends.*

(b) *One Span Rigid Frames. Fixed Ends.*

FIG. 116.—Typical One-Span Rigid Frames. (*See p.* 268.)

Single-span rigid frames have been used for spans up to 150 ft. The proposed rigid frame for the Brigitta Bridge in Vienna had a span of 186 ft. In the United States and in Great Britain a large number of rigid frames with spans of about 100 ft. have been built, and rigid frames of

spans up to 75 ft. are quite common. A great many of them have been built in recent years in Canada, for both highways and railroads.

General Characteristics of One-Span Rigid Frames. — One-span rigid frames are structures consisting of horizontal members one span long, each rigidly connected with the two vertical supporting members. The vertical members at their lower ends must be able to resist horizontal thrusts produced by the frame action. If the end of either vertical member is free to move horizontally, the construction is not a rigid frame but a statically determinate structure.

Three conditions may exist at the ends of the vertical members of a frame.

First, the ends may be restrained from any horizontal or vertical movement, but be free to turn (i.e., be unable to resist any bending moments acting at the ends); and in such case the frame is considered as hinged at the ends.

Second, in addition to being restrained as in the previous case, the ends of the vertical members may be fixed to the foundations so as to be able there to resist all bending moments to which they may be subjected. Then the frame is called " fixed " at the ends.

Third, the connection between the foundations and the vertical members may be such that the members at the ends may resist bending moments only in part. In this case, the ends are partly restrained, and the condition is intermediate between the first and second case.

The conditions at the ends affect to a large extent bending moments in the whole frame. The effect of fixing the ends is evident from comparison of the formulas on p. 278 with those on p. 312, and also from comparison of the bending-moment diagrams, Figs. 118, p. 277, and 120, p. 281, with Figs. 139 and 140, p. 313.

For partly restrained ends, it is necessary to provide for bending moments intermediate between those for a frame with fixed ends and for one with free ends. The simplest manner of solving the problem is by means of the fixed-point method, and then the lower fixed point in the vertical members is determined according to the expected degree of restraint at the ends.

Rigid Frames with Cantilevers. — Often the cost of a structure may be appreciably reduced by cantilevering the horizontal members at both ends, which permits the omission of heavy abutments. The structure then changes into a bridge with one large opening in the center and two small side openings. To finish up the bridge at the ends, a shallow apron with wing walls may be suspended from the end of each cantilever.

The beneficial effects of cantilevers are discussed on p. 280.

Rigid Frames with Counterweighted Cantilevers. — If it is desired to get an appreciable reduction in the horizontal thrusts of a rigid frame, the structure may be provided with counterweighted cantilevers placed at the level of the hinges. This type, shown in Fig. 135 (*b*), p. 304, has been developed in Hamburg where it is used to a large extent for crossings over navigable canals. Since all the structures there are supported on piles, the reduction of the horizontal thrust for dead loads reduces appreciably the total cost of the structures. By proper arrangement of cantilevers, it is possible to reduce the horizontal thrust for dead load to one-third of the value for a frame without cantilevers.[1] The counterweights consist of the earth fill carried by the cantilevers.

Rigid Frames with Hinged Ends vs. Rigid Frames with Fixed Ends. — Rigid frames with hinged ends have the following advantages over rigid frames with fixed ends: The design of a rigid frame with hinged ends is somewhat simpler because it has only one statically indeterminate value, namely, the horizontal thrust, whereas a frame with fixed ends has three statically indeterminate values. The arrangement of reinforcement in the vertical members of hinged frames is somewhat simpler. The effect of temperature changes, shrinkage, and any unequal settlement of foundations is smaller for frames with hinged ends. The foundations for hinged frames may be simpler and lighter because it is not necessary to take care of bending moments acting at the bottom of the vertical members. In some cases, it is easier to provide effective hinges than to secure a full restraint at the ends required for frames with fixed ends.

The disadvantages of frames with hinged ends are as follows: There may be some uncertainty about the effectiveness of the hinges. Fixed-end frames in many cases are more economical than frames with hinged ends. When the ends are fixed, after the structure is completed there is no necessity of periodical inspection of the hinges. The degree of restraint of the horizontal members is larger for frames with fixed ends than for frames with hinged ends; therefore, the use of frames with fixed ends may be advisable where it is desired to reduce the positive bending moments as much as possible. Compare bending moments in Fig. 118, p. 277, with those in Fig. 139, p. 313.

Types of Frames Treated. — So far as the design formulas are concerned, one-span frames are divided into rigid frames with hinged ends of vertical members; and rigid frames with fixed ends.

The following types of rigid frames with hinged ends are here treated:

1*a*. Right-angle rigid frames with constant and with variable

[1] For interesting description and discussion of frames of this type, see *Beton und Eisen*, 1928, p. 238.

moments of inertia. For this type, formulas are given for vertical loadings; for horizontal pressures, such as earth pressures; and finally for the effect of temperature changes and shrinkage. A numerical example for frames of this type is also given.

1*b*. Right-angle frame with cantilevers, with formulas for vertical loadings and a numerical example illustrating the use of these formulas.

1*c*. Right-angle frame with counterweighted cantilevers located at the level of the hinges.

1*d*. Formulas for fixed points for right-angle frames as well as the method of their application. Use of this method for frames with unequal heights of columns.

The treatment of rigid frames with fixed ends of vertical members consists of the following:

2*a*. Right-angle frame with fixed ends of columns. Formulas for vertical loadings; formulas for horizontal pressures; and finally formulas for the effect of temperature changes and shrinkage.

2*b*. Formulas for fixed points as well as their method of application.

In the section devoted to the description of rigid frames of these types, several designs from actual practice are described. Also numerous details of design are described and discussed.

GENERAL ASSUMPTIONS

Assumptions as to Moments of Inertia. — The formulas for rigid frames with hinged ends are worked out for the following two conditions:

1. Moments of inertia in each member are constant throughout its length, but the magnitude of the moments of inertia of the horizontal member is different from that of the vertical members.

2. Moments of inertia are variable in the horizontal member, but constant in the two vertical members.

For rigid frames with fixed ends formulas are given only for condition 1. For frames with variable moments of inertia the fixed point method should be used.

Horizontal Member. — Moments of inertia of a horizontal member may be considered as constant when the depth of the horizontal member is constant. Shallow haunches, with lengths not exceeding 0.2 of the clear span, do not affect materially the magnitude of the bending moments in the frame, so that the moments of inertia may be considered as constant.

Where the moments of inertia of the horizontal member vary appreciably, the frame may be considered as having variable moments of inertia. The constants α and β in such cases are the same as explained on p. 199. For straight and parabolic haunches, the constants may be taken from

the tables on p. 203. For other conditions they may be found as explained on p. 205.

Vertical Members. — Usually, vertical members of constant moments of inertia are used; therefore, all formulas here given are based on this assumption.

For frames with hinged ends, and especially where vertical members are not subjected to any direct horizontal pressures such as earth pressures, it is economical to use tapering vertical members. In such cases formulas based on constant moments of inertia of vertical members may be used, provided, however, that the value of I_h used in the formulas is made equal to the moment of inertia of the section of the vertical member at a height above the hinge equal to two-thirds of its theoretical height. In determining the theoretical shape of the frame, the substitute vertical member should be used.

Frame Axes and Theoretical Lengths of Members. — The frame axes and the theoretical lengths of frame members are determined in the same manner as explained on p. 226 in connection with multi-span frames. See also Fig. 124, p. 287.

Effect of Intersection Blocks upon Moments of Inertia. — As explained on p. 225, at the intersection of the frame members their moments of inertia are appreciably increased. The effect of this increase upon the horizontal member may be disregarded.

However, as far as the vertical members are concerned, the effect may be appreciable, especially where the height of the intersecting block forms a large proportion of the total height of the vertical member. The effect of the intersection block is taken care of in the formulas by the introduction of the ratios $\frac{h'}{h}$, where h is the theoretical height of the vertical member, and h' is its clear height as defined on p. 226.

PROCEDURE FOR DETERMINING BENDING MOMENTS IN RIGID FRAMES

All the preliminary work having been performed as explained on p. 194, and the theoretical dimensions of the frame having been fixed, bending moments and shears may be determined in the following manner:

A frame is usually subjected to vertical loadings, to temperature changes and shrinkage, and in some cases to horizontal earth pressures. Railroad structures are also subjected to horizontal longitudinal traction forces acting in either direction. Expected slight movements of the foundations also may have to be provided for.

Dead load and live load should be treated separately.

Usually, dead load may be considered as uniformly distributed along

the span of the horizontal member. The weight of the cross struts may be replaced by an equivalent uniformly distributed loading. When the dimensions of the horizontal member are variable, its variable weight may be replaced by an equivalent uniform loading. For more exact work, however, the horizontal member may be divided into sections, and the weight of each section may be computed and considered as a concentrated load.

For dead load, find bending moments at the top and bottom of the vertical members; and the bending-moment diagram in the vertical members is then a straight line. In the horizontal member, plot at the supports the negative bending moments, which are equal to the bending moments at the tops of the vertical members. The line connecting the points thus obtained is the closing line for the static bending-moment diagram. By plotting the ordinates of the static bending-moment diagram, the bending-moment diagram for dead load is completed.

Live load may be either uniformly distributed, or it may consist of concentrated wheel loads. For uniformly distributed live load, the maximum positive and maximum negative bending moments are found in the same manner as for dead load, considering the span as fully loaded. The diagram of maximum bending moments due to moving live loads for the horizontal member, however, will consist of two curves, one for positive bending moments and the other for negative bending moments, drawn as explained on p. 276.

For concentrated wheel loads, it is necessary to find the most unfavorable positions of the loads, as explained on p. 275. It should be noted that the most unfavorable position for maximum negative bending moments is different from the position for maximum positive bending moments. Bending-moment diagrams for positive and negative bending moments may be drawn as explained on p. 275.

Special bending-moment diagrams should be prepared for the effect of the changes of temperature and shrinkage as explained on p. 285 for frames with hinged ends; and on p. 316 for frames with fixed ends.

Bending moments in the frame due to earth pressures should be computed as explained on p. 283 for frames with hinged ends, and on p. 313 for frames with fixed ends. Part of the earth pressure should be considered as a fixed loading acting simultaneously on both sides; the balance should be considered as an accidental loading acting on either side or on both sides simultaneously.

For railroad structures, bending moments due to traction forces should be computed.

After the bending moments, shears, and thrusts for all the loading conditions are found, and bending-moment diagrams drawn, they should

be combined so as to get the most unfavorable results at all sections of the frame. The combined values should be used to determine the dimensions of the frame members and the amount of reinforcement. (See numerical example, p. 293.) All members are subjected to direct pressures and to bending moments. In vertical members, the direct pressures are comparatively large because they include the vertical reactions of the vertical loadings. In horizontal members, on the other hand, the direct pressures caused by the horizontal thrusts are usually small, and may be disregarded in design. (See also p. 295.)

Most Unfavorable Positions of Concentrated Moving Loads. — The following general rules for the position of concentrated moving loads in one-span rigid frames have been developed by the authors and used in their practice.

1. For maximum horizontal thrust and maximum negative bending moments at the ends of the horizontal member, the loads should be placed so that the position of the resultant coincides with the center of the span.

2. For maximum positive bending moments, the loads should be placed on the span so that the heavy load is on one side of the center of span and the resultant on the other side; and the distance of the heavy load from the center should be equal to $\dfrac{1 - 12C}{2(1 - 6C)}r$, where C is the frame constant used in the formulas for the horizontal thrust for uniformly distributed loading, and r is the distance of the resultant from the nearest heavy load. For hinged frames $C = C_1$ given by formulas on p. 278; for fixed frames $C = C_{10}$ in formula on p. 312. Values of r for trains of trucks are given on pp. 19 and 20 for different conditions.

Rule 2, while exact only for frames with hinged ends, is approximate, but accurate enough, for frames with fixed ends.

Largest Bending Moments at Intermediate Points for Moving Loads. — For moving loads to get diagrams of the largest positive and negative bending moments at intermediate points, it would be necessary to compute values at several points, in each case using the most unfavorable positions of the loading; to plot them; and to draw curves fitting the points thus obtained. Ordinarily the large amount of work this would entail is not warranted, and the following simple approximate methods may be used.

For positive bending moments, find the maximum positive bending moment for the most unfavorable position of loading. Plot this bending moment at the proper point, and draw a parabola between the point of maximum bending moment and the end of the span, using the maximum bending moment as the maximum ordinate. (See Fig. 117, p. 276.)

For negative bending moments, plot at the supports above the axis the maximum negative bending moment, and in the center of the span below the axis plot one-third of the largest positive bending moment computed for the same position of loading. Between these three points draw a parabola, which, above the axis, may be considered as the negative bending-moment diagram for moving loads.

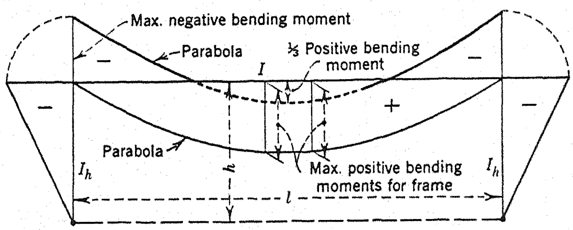


FIG. 117.—Diagrams of Largest Bending Moments for Live Load. (See p. 275.)

RIGHT-ANGLE FRAME, HINGED ENDS. FORMULAS FOR VERTICAL LOADS

Formulas and information are here given for determining bending moments, shears, and thrusts due to vertical loadings in one-span, right-angle rigid frames with hinged ends. Rigid frames with and without cantilevers are here taken care of.

Notation:

Let l = theoretical span length of frame.

h = theoretical height of frame.

h' = clear height of vertical member as defined on p. 226.

y = distance of any point of vertical member above the hinge.

I = constant moment of inertia of horizontal member; or minimum moment of inertia of member with variable moments of inertia.

I_h = moment of inertia of vertical member.

w = uniformly distributed vertical loading.

P_1, P_2, P_3 = concentrated vertical loads.

a_1, a_2, a_3 = distances of concentrated loads from left corner.

H_1 and H_2 = horizontal thrust at left and right hinge, respectively.

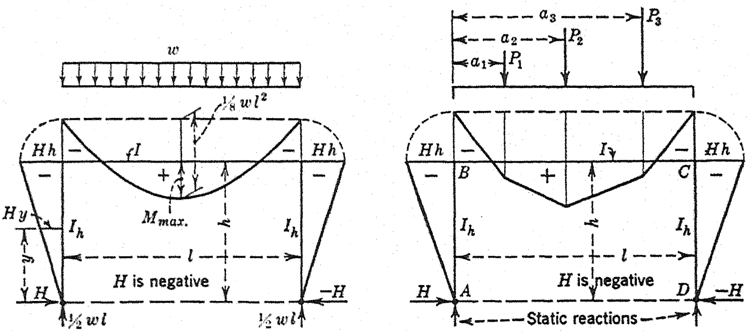
H = horizontal thrust for vertical loadings.

M_{sx} = static bending moment at any point x.

$M_{smax.}$ = maximum static bending moment.
M_1 = maximum negative cantilever bending moment.
C_1, C_2 = frame constants given in formulas (6) to (9), p. 278.
α, β = constants for horizontal member with variable moments of inertia.

Vertical Reactions. — All vertical reactions and external shears in the horizontal members of one-span frames with hinged ends are the same as for simply supported girder spans.

Shear Diagrams for Concentrated Moving Loads. — For frames with hinged ends, external shears at intermediate sections for moving loads are found in the same manner as for simply supported spans. See p. 28 for uniformly distributed moving loads, and pp. 22 to 24 for concentrated wheel loads.



(*a*) *Uniformly Distributed Loading.* (*b*) *Concentrated Loading.*

FIG. 118.—Right-angle Frame, Hinged Ends. Vertical Loading. (*See p.* 277.)

Horizontal Thrusts. — The horizontal thrusts acting at the bottom of the vertical members are the only statically indeterminate values for one-span frames with hinged ends. Separate formulas are given for frames without cantilevers and for frames with cantilevers.

Loads on Main Span of Frame. — For vertical loads on the main span of the frame the thrust at the left hinge acts from left to right, and is negative. At the right hinge, it acts from right to left and is positive. Numerically, both thrusts are equal, so that $H_1 = -H_2 = H$, and their value is given by the following formulas:

Horizontal Thrust at Left Hinge; Uniformly Distributed Loading. — (See Fig. 118 (*a*).)

$$H = -\frac{l}{h}C_1wl \tag{1}$$

Horizontal Thrust at Left Hinge; Concentrated Loads. (See Fig. 118 (b).)

$$H = -\frac{l}{h} C_2 \frac{a}{l}\left(1 - \frac{a}{l}\right) P \qquad \text{single load } P \tag{2}$$

$$H = -\frac{l}{h} C_2 \Sigma \frac{a}{l}\left(1 - \frac{a}{l}\right) P \qquad \text{several loads } P_1, P_2, P_3 \cdots \tag{3}$$

Loads on Cantilevers of Horizontal Member.—Horizontal thrusts due to loads on cantilevers are of opposite sign to the thrusts due to loads on the span.

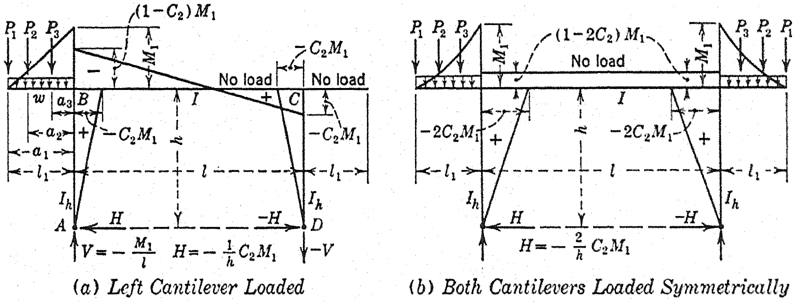


FIG. 119.—Right-angle Frame with Cantilevers, Hinged Ends. (See p. 278.)

Horizontal Thrust at Left Hinge Due to Cantilever Loads.— (See Fig. 119, above.)

$$H = -\frac{1}{h} C_2 M_1 \qquad \text{one cantilever loaded} \tag{4}$$

$$H = -\frac{2}{h} C_2 M_1 \qquad \text{both cantilevers symmetrically loaded} \tag{5}$$

M_1 is the maximum negative cantilever bending moment and it is computed as given on p. 119. Its value should be used in formulas (4) and (5) with its minus sign. H is positive.

Frame Constants C_1 and C_2 in Formulas (1) to (16).—
Moments of Inertia of Horizontal Member Constant:

$$C_1 = \frac{1}{4\left[3 + 2\left(\frac{h'}{h}\right)^3 \frac{Ih}{I_h l}\right]} \tag{6}$$

$$C_2 = 6C_1 \tag{7}$$

Moments of Inertia of Horizontal Member Variable:

$$C_1 = \frac{\beta}{4\left[2\alpha + \beta + 2\left(\frac{h'}{h}\right)^3 \frac{Ih}{I_h l}\right]} \tag{8}$$

$$C_2 = 6C_1 \tag{9}$$

where α and β are constants depending upon the shape of the horizontal member as explained on p. 199. For straight and parabolic haunches, constants are given on pp. 203 and 204. For other shapes see p. 205.

Bending Moments in Frame for Vertical Loadings. — In vertical members, bending moments produced by the horizontal thrusts may be represented by a straight line as shown in Figs. 118 and 119, p. 278. At the hinges, the bending moments are zero. At the corners, they reach their maximum, and are represented by the formula (11).

Bending Moments in Vertical Member for Vertical Loads:

$$M_y = Hy \quad \text{at any point } y \text{ above the hinge} \tag{10}$$

$$M_B = Hh \quad \text{maximum at corner } B \tag{11}$$

These formulas may be used for loads on the main span as well as for loads on the cantilevers. For loads on the main span, the thrust H is negative; therefore, bending moments Hh and Hy are also negative.

For downward loads on the cantilevers, the thrust H is positive; therefore, bending moments Hh and Hy are also positive.

The horizontal thrusts for loads on the main span produce, in the horizontal member, constant negative bending moments, which reduce all the positive static bending moments due to the loads. The resulting bending moments in the horizontal members are:

Bending Moments in Horizontal Member for Vertical Loads:

$$M_x = M_{sx} + Hh \qquad \text{at point } x \tag{12}$$

$$M_{\text{max.}} = M_{s\text{max.}} + Hh \tag{13}$$

For a frame with cantilevers, when the left cantilever is loaded, the bending moments in the horizontal member vary from a negative value M_{Br} at corner B, to a positive value M_{Cl} at corner C. The values are:

Bending Moments in Horizontal Member. Left Cantilever Loaded:

$$M_{Br} = (1 - C_2)M_1 \tag{14}$$

$$M_{Cl} = -C_2M_1 \tag{15}$$

Since M_1 is negative, M_{Br} is negative and M_{Cl} is positive.

When both cantilevers are symmetrically loaded, the horizontal member is subjected to the following constant negative bending moments.

Bending Moments in Horizontal Member. Both Cantilevers Loaded Symmetrically:

$$M_{Br} = M_{Cl} = (1 - 2C_2)M_1 \tag{16}$$

COMBINED BENDING MOMENTS. RIGHT-ANGLE FRAME WITH CANTILEVERS

Use of Frames with Cantilevers. — One-span rigid frames with cantilevers may be used advantageously for bridges with one large center opening and two small side openings. Also they may be used for structures where only one opening is needed, and in such cases cantilevers replace expensive abutments.

Cantilevers have the following beneficial effects upon the frame: The dead load on symmetrical cantilevers reduces the horizontal thrusts in the frame and the negative bending moments in the vertical members. By producing constant negative bending moments in the main span, the cantilever dead load still further reduces all positive bending moments in the horizontal member.

Combined Bending Moments in Frames with Cantilevers. — In frames with cantilevers, bending moments should be computed separately for the loads on the cantilevers and for the loads on the main span. Also they should be computed separately for dead load and for live load.

For cantilever bending moments and shears, concentrated wheel loads should be used even when in the main span equivalent uniformly distributed loading is substituted. For further explanation see p. 28.

Dead load should be assumed as acting simultaneously upon the main span and upon both cantilevers.

Maximum negative bending moment for live load at corner *B* is produced when a heavy load is placed at the end of the left cantilever, and the cantilever as well as the main span are fully loaded with vehicles spaced as closely as is permitted by the specifications. The right cantilever should not be loaded. If possible, the position of the loads on the main span should be the same as the most unfavorable position for a frame without cantilevers. (See p. 275.)

Maximum negative bending moments in the central portion of the span are produced by loads placed on both cantilevers, while the main span is not loaded.

For maximum positive bending moments in the main span, the main span only should be loaded, and the cantilevers not loaded.

For maximum negative bending moments in the vertical members, the main span should be fully loaded, and the cantilevers not loaded.

For maximum positive bending moments in vertical members, both cantilevers should be fully loaded, and the main span not loaded.

Bending moments for dead load should be combined with the various bending moments for live load so as to get the most unfavorable positive and negative bending moments at all sections of the frame. (See p. 167.) Special attention is called to rule 4 on p. 171.

RIGHT-ANGLE FRAME, HINGED ENDS. HORIZONTAL PRESSURES

A one-span rigid frame may be subjected to the following longitudinal horizontal forces.

1. Traction forces.
2. Earth pressures.
   (a) Uniformly distributed earth pressures represented by a rectangle. (See Fig. 120 (a), below.)
   (b) Earth pressure represented by a triangle. (See Fig. 120 (b).)

Traction Forces. — Where consideration of traction forces is necessary, proceed as follows: Compute magnitude of traction forces acting on the frame; consider that the resisting thrust at each hinge equals one-half of the total traction force; finally for these forces and reactions compute bending moments and shears in the frame.

Earth Pressures. — Here, also, the horizontal thrusts are the only statically indeterminate values. For each frame, one thrust for each loading is statically indeterminate, while the thrust at the other hinge may be found from the rules of statics.

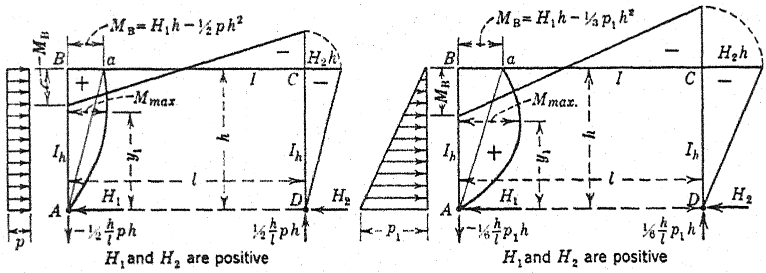


FIG. 120.—Right-angle Frame, Hinged Ends. Horizontal Pressures. (See p. 281.)

For one-sided horizontal pressures, the horizontal thrusts at both hinges act in the opposite direction to the direction of the pressures and their sum is equal to the sum of all the horizontal pressures.

For symmetrical horizontal pressures acting on both sides, the horizontal thrust at each hinge acts in the opposite direction to the direction of the pressure acting on that leg of the frame.

Notation. In addition to the notation on p. 276, let:

p = uniformly distributed horizontal pressure, per lineal foot of vertical member.

p_1 = maximum pressure at the bottom for triangular distribution, per lineal foot.

C_3, C_4 = frame constants for horizontal pressures.

C_h = frame constant for horizontal pressures.

Pressures Acting on Left Vertical Member Only. — Horizontal thrusts, when pressures act from the left, may be found from the following formulas:

Horizontal Thrusts, Uniformly Distributed Pressures Act from Left:

$$H_1 = (1 - C_3)ph \quad \text{Left} \tag{17}$$

$$H_2 = C_3 ph \quad \text{Right} \tag{18}$$

Horizontal Thrusts. Triangular Pressures Act from Left:

$$H_1 = (\tfrac{1}{2} - C_4)p_1 h \quad \text{Left} \tag{19}$$

$$H_2 = C_4 p_1 h \quad \text{Right} \tag{20}$$

Symmetrical Pressures Acting on Both Sides. — When pressures on both sides of the frame are symmetrical, the following formulas should be used:

Horizontal Thrusts, Symmetrical Uniformly Distributed Pressures:

$$H_1 = -H_2 = (1 - 2C_3)ph \tag{21}$$

Horizontal Thrusts, Symmetrical Triangular Pressures:

$$H_1 = -H_2 = (\tfrac{1}{2} - 2C_4)p_1 h \tag{22}$$

Frame Constants for Horizontal Pressures. — Frame constants C_3 and C_4 to be used in formulas (17) to (22), p. 282, depend upon the dimensions of the frame l and h; upon the moments of inertia of the frame members; upon whether moments of inertia are constant or variable; and finally upon the ratio $\dfrac{h'}{h}$ of clear height to the theoretical height of the vertical member. (See p. 226.) Often, when the depth of the intersecting block is comparatively small, it is permissible to use $\dfrac{h'}{h} = 1.0$. The constants are as follows:

Constants C_3 and C_4 for Constant Moments of Inertia and $\dfrac{h'}{h} = 1.0$:

$$C_3 = \frac{5\dfrac{Ih}{I_h l} + 6}{8\left(2\dfrac{Ih}{I_h l} + 3\right)} \tag{23} \qquad C_4 = \frac{0.225\dfrac{Ih}{I_h l} + 0.25}{2\dfrac{Ih}{I_h l} + 3} \tag{24}$$

Constants C_3 and C_4 for All Other Conditions:

$$C_3 = \frac{1}{8}\left\{2 + \left[1 - 2\left(1 - \frac{h'}{h}\right)^2\right]\frac{C_h}{1 - C_h}\frac{1 - 8C_1}{1 - 4C_1}\right\} \tag{23a}$$

$$C_4 = \frac{1}{12}\left\{1 + 0.7\left[1 - \left(1 - \frac{h'}{h}\right)^2\right]\frac{C_h}{1 - C_h}\frac{1 - 8C_1}{1 - 4C_1}\right\} \tag{24a}$$

in which C_1 is given in formula (6), p. 278, for constant moments of inertia, and in formula (8), p. 278, for variable moments of inertia; and

$$\frac{C_h}{1 - C_h} = \frac{\left(\frac{h'}{h}\right)^2\left(3 - 2\frac{h'}{h}\right)}{2\left(\frac{h'}{h}\right)^3 + \frac{I_h l}{Ih}q} \tag{25}$$

where $q = \dfrac{2 - 12C_1}{1 - 4C_1}$ for constant moments of inertia (26)

$$q = \frac{2\alpha' - 4C_1(2\alpha + \beta)}{1 - 4C_1} \text{ for variable moments of inertia} \tag{26a}$$

For $\dfrac{h'}{h} = 1$, formulas (23a), (24a) and (25) become appreciably simpler.

Bending Moments in Vertical Members of Frame Due to Earth Pressure. — In the leg of the frame not directly subjected to pressures, bending moments vary according to a straight line from zero at the hinge to a maximum at the top. (See Fig. 120, p. 281.)

In the loaded leg, i.e., the leg subjected to earth pressures, bending moments may be found in the following manner: From formulas (27) or (28) given below, find the bending moment acting at the top of the loaded vertical member, and plot it on a horizontal drawn at the top. Connect the point thus obtained with the hinge, and consider this straight line as the closing line for the static bending-moment diagram for the earth pressures. Starting from this line plot the static bending moments as given on p. 248.

The bending moments at the top of the loaded leg of frame are:

Bending Moment at Top of Loaded Leg of Frame (Corner B):

$$M_B = H_1h - \tfrac{1}{2}ph^2 \quad \text{Uniformly distributed pressures} \tag{27}$$

$$M_B = H_1h - \tfrac{1}{3}p_1h^2 \quad \text{Triangular pressures} \tag{28}$$

For one-sided pressures, these bending moments are positive, and for symmetrical pressures they are negative.

Maximum positive bending moments in loaded leg for earth pressures may be found from the following formulas:

Maximum Positive Bending Moment. Uniformly Distributed Pressures:

$$M_{\max.} = \tfrac{1}{2}H_1y_1 \tag{29}$$

where

$$y_1 = \frac{H_1}{p} \tag{30}$$

Maximum Positive Bending Moment. Triangular Distribution of Pressures:

$$M_{\max.} = H_1y_1 - \frac{1}{2}\left(1 - \frac{1}{3}\frac{y_1}{h}\right)p_1{y_1}^2 \tag{31}$$

where

$$y_1 = (1 - 1.41\sqrt{C_4})h \quad \text{for pressure on one side} \tag{32}$$

$$y_1 = (1 - 2\sqrt{C_4})h \quad \text{for symmetrical pressures on both sides} \tag{32a}$$

Bending Moments in Horizontal Members for Earth Pressures. — For one-sided pressures, bending moments in the horizontal member vary according to a straight line from a maximum positive value at the top of

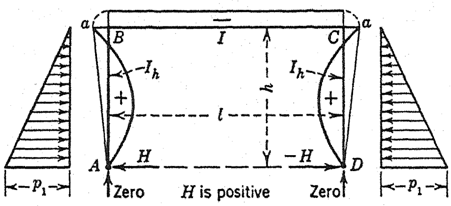


FIG. 121.—Right-angle Frame, Hinged Ends. Symmetrical Horizontal Pressures. (*See p.* 284.)

the leg subjected to earth pressures, to a maximum negative bending moment at the opposite end. (See Fig. 120, p. 281.)

For symmetrical earth pressures acting on both sides, the horizontal member is subjected to a constant negative bending moment. (See Fig. 121, above.)

RIGHT-ANGLE FRAME. TEMPERATURE CHANGES AND SHRINKAGE

Changes of temperature and shrinkage produce bending moments throughout the frame. Rise of temperature produces negative bending moments; fall of temperature and shrinkage produce positive bending moments. The effect of shrinkage may be assumed to be equivalent to a fall of temperature of 15° F. The bending moments are shown in Fig. 122, p. 285.

The only statically indeterminate value is the horizontal thrust which may be found from the following formulas.

In addition to the notation on p. 276, let:

α = coefficient of expansion per degree Fahrenheit. (See p. 374.)
t = change of temperature in degrees.
E = modulus of elasticity of material.

Then

Horizontal Thrust for Change of Temperature:

$$H = -2C_2\alpha Et\frac{I}{h^2} \quad \text{Rise of temperature} \tag{33}$$

$$H = +2C_2\alpha Et\frac{I}{h^2} \quad \text{Fall of temperature} \tag{33a}$$

When I is in feet to fourth power, h in feet, and E in pounds per square foot, the resulting H is in pounds.

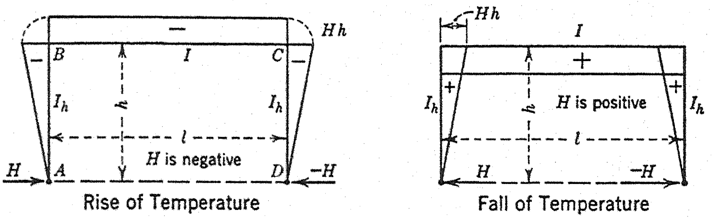


FIG. 122.—Right-angle Frame, Hinged Ends. Effect of Temperature Changes. (*See p.* 284.)

EXAMPLES OF DESIGN OF RIGID FRAMES WITH HINGED ENDS

The use of formulas for one-span right angle rigid frames with hinged ends is illustrated by two numerical examples. In the example that follows, a rigid frame without cantilevers is designed; in the second example on p. 298, bending moments, shears, and thrusts are found for a frame with cantilevers.

In the first example, after bending moments and shears are found, the dimensions of the frame and the amount of reinforcement are determined. Also the design at the hinges is given. The design of footings for frames is discussed on p. 436 in Chapter XVI.

EXAMPLE 1

Example 1. — Design a highway bridge of one span for which the clear span is 60 ft.; the vertical clearance in the center 22 ft.; the width of roadway 30 ft.; and the width of each of two sidewalks 7 ft.

Live load: 20-ton trucks spaced in trains as shown in Fig. 23, p. 60.

Impact ratio: $I = \dfrac{50}{l + 200}$.

Pavement 40 lb. per sq. ft.

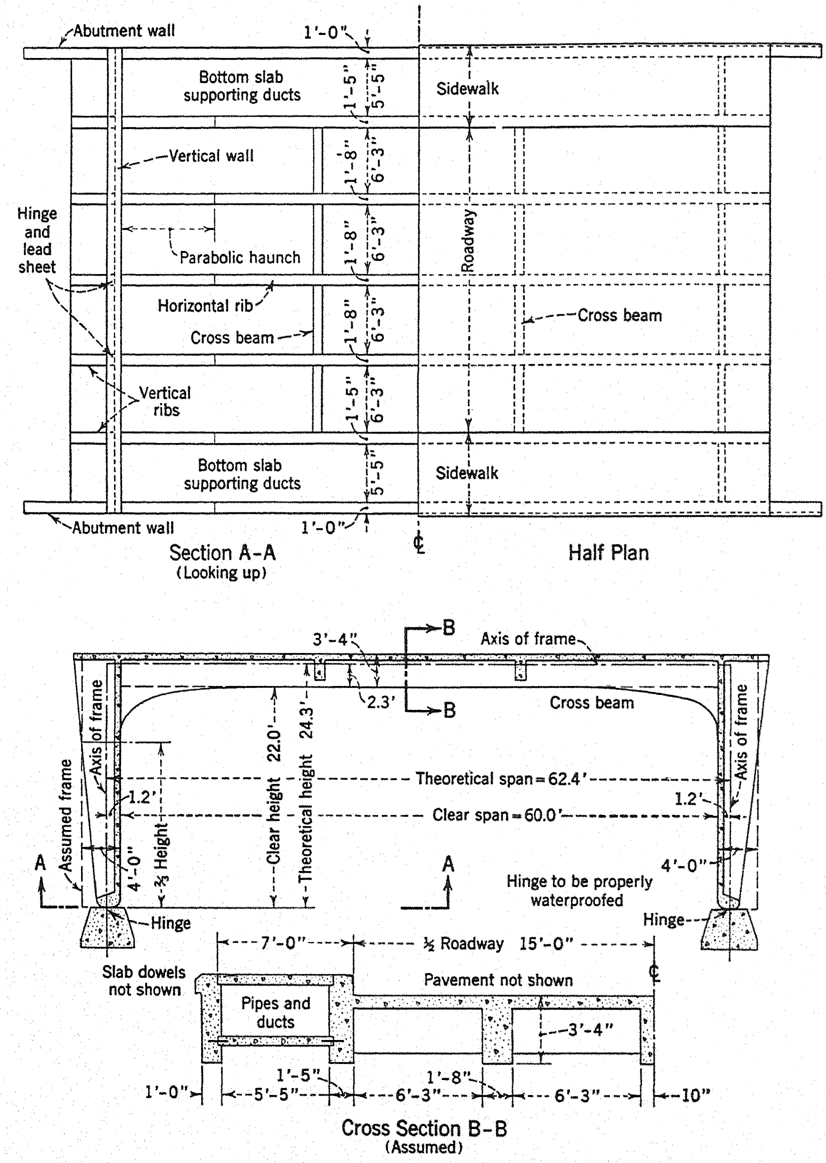


FIG. 123.—Rigid Frame Example. General Dimensions. (See p. 287.)

Solution. — In this case, there is a choice between a slab-frame design and a rib-frame design. In a slab design the quantities of concrete, and also the dead load, would be larger than in a rib design. However, the depth of construction in the center of the span for the slab frame could be made as small as 20 in., with a proper increase of the depths toward the supports. In an actual design, comparative estimates should be made, taking into consideration the difference in the cost of form-work for a slab frame and for a rib frame. Also, the beneficial effect of the smaller depth of construction should be taken into account. In some cases this may reduce the height and length of the fill approaches.

In this example, the solution using rib frames has been selected with the arrangement of frames shown in Fig. 123, p. 286. Only the frames supporting the roadway are designed here.

After the spacing of the frames is selected, the roadway slab is designed in the same manner as explained on p. 44 in connection with girder bridges.

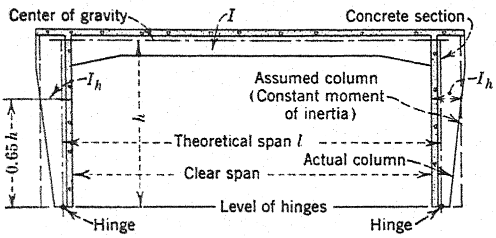


FIG. 124.—Actual and Theoretical Dimensions of Frame. (*See p.* 273.)

As a next step in design, preliminary dimensions of the frame are assumed either on the basis of preliminary computations or by judgment. The preliminary dimensions are used to determine the theoretical span and the height of the frame, the ratios of rigidity of the members, and the dead load. If it is found later that the assumed dimensions are not satisfactory, they may be changed without affecting the computations other than those for the dead load, provided, however, that the ratio of the ratios of rigidity of the horizontal member to those of the vertical members is maintained.

Assumed Concrete Dimensions. — The following concrete dimensions are assumed: Horizontal member: depth 40 in., breadth of stem 20 in. Vertical member: depth 48 in., breadth of stem 20 in. Spacing of frames, s = 7 ft. 11 in.

Theoretical Dimensions of Frame. — It is assumed that the axes of the frame coincide with the centers of gravity of the concrete dimensions of the members. The horizontal member is assumed to have constant moments of inertia because only short haunches are used. The tapering vertical members are replaced by members of constant depths in the manner explained on p. 273.

As indicated in Fig. 123, p. 286, the theoretical span and height are:

Theoretical span of frame, $l = 60 + 2 \times 1.2 = 62.4$ ft.

Theoretical height of frame, $h = 22 + 2.3 = 24.3$ ft.

The effect of the intersection block is here disregarded, making $h' = h$.

Ratio of Rigidity. — It is accurate enough to assume that the ratio of the moments of inertia of the horizontal member to that of the vertical member is equal to the ratio of the third powers of their depths, or $\dfrac{I}{I_h} = \left(\dfrac{40}{48}\right)^3$. Consequently $\dfrac{I}{I_h}\dfrac{h}{l} = \left(\dfrac{40}{48}\right)^3 \times \dfrac{24.3}{62.4} = 0.23.$

Frame Constants. — For the ratio of rigidity just computed, the frame constants, using formulas (6) and (7), p. 278, are:

$$C_1 = \frac{1}{4(3 + 2 \times 1.0 \times 0.23)} = 0.072; \quad C_2 = 6C_1 = 0.432$$

Dead Load. — For the slab as designed, and for the assumed dimensions of the rib, the dead load is $w = 1\,900$ lb. per lin. ft. of the frame.

Using formulas (11) to (13), the thrust and bending moments are:

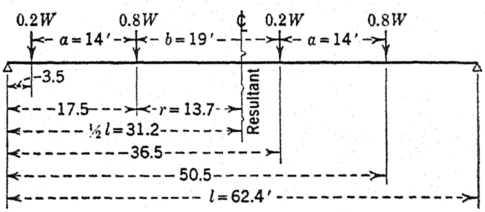
Horizontal thrust for dead load:

$$H = -\frac{62.4}{24.3} \times 0.072 \times 1\,900 \times 62.4 = -22\,000 \text{ lb.}$$

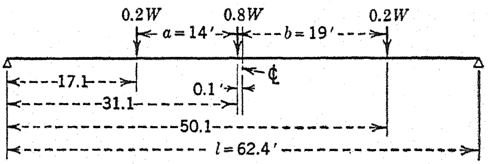
Bending moments for dead load:

$$M_B = M_C = -22\,000 \times 24.3 = -535\,000 \text{ ft-lb.}$$

$$M_{\max.} = \tfrac{1}{8}wl^2 + M_B = 925\,000 - 535\,000 = +390\,000 \text{ ft-lb.}$$



(*a*) *Position of Loads for Max. H*



(*b*) *Position of Loads for $M_{max.}$*

FIG. 125.—Example. Most Unfavorable Positions of Loading. (*See p.* 289.)

Live Load. — Using the impact ratio $I = \dfrac{50}{62.4 + 200} = 0.19$, the truck load including impact is $P = 47\,600$ lb. Two truck trains are placed side by side in the most unfavorable position for the rib under consideration. Then the proportion of each truck in a train carried by one rib is

$$W = \left(1.5 - \frac{10}{2 \times 7.82}\right)P = 0.86 \times 47\,600 = 40\,900 \text{ lb.}$$

Of this load 32 700 lb. is carried by the rear axle and 8 200 lb. by the front axle.

Positions of Live Loads for Maximum Thrust and Maximum Bending Moments. — Two positions of live load must be considered: one for maximum thrust and for maximum negative bending moment, and the other for maximum positive bending moment. (See p. 275.)

For maximum thrust, place on the frame two trucks for which $r = 13.7$ ft. as evident from the table on p. 20, item 4. The position of the resultant of the loads is made to coincide with the center of the span, as shown in Fig. 125 (*a*), p. 288.

For maximum positive bending moment, the distance of the heavy load from the center of the span is equal to $\frac{1 - 12C_1}{2(1 - 6C_1)} r = 1.64$ ft. (See p. 275.) For this position of two trucks, the last load would be outside of the frame. Therefore, use one truck and a front axle of the other truck, for which $r = 0.833$ ft. and the distance from the center of span to the heavy load is $\frac{1 - 12C_1}{2(1 - 6C_1)} r = \frac{0.136}{1.136} \times 0.833 = 0.1$ ft. The position of the loading is shown in Fig. 125 (*b*), p. 288.

Horizontal Thrusts for Live Load. — In the following table, computations are given for horizontal thrusts for the two positions of live load mentioned above and shown in Fig. 125, p. 288. The formula used for the thrusts is

$$H = -\frac{l}{h} C_2 \Sigma \frac{a}{l}\left(1 - \frac{a}{l}\right) P$$

The values for the left reactions are also worked out in the table.

VALUES OF $\Sigma P \frac{a}{l}\left(1 - \frac{a}{l}\right)$ AND V_A

Load P (including impact), lb.	a	$\frac{a}{l}$	$1 - \frac{a}{l}$	$\frac{a}{l}\left(1 - \frac{a}{l}\right)$	$P\frac{a}{l}\left(1 - \frac{a}{l}\right)$, lb.	Left Reaction $P\left(1 - \frac{a}{l}\right)$, lb.
			First Condition (*see Fig.* 125 (*a*))			
8 200	3.5	0.056	0.944	0.053	435	7 740
32 700	17.5	0.280	0.720	0.202	6 600	23 600
8 200	36.5	0.585	0.415	0.243	1 990	3 400
32 700	50.5	0.810	0.190	0.154	5 040	6 210
					$\Sigma P \frac{a}{l}\left(1 - \frac{a}{l}\right) = 14\,065$	$V_A = 40\,950$
			Second Condition (*see Fig.* 125 (*b*))			
8 200	17.1	0.274	0.726	0.199	1 640	5 950
32 700	31.1	0.498	0.502	0.25	8 180	16 420
8 200	50.1	0.802	0.198	0.158	1 300	1 620
					$\Sigma P \frac{a}{l}\left(1 - \frac{a}{l}\right) = 11\,120$	$V_A = 23\,990$

Using the values from the table, horizontal thrusts and the corresponding bending moments are found.

First Condition.

Horizontal thrust: $H = -\frac{62.4}{24.3} \times 0.432 \times 14\,065 = -15\,600$ lb.

Maximum negative bending moments:

$$M_B = M_C = -15\,600 \times 24.3 = -379\,000 \text{ ft-lb.}$$

Corresponding largest positive bending moment:

$$M = 40\,950 \times 17.5 - 8\,200 \times 14.0 - 379\,000 = 223\,000 \text{ ft-lb.}$$

Second Condition.

Horizontal thrust: $H = -\dfrac{62.4}{24.3} \times 0.432 \times 11\,120 = -12\,300$ lb.

Negative bending moments: $M_B = M_C = -12\,300 \times 24.3 = -299\,000$ ft-lb.

Maximum positive bending moment:

$$M_{\text{max.}} = 23\,990 \times 31.1 - 8\,200 \times 14.0 - 299\,000 = 332\,000 \text{ ft-lb.}$$

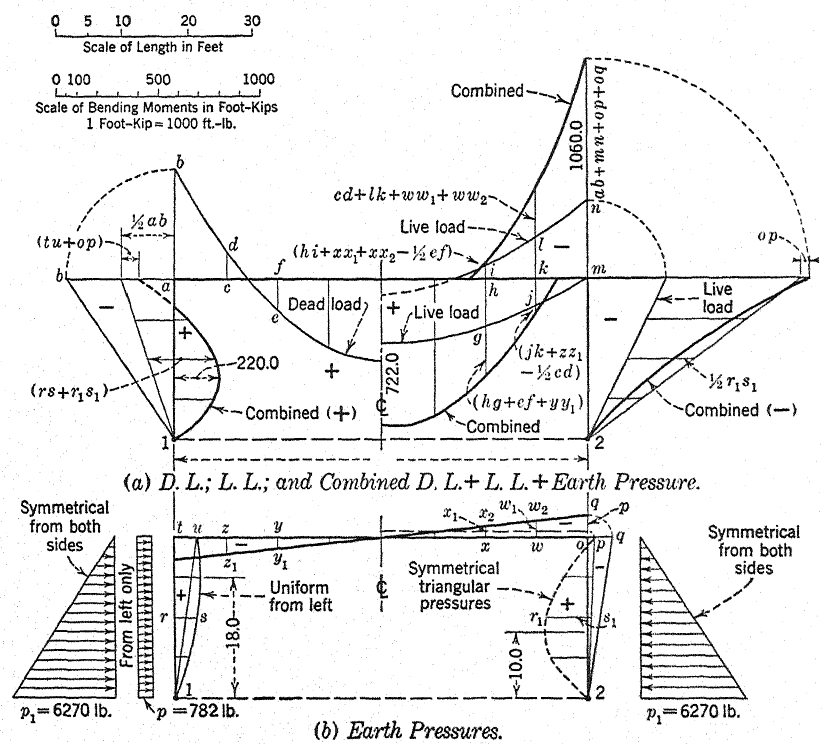


FIG. 126.—Example. Bending-Moment Diagrams for Rigid Frame. (*See p.* 293.)

Bending Moments for Live Load at Intermediate Points. — Bending moments for live loads at intermediate points are shown in the diagrams, Fig. 126, p. 290. These are drawn as suggested on p. 276.

Bending Moments for Earth Pressures

The effect of live loads placed on the road behind the abutment upon earth pressures is represented by a surcharge of 1.5 ft. The vertical distance from the hinge to the top of the surcharge is 27.33 ft. as shown in Fig. 127, p. 291. It is assumed that the weight of fill is 100 lb. per cu. ft., and the earth pressures are computed as given in "Concrete, Plain and Reinforced," Vol. I, p. 837. The pressure on the theoretical frame is represented by a trapezoid with top pressure of 782 lb. per lin. ft. and bottom pressure of 7 052 lb. For the sake of computations the trapezoid is divided into a triangle and a rectangle. The pressures then are:

Rectangular distribution, $p = 782$ lb. per lin. ft.

Triangular distribution, $p_1 = 6\,270$ lb. per lin. ft. at bottom.

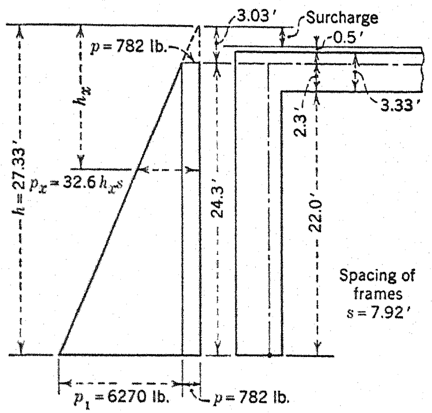


FIG. 127.—Example. Earth Pressures Acting on Rigid Frame. (*See p.* 291.)

Rectangular Distribution of Pressures, Acting from Left. — The frame constant, using formula (23), p. 282, is: for $\dfrac{I}{I_h}\dfrac{h}{l} = 0.23$, $C_3 = \dfrac{5 \times 0.23 + 6}{8(2 \times 0.23 + 3)} = 0.26$.

Horizontal thrusts for pressures from left (formulas (17) and (18), p. 282):

$$H_l = (1 - C_3)ph = 0.74 \times 19\,000 = 14\,100 \text{ lb.} \quad \text{Left hinge}$$
$$H_r = 19\,000 - 14\,100 = 4\,900 \text{ lb.} \quad \text{Right hinge}$$

Horizontal thrusts for pressures from the right on the right column, from symmetry:

$$H_{l1} = -4\,900 \text{ lb.} \quad \text{Left hinge;} \qquad H_{r1} = -14\,100 \text{ lb.} \quad \text{Right hinge}$$

Bending moments for pressures from the left:

$$M_B = H_l h - 0.5ph^2 = 14\,100 \times 24.3 - 0.5 \times 462\,000 = 112\,000 \text{ ft-lb.}$$
$$M_C = -H_r h = -4\,900 \times 24.3 = -119\,000 \text{ ft-lb.}$$
$$M_{\text{max.}} = \tfrac{1}{2}(1 - C_3)^2 ph^2 = 127\,000 \text{ ft-lb.}$$
$$y_1 = (1 - C_3)h = 18.0 \text{ ft.}$$

Bending moments at intermediate points in the loaded vertical member are obtained by plotting a parabola starting from the closing line indicated by $1 - u$ in Fig. 126, p. 290. In the right vertical member bending moments vary according to a straight line.

For earth pressures acting from the right, bending moments are symmetrical with those for the pressures acting from the left.

Triangular Pressures, Symmetrical on Both Sides. — As explained on p. 247, triangular pressures are assumed as acting on both sides simultaneously.

Frame constant, for $\frac{I}{I_h}\frac{h}{l} = 0.23$, $C_4 = \frac{0.225 \times 0.23 + 0.25}{2 \times 0.23 + 3} = 0.087.$

Using formulas (22) to (32), p. 282, thrusts and bending moments are found.

Horizontal thrust:

$$H_t = (\tfrac{1}{2} - 2C_4)p_1h = 0.326 \times 152\,300 = 49\,650 \text{ lb.} \quad \text{Left hinge}$$

Bending moments:

$$M_B = M_C = H_th - \tfrac{1}{3}p_1h^2 = 1\,206\,000 - 1\,233\,000 = -27\,000 \text{ ft-lb.}$$

$$M_{\text{max.}} = 49\,650 \times 10 - \tfrac{1}{2}\left(1 - \frac{1}{3} \times \frac{10}{24.3}\right)6\,270 \times 10.0^2 = 226\,500 \text{ ft-lb.}$$

$$y_1 = (1 - 2\sqrt{0.087}) \times 24.3 = 10.0 \text{ ft.}$$

The bending moments at intermediate points of the vertical member are found by plotting the static bending moment given on p. 248 from the closing line $2 - p$.

Bending-moment diagrams for this loading are shown in the right half of Fig. 126 (*b*). In the half not shown, the values are symmetrical.

Effect of Temperature Changes and Shrinkage

It is assumed that the range of temperature changes to be provided for amounts to ±30° F. Shrinkage will be taken care of by adding 15° to the fall of temperature.

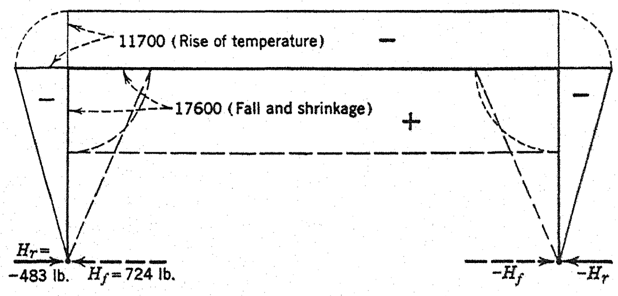


Fig. 128.—Example. Effect of Temperature Changes. (*See p.* 292.)

From formula (33), p. 285, for assumed $I = 6.95$ ft.^{4} and $\alpha E = 1\,584$ lb. per sq. ft., the horizontal thrusts at left hinge are:

$$\text{Rise: } H_r = -2 \times 0.432 \times 1\,584 \times 30 \times \frac{6.95}{24.3^2} = -483 \text{ lb.}$$

$$\text{Fall: } H_f = 2 \times 0.432 \times 1\,584 \times 45 \times \frac{6.95}{24.3^2} = 724 \text{ lb.}$$

Corresponding corner bending moments:

$$\text{Rise: } M_B = M_C = -483 \times 24.3 = -11\,700 \text{ ft-lb.}$$

$$\text{Fall: } M_B = M_C = 724 \times 24.3 = 17\,600 \text{ ft-lb.}$$

Bending-moment diagrams are shown in Fig. 128, above.

Combined Bending Moments

Bending moments for dead load, live load, earth pressures, and temperature changes are combined so as to get the most unfavorable conditions for all parts of the frame. In most specifications increased unit stresses are allowed when temperature effects are combined with the other loadings, which often makes their consideration unnecessary. To facilitate combinations of bending moments and shears for different conditions of loading, all results should be arranged in a table.

Combined bending moments in the frame, exclusive of the effect of temperature changes, are shown by heavy lines in Fig. 126 (*a*), p. 293. In combining bending moments no advantage was taken of the reduction of the positive bending moments by the symmetrical earth pressures for the reasons given on p. 247. For the same reasons the combined negative bending moments in the vertical member were reduced by one-half of the positive bending moments for the symmetrical earth pressures. See Fig. 126, p. 290.

Maximum combined negative bending moments at corners are

$$M_B = M_C = -535\,000 - 379\,000 - 119\,000 - 27\,000 = -1\,060\,000 \text{ ft-lb.}$$

and including temperature changes: $M_B = -1\,071\,700$ ft-lb.

Maximum combined positive bending moments in center of span is

$$M_{\text{max.}} = 390\,000 + 332\,000 = 722\,000 \text{ ft-lb.}$$

and including temperature changes and shrinkage: $M_{\text{max.}} = 739\,600$ ft-lb.

Maximum positive bending moment in vertical member, by combining earth pressures with one-half of the dead-load bending moments: $M_{\text{max.}} = 220\,000$ ft-lb. This was obtained by scaling from the diagram.

Effect of Wind Pressures and Traction Forces

In a rigid frame of the type used in this example, the effect of wind pressures may be entirely disregarded.

Since this structure is designed for highway traffic, traction forces do not need to be considered. See also discussion on p. 241.

External Shears, Vertical Reactions, and Direct Pressures

External Shears in Horizontal Members. —

Dead Load: $V_B = \frac{1}{2} \times 1\,900 \times 62.4 = 59\,300$ lb.

At intermediate points shears vary according to a straight line.

Live Loads: Using formulas from the table on p. 24, case 4, for $W = 40\,900$ lb.

$$x = 0 \qquad V_B = \left(2 - \frac{38.6}{62.4}\right) \times 40\,900 = 56\,400 \text{ lb.}$$

$$x_1 = 62.4 - 47 = 15.4 \qquad V_{x1} = \frac{55.4}{62.4} \times 40\,900 = 36\,300 \text{ lb.}$$

$$x_2 = 62.4 - 33 = 29.4 \qquad V_{x2} = \frac{30.2}{62.4} \times 40\,900 = 19\,800 \text{ lb.}$$

Earth Pressures. — Symmetrical earth pressures produce no shears in the horizontal member.

One-sided earth pressures produce in the horizontal member a constant shear. See Fig. 129, below. $R_A = \pm \frac{1}{2} ph \frac{h}{l} = \pm 3\,700$ lb.

Combined External Shears. —

$$V_B = 59\,300 + 56\,400 + 3\,700 = 119\,400 \text{ lb.}$$

At intermediate points, shears are shown in Fig. 129, below.

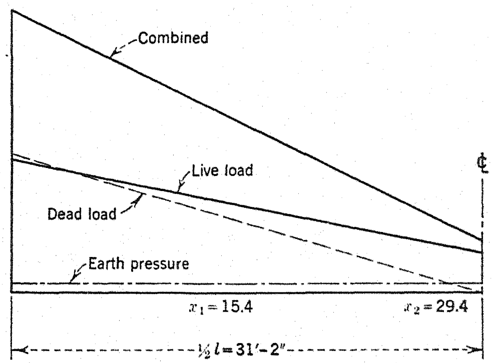


FIG. 129.—Example. Shear Diagrams for Horizontal Member. (*See p.* 294.)

External Shears in Vertical Members. — External shears in vertical members are due to horizontal thrusts and to horizontal earth pressures.

The following thrusts act at the end of the left vertical member:

Dead load:	$H_d = -22\,000$ lb.
Live load:	$H_l = -15\,600$ lb.
Temperature:	$H = -483$ lb. and 724 lb.
Earth pressures:	$H = 14\,100$ lb. (rectangular from left)
	$H = 49\,650$ lb. (symmetrical, triangular)
	$H = -4\,900$ lb. (rectangular from right)

Combined Horizontal Thrusts in Left Vertical Member. — In computing the maximum negative thrust, only one-half of the thrust due to the symmetrical earth pressure was used, because it is not certain that the full thrust will be actually realized. Temperature effects are neglected.

Maximum positive thrusts: $H_p = -22\,000 + 14\,100 + 49\,650 = 41\,750$ lb.
Maximum negative thrusts: $H_n = -22\,000 - 15\,600 - 4\,900 + \frac{1}{2} \times 49\,650 = -17\,680$ lb.

External Shears in Vertical Member at Intermediate Points. — For vertical loads, the external horizontal shears in the vertical member are constant, and are equal to the horizontal thrusts. The external shears due to earth pressures are combinations of the horizontal thrusts and the earth pressures.

The combined shear diagrams for the vertical member are shown in Fig. 130. Three curves are there shown for three combinations.

The first curve is for a combination of the dead load with the total earth pressures, marked in the figure as Case 1, of which the triangular pressures act on both sides

simultaneously, while the rectangular pressures act only on the member under consideration. At the bottom the shear equals the combined thrust $H_1 = 41\,750$ lb., and at any intermediate point the shear equals this thrust minus the aggregate earth pressures below that point. This condition gives maximum positive shears in the vertical member.

The second curve is for a combination of Case 1 with dead and live loads. For this condition the horizontal thrust is $H = 41\,750 - 15\,600 = 26\,150$ lb.

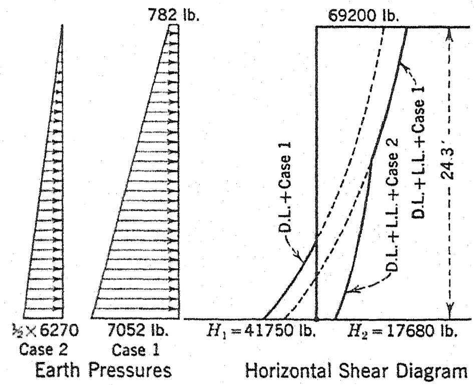


FIG. 130.—Example. External Shears in Vertical Member. (*See p.* 294.)

The third curve is for a combination of dead and live loads with one-half of the symmetrical pressures, and rectangular horizontal pressures acting on the opposite vertical member. Thrust for this case is $H_2 = -17\,680$ lb.

The shear diagram in Fig. 130, p. 295, is used to determine diagonal tension reinforcement in the vertical members.

Direct Pressures in Horizontal Members. — The horizontal member is subjected to direct pressures, which in this case are compression and are due to horizontal thrusts and earth pressures. The maximum value of the direct pressures is produced by dead and live loads combined with the symmetrical triangular earth pressures and the one-sided uniformly distributed earth pressures; and its value is 69 200 lb.

Direct Pressures in Vertical Members. — Combined direct pressures in the vertical members are also compression. The maximum value is produced by dead and live loads, and by the reaction of one-sided earth pressure. It varies from a minimum at the top to a maximum at the bottom. The difference between the two values is the weight of the vertical member and of the wall between the vertical ribs.

At top $R_T = 119\,400$ lb.

At bottom $R_B = 168\,200$ lb.

CONCRETE DIMENSIONS AND REINFORCEMENT OF FRAME

After all bending moments, shears, and thrusts are determined, and the bending-moment and shear diagram drawn, concrete dimensions of the members of the frame and the areas of reinforcement are found using the regular reinforced-concrete formulas.

Horizontal Member. — The horizontal member is subjected to direct pressures and bending moments. Bending moments at the different sections may be taken from the combined bending-moment diagram. To each loading condition there corresponds a definite direct pressure, which is constant throughout the whole

length of the member. Since the direct pressures are relatively small, ordinarily their effect is disregarded in designing the horizontal member; and the member is designed for bending moments, only. This greatly simplifies the work and is on the safe side as far as reinforcement is concerned. However, to take care of the compression stresses produced by the direct pressures, when using bending moments only, the sections of the frame subjected to the largest compression stresses should be designed for a somewhat smaller compression stress than the specified allowable stress. The difference may be made equal to double the average unit compression

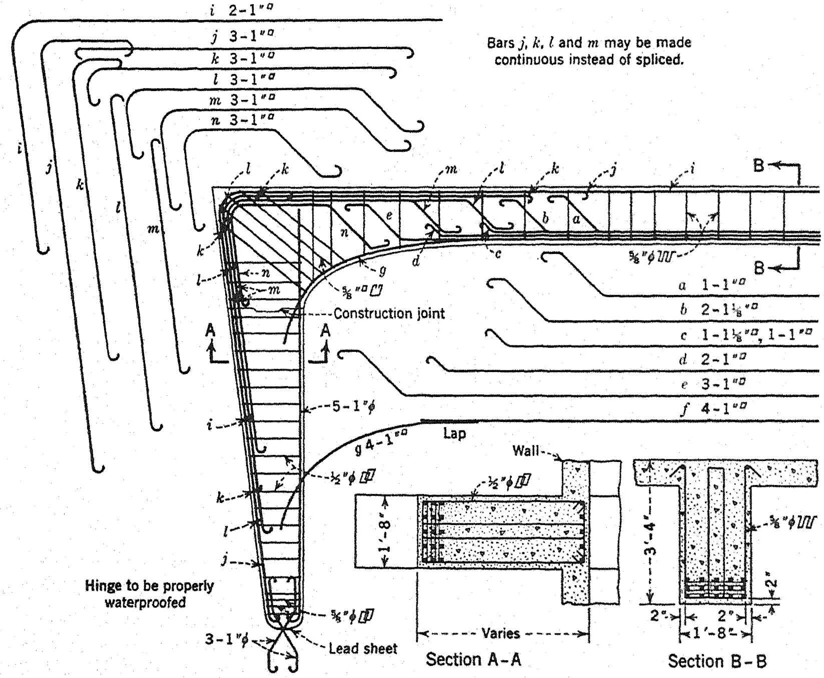


FIG. 131.—Example. Details of Right-angle Rigid Frame. (See p. 296.)

produced on the section under consideration by the direct pressure. In this example at the end of the member the allowable compression stress is reduced by 100 lb. per sq. in.

When it is desired to consider the effect of the direct pressures, the amount of tensile and compression reinforcement for the accepted concrete dimensions may be found from formulas (42) and (45), given in "Concrete, Plain and Reinforced," Vol. II, on pp. 233 and 234. To get maximum compression reinforcement at the corner use the maximum direct pressure with the maximum combined negative bending moment. To get the largest tension reinforcement, omit the symmetrical earth pressures because they produce large direct pressures and only small bending moments.

In this example, the points of bending of reinforcement were determined from the bending-moment diagram, Fig. 126, p. 290, and the shear diagram. The arrangement of reinforcement is shown in Fig. 131, p. 296. To avoid very long bars, which would be

difficult to transport and to handle on the job, the positive bending-moment reinforcement is not extended to the supports to act there as negative bending-moment reinforcement. Instead, separate sets of bars are used for the negative and for the positive reinforcement; and the bars in these sets overlap, thus producing the same effect as would have been obtained by long continuous bars.

Vertical Members. — At the bottom, the cross section of the vertical member is determined by diagonal tension and by compression stresses. Diagonal tension is produced by horizontal external shears as shown in Fig. 130, p. 295. Compression stresses are produced by the maximum direct pressure at the bottom of the member which is equal to 168 200 lb.

The area of contact of the vertical member with the footing is found by dividing the maximum direct pressure on the member by the allowable unit stress in bearing, $f_b = 1\,000$ lb. per sq. in. Therefore the bearing area is $A = \dfrac{168\,200}{1\,000} = 168.2$ sq. in. Making the width of the bearing area equal to the width of the member, i.e., 20 in., the other dimension is $\dfrac{168.2}{20} = 8.4$ in. The bearing area is arranged concentric with the cross section of the rib at the bottom. The balance of the cross section of the rib is rounded up at both sides of the bearing area, and the space between the footing and the edges of the rib is filled with plastic waterproofing material. A thin sheet of lead of the same dimensions as the bearing area, placed between the footing and the rib, permits the required rotating movements of the frame at the hinges.

To resist the horizontal thrusts at the hinge two sets of inclined crossing anchoring bars are used, which extend from the footing into the ribs. The cross section of the anchoring bars is obtained by dividing the maximum combined horizontal thrust by the allowable unit stress in steel in direct shear. Therefore $A_s = \dfrac{41\,750}{12\,000} = 3.5$ sq. in.

Use at each hinge six 1-in. round bars, three in each direction. The bars are arranged so that the two sets of bars cross in the center of the bearing area.

To prevent splitting of the vertical member at the bottom due to the concentrated reactions, closely spaced hoops are used at the bottom. The amount of the reinforcement in the hoops is computed by assuming that a lateral tension is developed equal to 0.3 of the maximum direct pressure on the vertical member. This lateral tension is resisted by the hoops located within a vertical distance of 15 in. This rule was developed in Europe, and its correctness has been corroborated by tests. It is recommended for use until a more satisfactory method is developed. Direct pressure is 168 200 lb.; lateral tension $0.3 \times 168\,200 = 50\,460$ lb. Therefore the required area of hoops is $A_s = \dfrac{50\,460}{16\,000} = 3.2$ sq. in.

Use four double hoops of $\frac{5}{8}$-in. round bar, spaced 5 in. on centers.

In the balance of the vertical member, the hoops are governed by diagonal tension in the same manner as stirrups in beams, but at the same time the spacing of the hoops should not be smaller than is customary in column design. The diagram of external shears is shown in Fig. 130, p. 295.

The dimensions of the vertical member at the top are determined by bending moments. Also, to maintain the assumed ratio of rigidity of the frame, the final dimensions of the vertical member must be such that the moment of inertia of the cross section at a distance above the hinge equal to two-thirds of the theoretical

height is equal to the assumed value. In this example, for this purpose the depth of the section must be 4 ft.

The vertical member is subjected to direct pressure and to bending moments, and the conditions are similar to those discussed in connection with the horizontal member, except that the effect of the direct pressure is very much larger and should be taken into consideration. To simplify the work, at two or three sections the amount of tension and compression reinforcement should be found by formulas for direct pressure and bending moments mentioned in connection with the horizontal member. At the other points, the amount of steel may be found from the formula $A_s = \frac{M}{jdf_s}$ and reduced by a proper percentage.

The reinforcement of the member is shown in Fig. 131, p. 296. The bars resisting negative bending moments are marked by letters from i to n, inclusive. This reinforcement must be extended around the corner in order to transfer tension stresses from the horizontal member to the vertical member. To avoid the use of long unwieldy bars, and also the necessity of placing all bars before pouring concrete in the vertical member, bars j, k, l, and m are made up of two parts and properly spliced. The points of splicing are arranged so that not more than three bars are spliced at any one section. Also, the splices are placed within the intersection block, where the stresses are smaller than elsewhere in the members. With this arrangement of bars, the vertical members may be poured up to the construction joint located at the bottom of the haunch before the horizontal parts of the spliced bars are in place.

At the construction joint, a key is provided in the concrete of sufficient dimensions to take care of the direct shears. This joint should be properly protected by waterproofing to prevent leakage of water.

Footing. — The design of footings for frames is in chapter discussed on p. 436.

EXAMPLE 2

The purpose of the example that follows is to show the effect of cantilevers upon bending moments in a right-angle rigid frame with hinged ends.

Example 2. — Find bending moments and shears in a right-angle frame with cantilevers, the main dimensions of which are the same as those in Example 1. To get a direct comparison between the bending moments for a frame with and without cantilevers, the dimensions of the frame here used are the same as in Example 1, p. 285; also, the same cross section of the bridge is assumed. In an actual design, it would be more economical to use lighter vertical members in the design with cantilevers because the bending moments there are much smaller than in a frame without cantilevers.

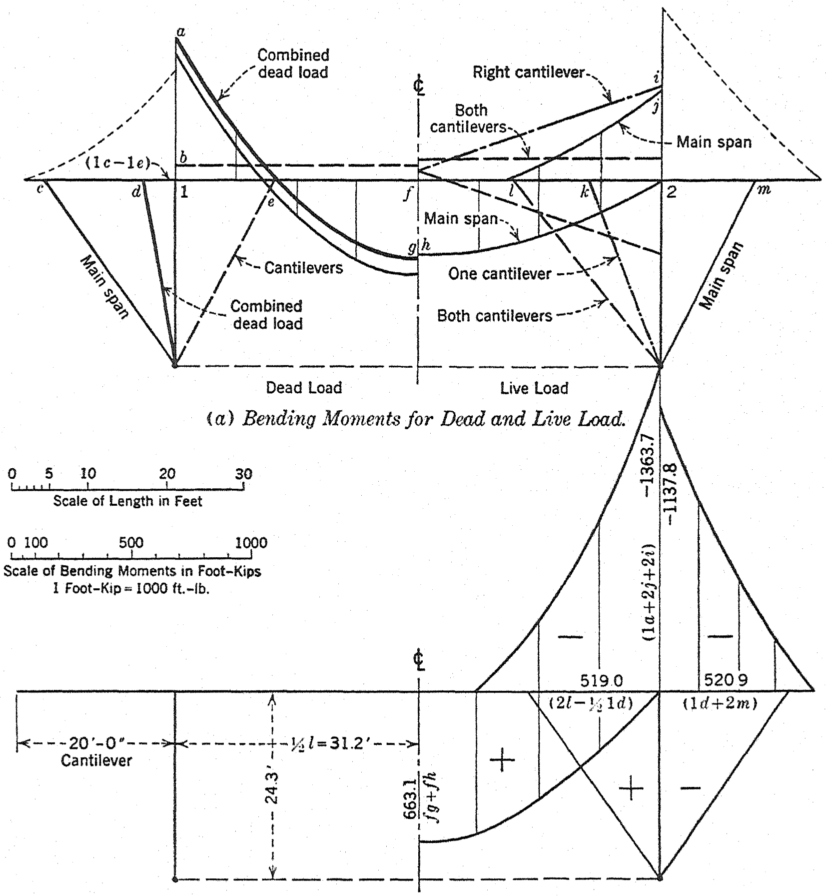
The main dimensions of the frame are: span 62.4 ft.; theoretical height 24.3 ft.; length of cantilever 20.0 ft. measured from vertical frame axis. (See Fig. 132, p. 299.)

Live load and impact are the same as in the previous example.

At the end of each cantilever is suspended an apron with wing walls for retaining the embankment, weighing 6 000 lb. per cantilever.

Solution. — Since the bending moments for loads on the main span of the frame are already known, it is only necessary to find bending moments for the loads on the cantilevers. As found on p. 288, the frame constant is $C_2 = 0.432$.

Dead Loads on Cantilevers. — The unit dead load on the cantilever is taken as $w = 1\,690$ lb.; the concentrated dead load, $P = 6\,000$ lb., is distant 19.5 ft. from the column axis.



(a) *Bending Moments for Dead and Live Load.*

(b) *Combined Bending Moments.*

FIG. 132.—Example of Rigid Frame with Cantilevers. Bending-Moment Diagrams. (*See p.* 298.)

Maximum cantilever bending moment for dead load:

$$M_{1d} = -(\tfrac{1}{2} \times 1\,690 \times 20^2 + 6\,000 \times 19.5) = -455\,000 \text{ ft-lb.} \quad \text{or} \quad 455.0 \text{ ft-kips}$$

Horizontal thrust, both cantilevers loaded, formula (5), p. 278:

$$H = \frac{2}{24.3} \times 0.432 \times 455\,000 = 16\,180 \text{ lb.} \quad \text{or} \quad 16.2 \text{ kips}$$

Bending moments in frame for dead load on both cantilevers, using formulas (11) and (16), p. 279;

In main span $M_{Br} = M_{Cl} = -(1 - 2 \times 0.432) \times 455\,000 = -61\,900$ ft-lb.
In vertical members $M_B = M_C = 16\,180 \times 24.3 = 393\,100$ ft-lb.

Combined Bending Moments for Dead Load. — Add the bending moments just found to the bending moments for dead load on the main span of frame from p. 288.

Combined maximum negative bending moments in vertical members for dead load:

$$M_B = M_C = -535\,000 + 393\,100 = 141\,900 \text{ ft-lb.}$$

Combined maximum bending moments in horizontal member:

Positive: $M_{\text{max.}} = 390\,000 - 61\,900 = 328\,100$ ft-lb.
Negative: $M_{Br} = M_{Cl} = -(535\,000 + 61\,900) = -596\,900$ ft-lb.
Combined horizontal thrust: $H = -22\,000 + 16\,180 = -5\,820$ lb.

Bending-moment diagrams for dead load are shown in the left half of Fig. 132 (*a*), p. 299.

Live Loads. — In this case, bending moments are found separately for live loads on cantilevers, and these are combined with the previously computed bending moments for live loads on the main span. In some combinations this method gives somewhat too unfavorable results, because the most unfavorable loading on the cantilever may not be possible with a simultaneous most unfavorable position of loads on the main span.

When the rear axle of a design truck is placed at the end of the cantilever, it can accommodate longitudinally one truck. Loads on axles are found on p. 288. Loading: $P_1 = 32\,700$ lb., $a_1 = 19.5$ ft.; $P_2 = 8\,200$ lb., $a_2 = 19.5 - 14 = 5.5$ ft. Bending moment on cantilever,

$$M_{1l} = -(32\,700 \times 19.5 + 8\,200 \times 5.5) = -682\,800 \text{ ft-lb.}$$

Using formulas (4) to (15), p. 279, the horizontal thrust and bending moments due to the live loads on the left cantilever are:

$$H = \frac{1}{24.3} \times 0.432 \times 682\,800 = 12\,100 \text{ lb.}$$
$$M_{Br} = -(1 - 0.432) \times 682\,800 = -387\,800 \text{ ft-lb.}$$
$$M_{Cl} = 0.432 \times 682\,800 = 295\,000 \text{ ft-lb.}$$
$$M_B = M_C = M_{Cl} = 295\,000 \text{ ft-lb.}$$

Symmetrical live loads on both cantilevers

$$H = 2 \times 12\,100 = 24\,200 \text{ lb.}$$
$$M_{Br} = M_{Cl} = -387\,800 + 295\,000 = -92\,800 \text{ ft-lb.}$$
$$M_B = M_C = 2 \times 295\,000 = 590\,000 \text{ ft-lb.}$$

Bending-moment diagrams for live load on cantilevers as well as the combined bending-moment diagram for live loads are shown in the right half of Fig. 132, p. 299.

Combined Bending Moments for Dead and Live Loads. — Maximum negative bending moments in the corners are obtained by combining the bending moment for the combined dead load with the bending moments for live load on the main span from p. 290 and for live loads, on the cantilever next to the corner under consideration.

$$M_{Br} = -(596\,900 + 379\,000 + 387\,800) = -1\,363\,700 \text{ ft-lb.}$$

Maximum positive bending moment is obtained by adding the bending moment for the combined dead load to the bending moment for live load on the main span from p. 290.

$$M_{\text{max.}} = 328\,100 + 332\,000 = 660\,100 \text{ ft-lb.}$$

In the vertical member, maximum negative bending moment at the top is obtained by adding the bending moment for the combined dead load to the bending moment for live load on the main span.

$$M_B = -(141\,900 + 379\,000) = -520\,900 \text{ ft-lb.}$$

Maximum positive bending moment at the top of the vertical member is obtained by adding to the bending moments for symmetrical live loads on both cantilevers, with its sign, one-half the bending moment for the dead load. (See p. 171, rule 4.)

$$M_B = 590\,000 - \tfrac{1}{2} \times 141\,900 = 519\,000 \text{ ft-lb.}$$

Horizontal Thrusts.

Maximum positive thrust $H = 24\,200 - \frac{1}{2} \times 5\,820 = 21\,300$ lb.
Maximum negative thrust $H = -(5\,820 + 15\,600) = -21\,420$ lb.

See p. 171 for instructions about combining values of opposite signs.

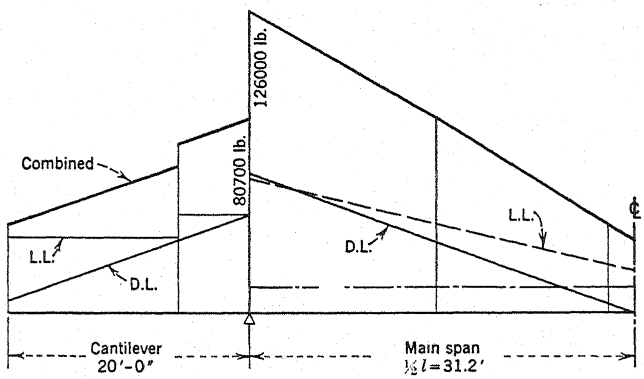


FIG. 133.—Frame with Cantilevers. External Shears. (*See* p. 301.)

External Shears in Horizontal Member. — External shears for dead load and for live load on the main span are the same as for a frame without cantilevers.

Since dead load on the cantilevers is symmetrical, it produces no shears in the main span.

Live load on the left cantilever produces positive shear in the main span, which is equal to the maximum cantilever bending moment divided by the span length. This shear is constant throughout the span.

$$V_B = \frac{682\,800}{62.4} = 10\,900 \text{ lb.}$$

Combined end shear.

$$V_B = 59\,300 + 56\,400 + 10\,900 = 126\,600 \text{ lb.}$$

The external shear diagram is shown in Fig. 133, above.

External Shear in Vertical Member. — External shears in vertical members are due to horizontal thrusts, and are constant throughout their length.

Direct Pressure in Horizontal Member. — Direct tension is equal to the combined positive thrusts. It is: $H = 21\ 300$ lb.

Direct compression, equal to the combined negative thrusts, is $H = -21\ 420$ lb.

Direct Compression in Vertical Member. — Direct compression in a vertical member is equal to the dead-load reactions of the horizontal member, reaction for the live load, with loads on the main span and on the cantilever next to the vertical member under consideration, and the weight of the vertical member.

Earth Pressures. — When designing the apron and the wing walls which are attached to the ends of the cantilevers, full effect of the earth pressures acting upon them should be used. The vertical ribs which are fixed to the ends of the frame ribs, and to which are attached the apron wall and the wing walls, act as cantilevers fixed at the top and free at the bottom, and subjected to the earth pressure transferred to them by the walls. The passive pressure of the fill in front of the wall should not be considered as reducing the active earth pressures. The walls should be considered as continuous vertical slabs spanning between the ribs and loaded by the vertical ribs.

So far as the structure as a whole is concerned, the effect of the earth pressures acting on the aprons may be disregarded because the walls are usually well imbedded in the fill so that the active earth pressures on one side are counteracted by the passive pressures on the other side of the apron. If, however, it is considered wise to neglect the effect of the passive pressures, the active pressures produce in the horizontal cantilever arm of the frame constant negative bending moments which should be added to the other cantilever bending moments in computing the bending moments in the frame.

Concrete Dimensions and Reinforcement. — (See Fig. 134, p. 303.) After the bending moments and shears are computed, concrete dimensions and the cross sections of reinforcement are found in the same manner as for a frame without cantilevers. (See p. 295.) In this case the direct pressures in the horizontal member are very much smaller so that their effect may be disregarded, and the horizontal member designed for bending moments only.

The depths of cross sections of the cantilevers are here made large enough to keep the compression stresses due to negative bending moments within working limits without the use of compression reinforcement. In cantilevers, the same allowable unit stresses are used as specified for beams at the supports.

It should be noted that in this example the cross section of the vertical member is rectangular, while in Example 1 the vertical members are T-beams in cross section. Since it has been assumed that the moment of inertia of the vertical member in both cases are the same, a much larger depth must be used in this case than in Example 1. Compare Fig. 131, p. 296, with Fig. 134, p. 303.

RIGHT-ANGLE FRAME WITH COUNTERWEIGHTED CANTILEVERS AT HINGES

Horizontal thrusts in a rigid frame may be appreciably reduced by the use of counterweighted cantilevers located at the bottom of the frame. The use of this design is discussed on p. 271. The counterweights usually consist of the fill on the cantilevers.

In addition to notation on p. 276, let M_1 = maximum negative bending moment of loads on cantilever.

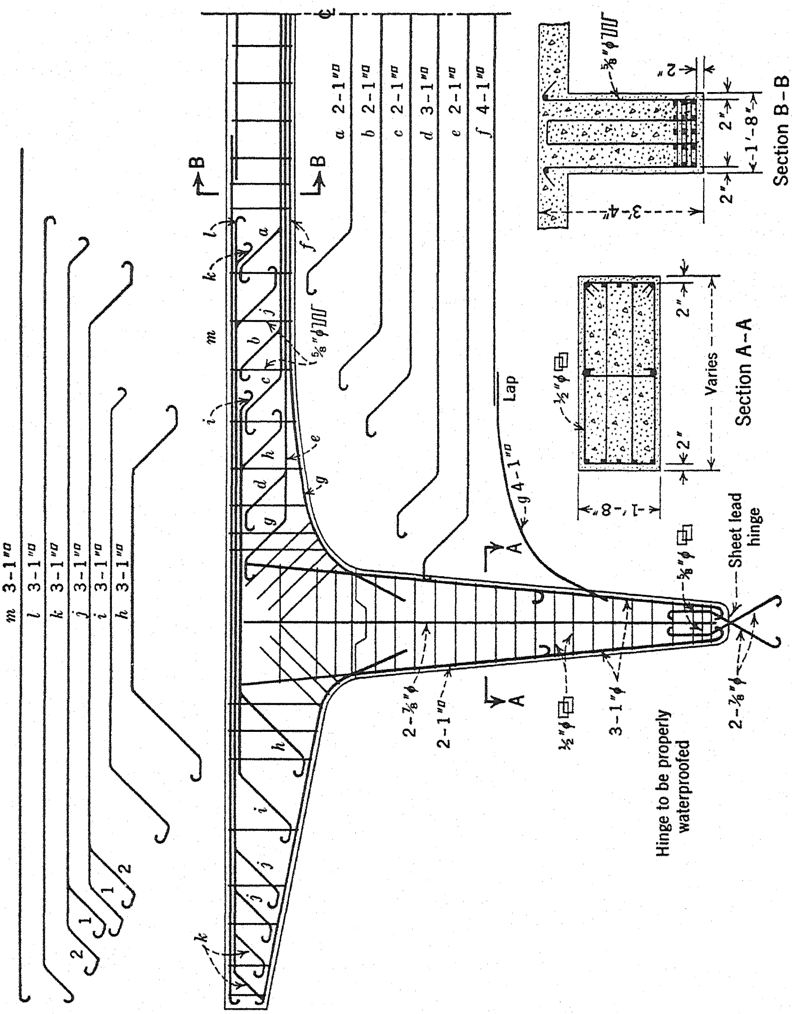


Fig. 134.—Example. Details of Rigid Frame with Cantilevers. (See p. 302.)

Here also the horizontal thrust is the only statically indeterminate value. For the loads on the cantilevers, the thrusts are computed from the formulas:

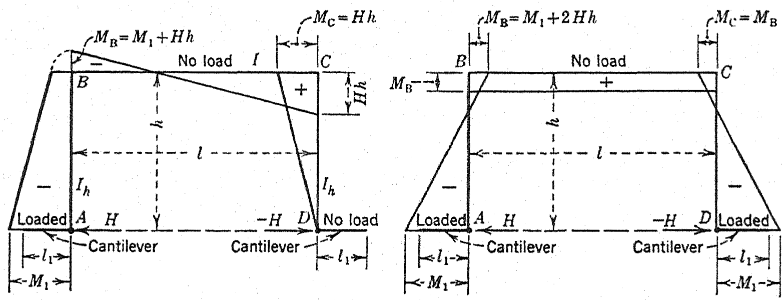
Horizontal Thrust, One Cantilever Loaded:

$$H = -C_2\left(\frac{I}{I_h}\frac{h}{l} + 1\right)\frac{M_1}{h} \tag{34}$$

Horizontal Thrust, Both Cantilevers Loaded Symmetrically:

$$H = -2C_2\left(\frac{I}{I_h}\frac{h}{l} + 1\right)\frac{M_1}{h} \tag{35}$$

In both cases the frame constant C_2 is as given on p. 278.



(*a*) *Load on Left Cantilever* (*b*) *Symmetrical Loads on Both Cantilevers*

FIG. 135.—Rigid Frame with Counterweighted Cantilevers at Level of Hinges. (*See p.* 304.)

For cantilever loads, horizontal thrusts at the left hinge are positive and at the right hinge negative. Therefore they act in the opposite directions to the thrusts due to vertical downward loads on the main span. For dead loads advantage may be taken of this for a reduction of the horizontal thrusts.

In Fig. 135, p. 304, bending moments are shown in the frame for loads on one and on both cantilevers.

RIGHT-ANGLE FRAME, HINGED ENDS. FIXED-POINT METHOD

In unusual cases, where the formulas in the preceding pages do not apply, the fixed-point method may be resorted to. When the frame, the loading, or both, are not symmetrical, the results obtained directly by means of the fixed-point method must be corrected as explained under separate headings for vertical and for horizontal loadings.

For explanation of the fixed point method as applied to rigid frames see Chapter XI, p. 220.

For vertical loadings one-span rigid frames need only the left and the right fixed point in the horizontal member, f and f'.

For horizontal loadings such as earth pressure and for the effect of temperature changes the upper fixed points in the vertical members are also needed. They are f'_{h1} and f'_{h2}. The lower fixed points coincide with the hinges.

In one-span rigid frames with cantilevers, in addition to the fixed points it is necessary to find the ratios of transference for bending moments from the cantilevers to the horizontal members. With two cantilevers there are two ratios per frame, r'_1, and r_2.

The sequence for determining the various values is as follows:

1. Find the left fixed point f, in the horizontal member using formula (36), below.

2. Find the upper fixed point in the right vertical member f'_{h2} using formula (37), below.

3. For frame with cantilevers find the left ratio of transference at the second vertical member r_2, i.e., the ratio of transference from the right cantilever to the horizontal member, using formula (38).

The formulas that follow are arranged in the order just named.

In addition to the notation on pp. 276 and 281, let

h_1 = height of left vertical member.

h_2 = height of right vertical member.

h'_2 = clear height of right vertical member as defined on p. 226.

I_{h1}, I_{h2} = moments of inertia of vertical members.

I = moment of inertia of horizontal member.

Fixed Points in Frame, Hinged Ends.—
Left Fixed Point in Horizontal Member:

$$f = 4C_1l \tag{36}$$

where C_1 is the constant from formulas (6) or (8), p. 278, in which for $\frac{I}{I_h}\frac{h}{l}$ substitute $\frac{I}{I_{h1}}\frac{h_1}{l}$.

Upper Fixed Point in Right Vertical Member with Height h_2:

$$f'_{h2} = \frac{\left(\frac{h'_2}{h_2}\right)^2\left(3 - 2\frac{h'_2}{h_2}\right)}{3\left(\frac{h'_2}{h_2}\right)^2 + \frac{I_{h2}}{I}\frac{l}{h_2}q} h_2 \tag{37}$$

For rigid frames with cantilevers:

Ratio of Transference from Right Cantilever to Horizontal Member:

$$r_2 = 2\left(\frac{h'_2}{h_2}\right)^3 \frac{1}{2\left(\frac{h'_2}{h_2}\right)^3 + \frac{I_{h2}}{I}\frac{l}{h_2}q} \tag{38}$$

In formulas (37) and (38), values of q are:

$$q = 2 - \frac{1}{\frac{l}{f} - 1} \quad \text{for constant moments of inertia} \tag{39}$$

$$q = 2\alpha' - \frac{1}{\frac{l}{f} - 1}\beta \quad \text{for variable moments of inertia} \tag{39a}$$

The upper fixed points are needed only for finding bending moments for horizontal pressures acting on the frame.

For symmetrical frames, all fixed points are symmetrical.

For unsymmetrical frames, the right fixed point, the upper fixed point in the first leg, and the ratio of transference from the first cantilever to the right are found by starting at the right end and proceeding toward left in the same manner as explained on p. 305.

Notation:

M'_B and M'_C = uncorrected bending moments at corners B and C found by fixed-point method.

H'_1 and H'_2 = uncorrected thrusts.

H_{c1} and H_{c2} = correcting thrusts.

H_1 and H_2 = final corrected thrusts.

M_B and M_C = final corrected bending moments.

CORRECTIONS FOR VERTICAL LOADS. RIGHT-ANGLE FRAME, HINGED ENDS

For each condition of vertical loading, find the uncorrected bending moments in the frame by means of the fixed-point method in the usual manner. This determines the negative bending moments in the corners, M'_B and M'_C.

For each loading, uncorrected, find the horizontal thrust at each vertical member by dividing the corner bending moment by the height of the member, $H'_1 = \frac{M'_B}{h_1}$ and $H'_2 = -\frac{M'_C}{h_2}$. When the thrusts at both ends are numerically equal to each other, no corrections are needed.

When the thrusts at both ends are not numerically equal, a correcting thrust must be added at each end; and the value of these correcting

thrusts may be found from the following formulas in which H'_1 and H'_2 should be used with their signs.

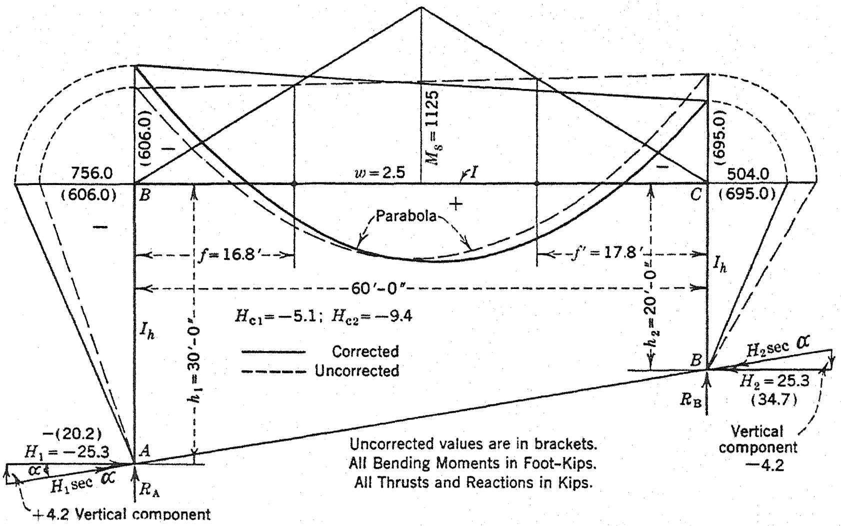


FIG. 136.—Right-angle Frame, Hinged Ends. Fixed-Point Method. (*See* p. 310.)

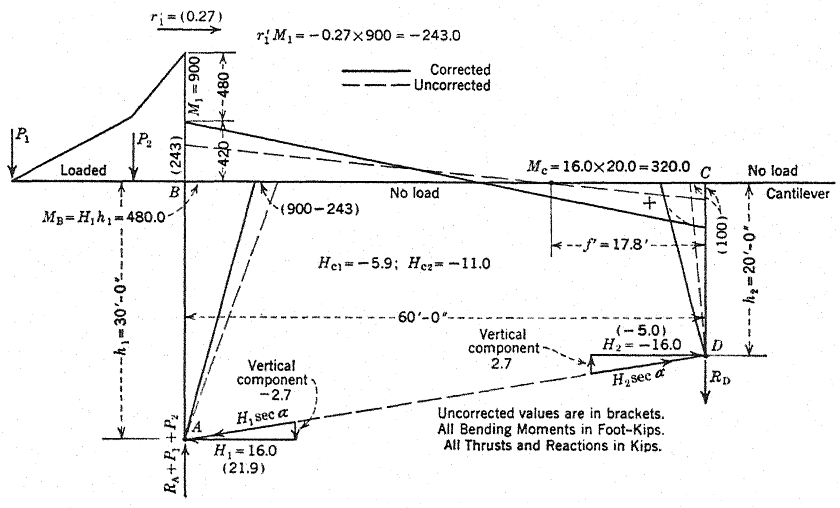


FIG. 137.—Right-angle Frame with Cantilevers, Hinged Ends. Fixed-Point Method. (*See* p. 310.)

Correcting Thrusts for Vertical Loadings:

$$H_{c1} = -\frac{1}{2}\left(H'_1 + H'_2\right)\left(\frac{h_2^2}{h_1^2 + h_2^2} + \frac{h_2}{h_1 + h_2}\right) \tag{40}$$

$$H_{c2} = -(H'_1 + H'_2) - H_{c1} \tag{40a}$$

Final corrected thrusts H_1 and H_2 are found by adding H'_1 to H_{c1} and H'_2 to H_{c2}.

Corner bending moments are then found for the corrected thrusts by multiplying each thrust by the corresponding height of the vertical member; and the bending-moment diagram is completed in the known manner.

This method is exact for frames having vertical members of the same heights, i.e., when $h_1 = h_2$; but in ordinary cases it gives results accurate enough for practical purposes even for unequal heights of vertical members. This method was used in the examples shown in Figs. 136 and 137, p. 307; and the resulting thrusts there are less than 4 per cent in error as compared with accurate results.

CORRECTIONS FOR HORIZONTAL PRESSURES. RIGHT-ANGLE FRAME, HINGED ENDS

Method of procedure and formulas are here given for correcting bending moments for horizontal pressures. Separate formulas are given for rectangular and triangular distribution of pressures.

As in the previous case, the results of this method are exact for frames with equal heights of vertical members. For unequal heights, however, the errors are small, and the method is accurate enough for practical purposes.

To find bending moments due to unsymmetrical earth pressures, proceed as follows:

Find all fixed points as explained on p. 305.

Using the values for the fixed points, find the uncorrected bending moments at the corners, M'_B and M'_C. Formulas for these values are given on p. 308 for uniformly distributed pressures and for triangular distribution of pressures.

From the requirement that the sum of the thrusts acting on the frame must be equal to the sum of the horizontal pressures, formulas for the correcting thrusts H_{c1} and H_{c2} are found. Formulas (42) and (43) are for uniformly distributed pressures; and formulas (48) and (49) for triangular distribution. The final corner bending moments M_B and M_C are now found as well as the final thrusts H_1 and H_2, using appropriate formulas.

Rectangular Distribution of Earth Pressures. — Formulas are here given for uncorrected corner bending moments as determined by the fixed-point method for correcting thrusts and for final bending moments and thrusts for uniformly distributed horizontal pressures. The pressures are assumed to act from left to right on the left vertical member. p is uniformly distributed pressure.

In the formulas, it is assumed that the clear height h'_1 (as defined on p. 226) is practically equal the theoretical height h_1. For other conditions multiply the corner bending moments in formulas (41) and (41*a*) by $\left[1 - 2\left(1 - \frac{h'_1}{h_1}\right)^2\right]$.

Uncorrected Corner Bending Moments as Determined by Fixed-Point Method:

$$M'_B = -\frac{1}{4}\frac{f'_{h1}}{h_1 - f_{h1}} ph_1^2 \qquad (41) \qquad M'_C = -M'_B \frac{f'}{l - f'} \qquad (41a)$$

Correcting Thrusts:

$$H_{c1} = \frac{1}{2}\left[\frac{1}{2}ph_1 + \frac{M'_C}{h_2} - \frac{M'_B}{h_1}\right]\left(\frac{h_2^2}{h_1^2 + h_2^2} + \frac{h_2}{h_1 + h_2}\right) \qquad (42)$$

$$H_{c2} = \left(\frac{1}{2}ph_1 + \frac{M'_C}{h_2} - \frac{M'_B}{h_1}\right) - H_{c1} \qquad (43)$$

Final Bending Moments at Corners B and C:

$$M_B = M'_B + H_{c1}h_1 \qquad (44) \qquad M_C = M'_C - H_{c2}h_2 \qquad (44a)$$

Final Thrusts:

$$H_1 = ph - H_2 \qquad (45) \qquad H_2 = -\frac{M'_C}{h_2} + H_{c2} \qquad (45a)$$

Triangular Distribution of Horizontal Pressures. — Similar formulas are here given for the triangular distribution of pressures, where p_1 is the maximum pressure at the bottom. As in the previous case, the pressures act on the left member from left to right. Also $h'_1 = h_1$. For other ratios of $\frac{h'_1}{h_1}$ multiply M'_B and M'_C by $\left[1 - \left(1 - \frac{h'_1}{h_1}\right)^2\right]$.

Uncorrected Corner Bending Moments as Determined by Fixed-Point Method:

$$M'_B = -\frac{1}{8.57}\frac{f'_{h1}}{h_1 - f'_{h1}} p_1h_1^2 \quad (46) \qquad M'_C = -M'_B \frac{f'}{l - f'} \qquad (47)$$

Correcting Thrusts:

$$H_{c1} = \frac{1}{2}\left(\frac{1}{6}p_1h_1 + \frac{M'_C}{h_2} - \frac{M'_B}{h_1}\right)\left(\frac{h_2^2}{h_1^2 + h_2^2} + \frac{h_2}{h_1 + h_2}\right) \qquad (48)$$

$$H_{c2} = \left(\frac{1}{6}p_1h_1 + \frac{M'_C}{h_2} - \frac{M'_B}{h_1}\right) - H_{c1} \qquad (49)$$

Final Bending Moments at Corners B and C:

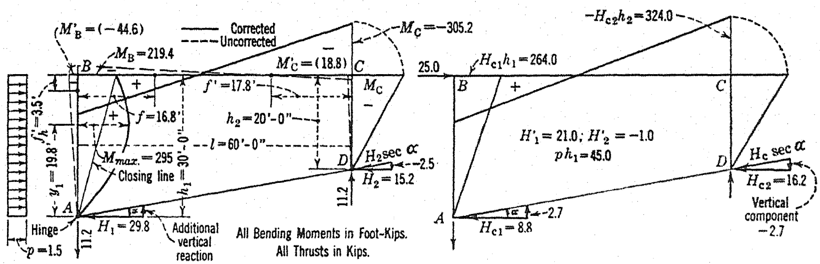
$$M_B = M'_B + H_{c1}h_1 \quad (50) \qquad M_C = M'_C - H_{c2}h_2 \quad (51)$$

Final Thrusts:

$$H_1 = \frac{1}{2}p_1h - H_2 \quad (52) \qquad H_2 = -\frac{M'_C}{h_2} + H_{c2} \quad (53)$$

Examples of Use of Fixed-Point Method for One-Span Frames.— The use of the fixed-point method with the necessary corrections is illustrated by the examples shown in Figs. 136 to 138, pp. 307 and 310.

In all cases, an unsymmetrical frame is used of the following dimensions: span length, l = 60.0 ft.; heights of vertical members, h_1 = 30.0 ft.; h_2 = 20.0 ft.; fixed points in horizontal member, f = 16.8 ft. and f' = 17.8 ft.; in vertical member f'_{h1} = 3.5 ft.; f'_{h2} = 1.7 ft.



(*a*) *Uncorrected and Final Bending Moments.* (*b*) *Correcting Bending Moments.*

FIG. 138.—Right-angle Frame, Hinged Ends. Horizontal Pressures. (*See* p. 311.)

It should be noted that the final thrusts are acting along an inclined line obtained by connecting the two hinges. Their values are equal to the horizontal thrusts multiplied by the secant of the angle of inclination.

In Fig. 136, p. 307, solution is given for vertical loads on the main span. The uncorrected thrusts are $H'_1 = -20.2$ kips, and $H'_2 = 34.7$ kips. The correcting thrusts are $H_{c1} = -5.1$ kips and $H_{c2} = -9.4$. The final thrusts $H_1 = -H_2 = -25.3$ kips. The bending moments are clearly shown in the figure.

In Fig. 137, p. 307, the bending-moment diagram is shown for a loading on the left cantilever. Here, the uncorrected ratio of transference is $r'_1 = 0.27$, so that the bending moment transferred to the horizontal member is $-0.27 \times 900.0 = -243.0$ kips. The diagram determined by the fixed-point method is shown by dash lines, and the corresponding values are shown in brackets. The correcting thrusts are $H_{c1} = -5.9$ kips and $H_{c2} = -11.0$ kips, and the final thrusts are $H_1 = -H_2 = 16.0$ kips. The balance of the work is clearly shown in the figure, the final bending moments being indicated by solid lines.

In Fig. 138, p. 310, is shown the work for horizontal pressures acting on the left vertical member. The uncorrected bending moment at B is $M'_B = -44.6$ ft-kips, and the uncorrected diagram is shown by dash lines. The parabola for the uncorrected diagram is not shown. The correcting thrusts are $H_{c1} = 8.8$ kips and $H_{c2} = 16.2$ kips, and the corresponding diagram is shown in Fig. 138 (b), p. 310. This diagram could be dispensed with; it is drawn here only to show the effect of the correcting thrusts. The final thrusts are $H_1 = 29.8$ kips and $H_2 = 15.2$ kips, and the final bending-moment diagram is shown in Fig. 138 (a) by solid lines.

RIGHT-ANGLE RIGID FRAMES. FIXED ENDS

Definition. — A right-angle rigid frame with fixed ends is a frame in which the ends of the vertical members are fixed by the foundation in such manner that when loaded the tangent to the deflection curve at each end coincides with the original axis of the vertical member.

Assumptions as to Moments of Inertia. — In the formulas in Tables I and II, pp. 312 and 314, frame constants are given for the assumption that the moments of inertia are constant throughout the length of each member, but the constant moments of inertia of the horizontal member, I, are different from the constant moments of inertia I_h of the vertical members.

For variable moments of inertia of the horizontal member the fixed-point method should be used as described on p. 316.

Formulas for Vertical Loadings on Main Span. — In Table I, p. 312, formulas are given for vertical reactions, horizontal thrusts, and bending moments at critical sections for vertical loadings on the span of the frame. Separate formulas are given for uniformly distributed loading and for concentrated loads.

For concentrated loads, the formulas are exact only for constant moments of inertia of the horizontal member. For variable moments of inertia, results for *single* loads may err as much as ± 15 per cent; but for three or more loads distributed over the span the errors cancel so that the final results are accurate enough for practical purposes.

When concentrated wheel loads are used, they should be placed on the span in the most unfavorable positions. The position for maximum thrust and maximum negative bending moment is different from the position for maximum positive bending moment. The criterions for unfavorable positions of loads are given on p. 275.

The maximum bending moments at the critical sections for moving live loads having been found, the bending-moment diagrams may be drawn as explained on p. 275.

TABLE I

RIGHT-ANGLE FRAME. FIXED ENDS.

VERTICAL LOADING (SEE FIG. 139, P. 313)

Item	Notation	Uniform Loading	Concentrated Load P	Several Concentrated Loads
			Vertical Reactions	
1	V_A	$\frac{1}{2}wl$	$\left[\left(1-\frac{a}{l}\right)+C_{11}C_a\right]P$	$\Sigma\left[\left(1-\frac{a}{l}\right)+C_{11}C_a\right]P$
2	V_D	$\frac{1}{2}wl$	$P-V_A$	$\Sigma P-V_A$
			Horizontal Thrust, Left End	
3	H	$-\frac{l}{h}C_{10}wl$	$-6.0\frac{l}{h}C_{10}\frac{a}{l}\left(1-\frac{a}{l}\right)P$	$-6.0\frac{l}{h}C_{10}\Sigma\frac{a}{l}\left(1-\frac{a}{l}\right)P$
			Bottom Bending Moments	
4	M_A	$\frac{1}{3}C_{10}wl^2$	$-\frac{1}{3}Hh-\frac{1}{2}C_{11}C_aPl$	$-\frac{1}{3}Hh-\frac{1}{2}C_{11}l\Sigma C_aP$
5	M_D	$\frac{1}{3}C_{10}wl^2$	$-\frac{1}{3}Hh+\frac{1}{2}C_{11}C_aPl$	$-\frac{1}{3}Hh+\frac{1}{2}C_{11}l\Sigma C_aP$
			Corner Bending Moments	
6	M_B	$-2M_A$	M_A+Hh	M_A+Hh
7	M_C	$-2M_A$	M_D+Hh	M_D+Hh
			Maximum Positive Bending Moment	
8	$M_{\max.}$	$\frac{1}{8}wl^2-M_B$	M_B+aV_A	See note

Frame Constants: (Notation on p. 276.)

$$C_{10}=\frac{1}{4\left(\frac{Ih}{I_hl}+2\right)};\qquad C_{11}=\frac{1}{1+6\frac{Ih}{I_hl}};\qquad C_a=\frac{a}{l}\left(1-\frac{a}{l}\right)\left(1-2\frac{a}{l}\right)$$

In item (3) column (5)

$$\Sigma\frac{a}{l}\left(1-\frac{a}{l}\right)P=\frac{a_1}{l}\left(1-\frac{a_1}{l}\right)P_1+\frac{a_2}{l}\left(1-\frac{a_2}{l}\right)P_2+\frac{a_3}{l}\left(1-\frac{a_3}{l}\right)P_3+\cdots$$

In items (4) and (5)

$$\Sigma C_aP=\frac{a_1}{l}\left(1-\frac{a_1}{l}\right)\left(1-2\frac{a_1}{l}\right)P_1+\frac{a_2}{l}\left(1-\frac{a_2}{l}\right)\left(1-2\frac{a_2}{l}\right)P_2+\frac{a_3}{l}\left(1-\frac{a_3}{l}\right)\left(1-2\frac{a_3}{l}\right)P_3+\cdots$$

NOTE: To get maximum positive bending moment for several concentrated loads, draw a bending-moment diagram as in Fig. 139, p. 313.

Since H is negative, Hh and Hy are negative. Horizontal thrust at the right end is equal to $-H$.

When loads P are in pounds, w in pounds per lineal foot, and the dimensions in feet, the reactions and thrusts are in pounds and the bending moments in foot-pounds.

Bending-moment diagrams for vertical loadings are shown in Fig. 139, below.

Formulas for Horizontal Earth Pressures. — The discussion of earth pressure on p. 248 applies also to one-span frames with fixed ends. In Table II, p. 314, formulas are given for vertical reactions, horizontal

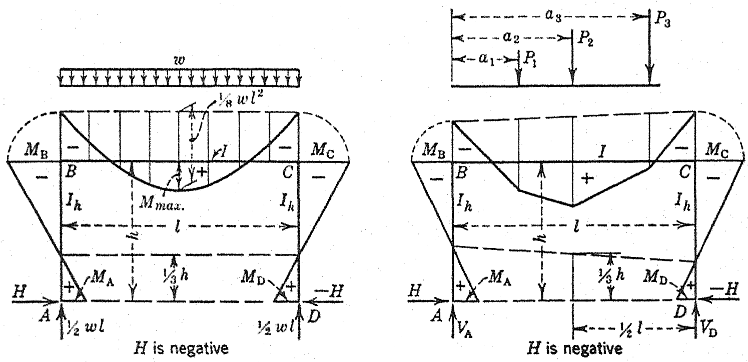


FIG. 139.—Right-angle Frame, Fixed Ends. Vertical Loadings. (*See p.* 312.)

thrusts, and bending moments due to horizontal earth pressures. Separate formulas are given for one-sided uniformly distributed earth pressures; and for triangular distribution of pressures acting on one side as well as acting symmetrically on both sides of the frame.

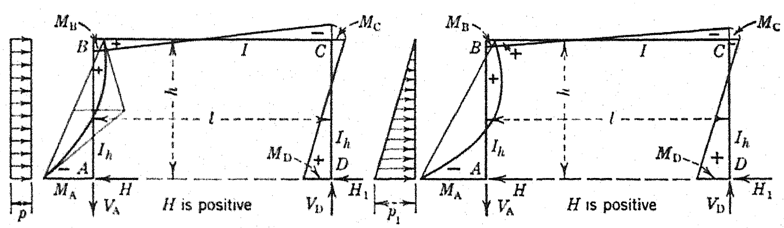


FIG. 140.—Right-angle Frame, Fixed Ends. Horizontal Pressures. (*See p.* 313.)

After the bending moments and thrusts are found, they are combined with the bending moments for vertical loads in the same manner as shown in the numerical example for frames with hinged ends. (See p. 293.)

Bending-moment diagrams for horizontal earth pressures are shown in Fig. 140, above.

TABLE II

RIGHT-ANGLE FRAME. FIXED ENDS.
HORIZONTAL PRESSURES (SEE FIG. 140, P. 314)

Item	Notation	Location	Uniform Pressure p	Triangular Pressure p_1	
			on Left Member	on Left Member	Symmetrical on Both Members
			Vertical Reactions		
1	V_A	Left	$-8C_{12}\frac{h}{l}ph$	$-2C_{12}\frac{h}{l}p_1h$	0
2	V_D	Right	$-V_A$	$-V_A$	0
			Horizontal Thrusts		
3	H	Left	$C_{10}\left(3\frac{Ih}{I_hl}+6.5\right)ph$	$\frac{1}{10}C_{10}\left(17\frac{Ih}{I_hl}+36\right)p_1h$	$\frac{1}{10}C_{10}\left(14\frac{Ih}{I_hl}+32\right)p_1h$
4	H_1	Right	$ph-H$	$\frac{1}{10}C_{10}\left(3\frac{Ih}{I_hl}+4\right)p_1h$	$-H$
			Bending Moments at Bottom		
5	M_A	Left	$-\left[\frac{1}{6}C_{10}\left(7\frac{Ih}{I_hl}+15\right)-4C_{12}\right]ph^2$	$-\left[\frac{1}{30}C_{10}\left(13\frac{Ih}{I_hl}+28\right)-C_{12}\right]p_1h^2$	$-\frac{1}{30}C_{10}\left(6\frac{Ih}{I_hl}+16\right)p_1h^2$
6	M_D	Right	$\left[\frac{1}{6}C_{10}\left(5\frac{Ih}{I_hl}+9\right)-4C_{12}\right]ph^2$	$\left[\frac{1}{30}C_{10}\left(7\frac{Ih}{I_hl}+12\right)-C_{12}\right]p_1h^2$	M_A

Item	Notation	Location	Uniform Pressure p	Triangular Pressure p_1	
			on Left Member	on Left Member	Symmetrical on Both Members
			Corner Bending Moments		
7	M_B	Left	$M_A + Hh - \frac{1}{2}ph^2$	$M_A + Hh - \frac{1}{3}p_1h^2$	$M_A + Hh - \frac{1}{3}p_1h^2$
8	M_C	Right	$M_D - H_1h$	$M_D - H_1h$	M_B
			Bending Moment in Loaded Member, y and y_1 from top		
9	M_y	y	$M_B - Hy - \frac{1}{2}py^2 + phy$	$M_B - Hy - \frac{1}{6}\frac{y}{h}p_1y^2 + \frac{1}{2}p_1hy$	
10	$M_{max.}$	y_1	$M_B - Hy_1 - \frac{1}{2}py_1^2 + phy_1$	$M_B - Hy_1 - \frac{1}{6}\frac{y_1}{h}p_1y_1^2 + \frac{1}{2}p_1hy_1$	
11	y_1	From top	$h\left(1 - \frac{H}{ph}\right)$	$h\sqrt{1 - \frac{2H}{p_1h}}$	

Frame Constants:

$$C_{10} = \frac{1}{4\left(\frac{Ih}{I_hl} + 2\right)} \qquad C_{11} = \frac{1}{1 + 6\frac{Ih}{I_hl}} \qquad C_{12} = \frac{1}{8}C_{11}\frac{Ih}{I_hl}$$

If p and p_1 are in pounds per lineal foot, and all dimensions in feet, the reactions and thrusts are in pounds, and the bending moments in foot-pounds.

Formulas for Effect of Temperature Changes. — Using notation on p. 285, the horizontal thrusts and bending moments due to changes of temperature are as follows. (See Fig. 141 below.)

Horizontal Thrusts:

$$H_t = \pm 3\alpha Et \frac{I}{h^2} \frac{\left(1 + 2\frac{Ih}{I_h l}\right)}{\frac{Ih}{I_h l}\left(2 + \frac{Ih}{I_h l}\right)} \tag{54}$$

Bending Moments:

$$M_A = M_D = -H_t y_t \tag{55}$$

$$M_B = M_C = H_t(h - y_t) \tag{56}$$

where

$$y_t = \frac{1 + \frac{Ih}{I_h l}}{1 + 2\frac{Ih}{I_h l}} h \tag{57}$$

For rise of temperature the thrust H_t is negative, and for the fall of temperature it is positive. In formulas (55) and (56), H_t should be used with its sign.

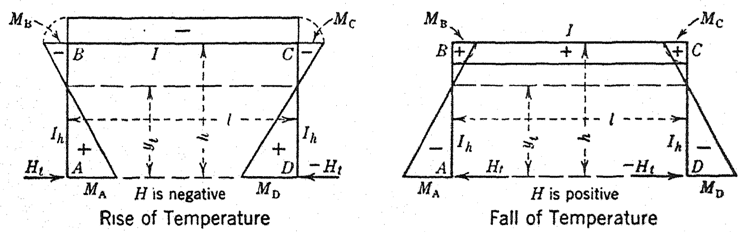


FIG. 141.—Right-angle Frame, Fixed Ends. Effect of Temperature Changes. (*See p.* 316.)

FIXED-POINT METHOD. RIGHT-ANGLE FRAME, FIXED ENDS

In cases where the formulas in the preceding pages do not apply, the fixed-point method may be resorted to. When the frame, the loading, or both are not symmetrical, the results must be corrected as explained under separate headings for vertical and for horizontal loadings.

For explanation of the fixed-point method as applied to rigid frames see Chapter XI, p. 220.

A one-span rigid frame with fixed ends has two fixed points in the horizontal member, f and f'; and two fixed points in each vertical member. These are f_{h1}, f'_{h1}, f_{h2} and f'_{h2}.

For frames with cantilevers it is also necessary to find the ratios of transference for the bending moments from the cantilevers to the horizontal member. These are r'_1 for the left cantilever and r_2 for the right cantilever.

The sequence for finding the fixed points is the same as given for frames with hinged ends on p. 305, except that it is necessary first to compute the lower fixed points in the vertical members from formulas (58) and (59).

Notation:

In addition to notation on p. 305.

f, f' = left and right fixed point in horizontal member.
f_{h1}, f'_{h1} = lower and upper fixed point in left vertical member.
f_{h2}, f'_{h2} = lower and upper fixed point in right vertical member.

Fixed Points in Frame with Fixed Ends.—

Lower Fixed Point in Vertical Member:

$$f_{h1} = \frac{1}{9}\left(2 + \frac{h'_1}{h_1}\right)h_1 \quad (58) \qquad f_{h2} = \frac{1}{9}\left(2 + \frac{h'_2}{h_2}\right)h_2 \quad (59)$$

Left Fixed Point in Horizontal Member: Use formulas (14) and (15), p. 230, for constant moments of inertia; and formulas (16) and (17) for variable moments of inertia.

Upper Fixed Point in Right Vertical Member: Use formula (37), p. 305, for frames with hinged ends.

For frames with cantilevers:

Ratio of Transference from Right Cantilever to Horizontal Member: Use formula (3), p. 228, for constant moments of inertia and formula (7), p. 228, for variable moments of inertia. The ratio is computed by considering the cantilever as the second span of a two-span frame.

The same remarks apply here as given on p. 306 in connection with fixed points for frames with hinged ends.

CORRECTIONS FOR VERTICAL LOADS

For each condition of vertical loading, find the corner bending moments in the frame by means of the fixed points in the usual manner. This determines the uncorrected bending moments M'_A, M'_B, M'_C, and M'_D.

For each loading, find the uncorrected thrusts in the vertical members

$$H'_1 = \frac{1}{h_1 - f_{h1}} M'_B \quad (60) \qquad H'_2 = -\frac{1}{h_2 - f_{h2}} M'_C \quad (61)$$

When the thrusts at both ends are numerically equal to each other,

no corrections are needed. Otherwise, a correcting thrust must be added at each end, and the value of these thrusts may be found from the following formulas:

Correcting Thrusts for Vertical Loadings:

$$H_{c1} = -(H'_1 + H'_2)\frac{h_2^2}{h_1^2 + h_2^2} \quad (62) \qquad H_{c2} = -(H'_1 + H'_2) - H_{c1} \quad (63)$$

The points of application of these thrusts are at distances above the bottom y_{t1} for the left member and y_{t2} for the right member, the values of which may be determined from the following formulas.

Distances of Points of Application of Correcting Thrusts:

$$y_{t1} = \frac{\dfrac{f}{l-f}\dfrac{f_{h1}}{h_1 - f_{h1}} + C\dfrac{f_{h1}}{f'_{h1}}}{\dfrac{f}{l-f}\dfrac{h_1}{h_1 - f_{h1}} + C + C\dfrac{f_{h1}}{f'_{h1}}} h_1 \quad (64)$$

$$y_{t2} = \frac{\dfrac{f'}{l-f'}\dfrac{f_{h2}}{h_2 - f_{h2}} + \dfrac{1}{C}\dfrac{f_{h2}}{f'_{h2}}}{\dfrac{f'}{l-f'}\dfrac{h_2}{h_2 - f_{h2}} + \dfrac{1}{C} + \dfrac{1}{C}\dfrac{f_{h2}}{f'_{h2}}} h_2 \quad (65)$$

where

$$C = \left(\frac{h_2}{h_1}\right)^2 \frac{I_{h1}}{I_{h2}}\frac{f'_{h1}}{f'_{h2}}\frac{h_2 - (f_{h2} + f'_{h2})}{h_1 - (f_{h1} + f'_{h1})} \quad (66)$$

For symmetrical frames with $h_1 = h_2$, the constant becomes $C = 1$. The correcting bending moments are:

Correcting Bending Moments:

In left member: $M_{Ac} = -y_{t1}H_{c1}$ (67) $M_{Bc} = (h_1 - y_{t1})H_{c1}$ (68)

In right member: $M_{Dc} = y_{t2}H_{c2}$ (69) $M_{Cc} = -(h_2 - y_{t2})H_{c2}$ (70)

These bending moments are added to the previously determined bending moments, and the diagram is completed as shown in Fig. 142, p. 319.

This method is exact for frames with both vertical members of the same height, i.e., when $h_1 = h_2$. For unequal heights, in ordinary cases, it gives results accurate enough for practical purposes. In the example in Fig. 142, p. 319, with the quite appreciable difference in height of columns the maximum error amounts only to 3 per cent.

When exact results are desired, the correcting thrusts and bending moments should be found as explained on p. 220 in connection with multi-span frames, and illustrated in the example on p. 258.

CORRECTIONS FOR HORIZONTAL PRESSURES. RIGHT-ANGLE FRAME WITH FIXED ENDS

The method of procedure and formulas are here given for correcting thrusts and bending moments for horizontal pressures. Separate formulas are given for triangular and for rectangular distribution of pressures.

As explained in connection with the previous case, the method here described is exact only for equal heights of vertical members, but is accurate enough for practical purposes for all other conditions.

To find bending moments proceed as follows:

Find all fixed points as explained on p. 317.

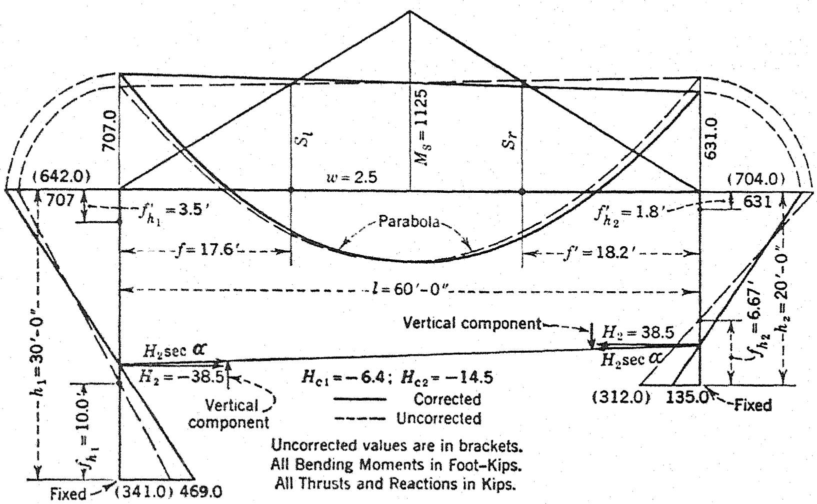


FIG. 142.—Right-angle Frame, Fixed Ends. Fixed-Point Method. *(See p. 318.)*

Find bending moments in the frame using the fixed points in the same manner as explained on p. 231 in connection with multi-span frames. These are the corner bending moments M'_A, M'_B, M'_C and M'_D. Also find uncorrected thrusts in the vertical members, H'_1 and H'_2.

Find the correcting thrusts from formulas (71) and (72), p. 320, for uniformly distributed earth pressure, and from formulas (79) and (80) for triangular distribution. Finally, find the final bending moments and thrusts.

Rectangular Distribution of Earth Pressures. — The pressures are assumed to act from left to right on the left vertical member.

Correcting Thrusts in Terms of Uncorrected Thrusts H'_1 and H'_2:

$$H_{c1} = \left[ph_1 - (H'_1 + H'_2)\right]\frac{h_2^2}{h_1^2 + h_2^2} \tag{71}$$

$$H_{c2} = ph_1 - (H'_1 + H'_2) - H_{c1} \tag{72}$$

Final Bending Moments at Corners:

$$M_A = M'_A - y_{t1}H_{c1} \tag{73}$$
$$M_B = M'_B + (h_1 - y_t)H_{c1} \tag{74}$$
$$M_D = M'_D + y_{t2}H_{c2} \tag{75}$$
$$M_C = M'_C - (h_2 - y_{t2})H_{c2} \tag{76}$$

where y_{t1} and y_{t2} are given in formulas (64) and (65), p. 318.

Final Thrusts:

$$H_1 = H'_1 + H_{c1} \tag{77}$$
$$H_2 = H'_2 + H_{c2} \tag{78}$$

An example of the use of this method is shown in Fig. 143, below.

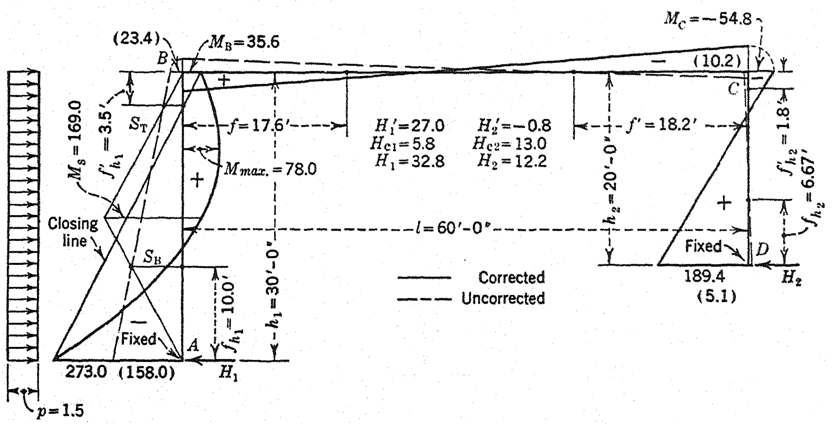


FIG. 143.—Right-angle Frame, Fixed Ends. Fixed-Point Method for Earth Pressures. *(See p. 320.)*

Triangular Distribution of Horizontal Pressures. — As in the previous case, pressures are assumed to act from left to right on the left vertical member. p_1 is the maximum pressure at the bottom.

Correcting Thrusts in Terms of Uncorrected Thrusts H'_1 and H'_2:

$$H_{c1} = \left[\frac{1}{2}p_1h_1 - (H'_1 + H'_2)\right]\frac{h_2^2}{h_1^2 + h_2^2} \tag{79}$$

$$H_{c2} = \frac{1}{2}p_1h_1 - (H'_1 + H'_2) - H_{c2} \tag{80}$$

Final bending moments are found in the same manner as for the rectangular distribution.

DESCRIPTION OF ONE-SPAN RIGID FRAMES

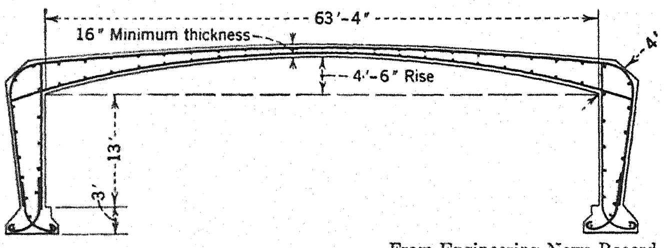
Slab Frames. — The simplest design of rigid frames consists of a solid horizontal slab rigidly connected with vertical walls. Such structures are now used for spans of considerable lengths, particularly in grade separation work for carrying roadways over tracks or tracks over roadways.

The economy of this type lies in the low cost of formwork and in the small depth of construction. The last feature is of particular importance in grade separation, because by reducing the depth of construction the height of the approach fill and its length may be materially reduced.

The solid slab may be of uniform thickness, throughout; the slab at the supports may be increased in thickness by the introduction of haunches; or the bottom of the slab may be either segmental or parabolic. Where the haunches are of appreciable lengths and depths, the frame must be considered in design as having variable moments of inertia. Occasionally, to conform to the grade of the roadway, the top of the frame also may be curved. Where the curvature is small, a frame with horizontal member may be substituted. For more exact work, frames with curved tops may be treated in the same manner as arches.

For frames with variable moments of inertia of the horizontal member the depth of slab in the center of the span may be as small as $\frac{1}{35}$ of the span for hinged frames and $\frac{1}{40}$ of the span for frames fixed at the ends. The maximum depth at the support should be made about $\frac{1}{15}$ of the span.

The depth of the vertical member at the top should be about equal to the depth of the horizontal slab at the support. For frames with hinged ends the thickness of slab at the bottom of the vertical member may be accepted from $\frac{1}{25}$ to $\frac{1}{30}$ of the span. Stresses at all critical sections must be checked.



From Engineering News-Record.

FIG. 144.—Special Design of Solid-Slab Frame. *(See p. 321.)*

Examples of Solid Slab Frames. — The design shown in Fig. 144, p. 321, is one of a number of rigid frames erected by the Westchester Park Commission of New York State. With a span of 63 ft. 4 in., the minimum depth of the slab at the center is only 1 ft. 4 in., giving a ratio of

depth to span of 1 : 47.5. The top of the frame is parabolic, and the rise of the intrados is 4 ft. 6 in. The maximum depth of the slab at the supports is 4 ft.

The Canadian National Railway erected in recent years a number of one-span rigid frames carrying tracks across roadways. One of these, the structure near Vaudreuil, Quebec, has a clear span of 72 ft. and a depth of construction of only 3 ft. 9 in. In this design, no cross ties or ballast were used to support the rails, as these are attached to longitudinal ties imbedded in the concrete.

Rib Frames. — For long spans, the solid slab frames are not economical because the increased cost of materials and the large dead load may more than offset the advantages of the simple formwork. In such cases, the girder or rib frames give a more satisfactory solution of the problem.

The limiting spans for which it is more economical to use rib frames depend upon the relation between the cost of materials and the cost of formwork. Also, the question of the headroom may influence to a large extent the type of design to be adopted. In the United States, where cost of formwork is comparatively high, solid slab frames have been used for spans up to, and even exceeding, 70 ft. In Europe, on the other hand, solid slab frames are used only for short spans.

The design on p. 116 shows a typical arrangement of frames in a rib-frame design. The spacing of the frames is governed by the same economic considerations as the spacing of girders in girder bridges. The floor design may be of one of the three types described on p. 44. Often type 3, using floor beams and slabs reinforced in two directions, gives the most satisfactory results.

The cross sections of the ribs may be constant throughout the span, in which case the frames are considered as having constant moments of inertia of the horizontal member. Short, shallow haunches do not affect the bending moments to any considerable extent. However, when their depths and lengths are large, the frame should be considered as having variable moments of inertia.

In frames with fixed ends, the cross sections of the vertical members are almost always constant. In frames with hinged ends, on the other hand, the cross section of the vertical members are small at the hinges, and increase toward the top. For the purpose of computations, such vertical members are replaced by an imaginary member with constant moments of inertia in the manner explained on p. 273.

For narrow bridges, through rigid frames may be used with the frames placed one at each side of the bridge. Parts of the wing walls of U-abutments may be used for vertical members of the frame.

Vertical Members of Rigid Frames Supporting Abutment Walls. — In one-span frames, walls used to retain the embankment are often connected with the vertical members. The frame then must be designed to resist the bending moments and shears produced in all its members by the earth pressures.

The walls may be placed in front of the ribs so that only a plane vertical surface is exposed to view. Then the wall at the top acts as a compression flange for the vertical ribs, which are there subjected to negative bending moments. (See Fig. 145, below.)

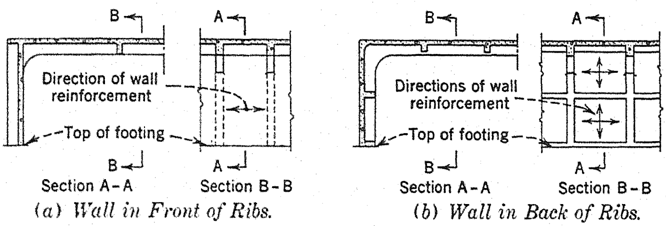


FIG. 145.—Abutment Walls of Rigid Frames. (*See p.* 323.)

The wall also may be placed back of the ribs, so that in the completed structure the ribs are exposed to view. This arrangement makes waterproofing easier because the area of concrete in contact with the soil is a plane surface.

In design, the wall must be considered as a slab spanning between the ribs and resisting the horizontal earth pressures. The thickness of the wall may be uniform, or it may be smallest at the top and increase toward the bottom. For wide spacings of frames, cross beams may be introduced between the frames to support the wall. Also, walls reinforced in two directions and supported along four sides may be used as shown in Fig. 145, above.

Long-Span Rigid Frame. — An example of a long-span rigid frame is furnished by the Victoria Bridge[3] in Bromberg, Germany. The clear span of the frame is 118.4 ft. The structure consists of seven frames, the horizontal members of which form narrow ribs in the center of the span. The width of the ribs increases toward the supports, and near the supports all ribs merge into a solid slab. The vertical members at the top also form a solid wall for part of their height. In the lower part they separate into separate ribs.

Proposed Design for Brigitta Bridge in Vienna. — Of interest is the design proposed for the Brigitta Bridge and shown in Fig. 146, p. 324. Its span is 186.0 ft., and the cantilevers are 37.0 ft. long.

[3] See Deutsche Bauzeitung year 1913. Heft 15 to 18.

The underside of the ribs is curved, and in addition a lower slab is introduced at the supports to resist compression due to the negative

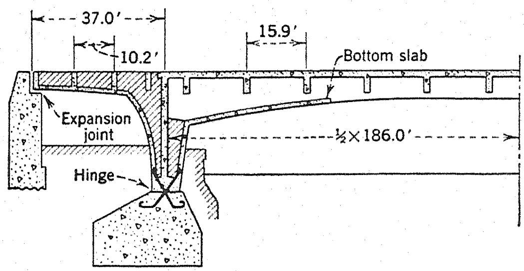
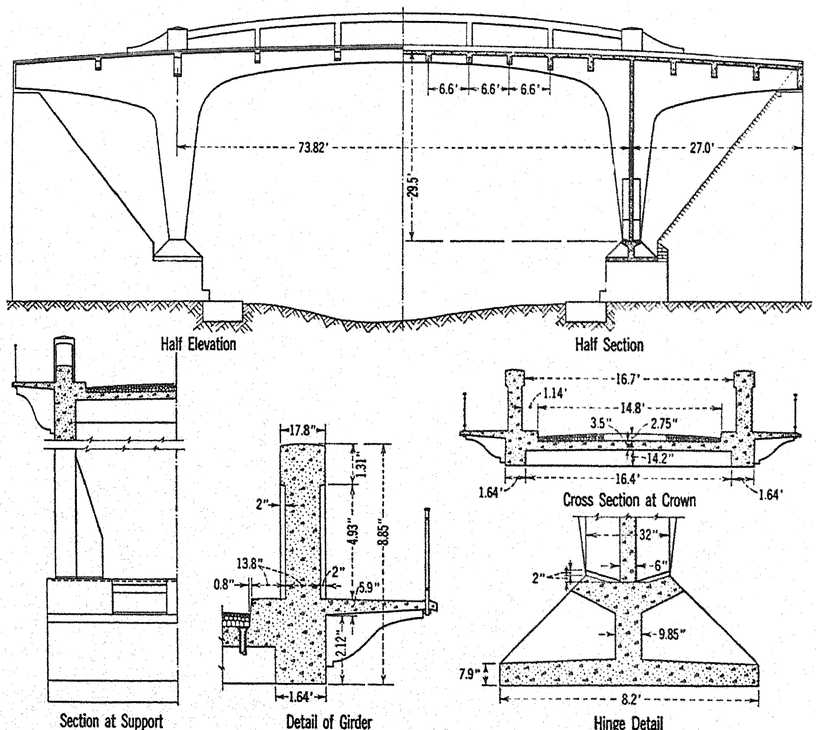


FIG. 146.—Design for Brigitta Bridge in Vienna, by Ast & Co. (See p. 323.)

bending moments. In the cantilever the lower slab extends for the whole length of the cantilever; and as no top slab is there used, the roadway is placed on fill.



Ch. Fatio, Civil Engineer

FIG. 147.—Rigid Frame with Cantilevers across Le Dropt River. (See p. 325.)

Through Bridge of Rigid-Frame Design. — The Valhalla Bridge[4] built by the Westchester, New York, Park Commission is an example of a through rigid-frame design. The ribs of the two frames forming the structure extend above the roadway, and their vertical members form wing walls of the abutments.

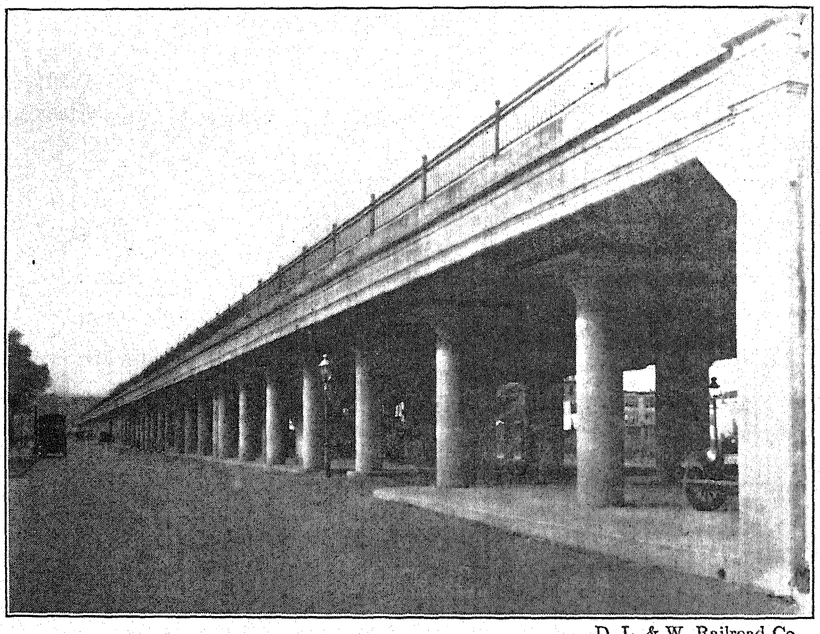
Through Bridge with Cantilevers. — In the design of the bridge across Le Dropt River, shown in Fig. 147, p. 324, the roadway is carried by floor beams extending between two through frames. The sidewalk is cantilevered outside of the through frames.

[4] See *Engineering News-Record*, July 2, 1925, p. 17.

CHAPTER XIV

FLAT-SLAB BRIDGES

In flat-slab bridges, treated in this chapter, the floor construction consists of a reinforced-concrete slab extending in four directions and supported directly by isolated reinforced-concrete columns without the aid of beams and girders. An illustration of a bridge of this type as used for a railroad viaduct is shown in Fig. 148, below.



D. L. & W. Railroad Co.

FIG. 148.—Flat-Slab Viaduct at Brick Church, N. J. (*See p.* 326.)

For building construction, floors of flat-slab type are widely used and are very economical, particularly for structures designed for heavier live loads. In bridge construction, on the other hand, flat-slab floors have not been used to as great an extent as their merits would justify. One reason for this is that details of flat-slab design, such as the proper arrangement of columns, have not up to this time been fully developed. Another reason is that the methods of flat-slab design were developed

for building construction and are not directly applicable to flat-slab bridges which not only are likely to need unusual spacings of columns but also must be designed for moving live loads.

The purpose of the present chapter is to supply all the missing information for safe design of flat-slab bridges. Many formulas here given will also be found useful in types of building construction in which, because of unusual arrangement of columns, ordinary flat-slab formulas do not apply. The practical use of flat slabs in bridge design is here discussed and the methods of design are illustrated by a numerical example. Several illustrations of existing structures are here shown.

Advantages of Flat-Slab Design for Bridges. — Flat-slab design is suited both to highway and railway bridges. For multi-span structures of spans up to 40 ft., when the arrangement is such as to permit the use of flat-slab designs, this type offers the most economical solution of the problem. In all cases, the cost of formwork as well as that of vertical supports are appreciably lower than for any other type; in many cases, even the cost of materials required for the flat-slab structure is lower. Flat slabs are built monolithic with the supporting columns; therefore, no expensive expansion bearings are needed. Since the superstructure is of uniform cross section and is reinforced in two directions, it is able to resist admirably the effects of temperature changes and shrinkage, so that for bridges of moderate lengths no intermediate expansion joints are needed. Of great importance also is the fact that a flat-slab design is likely to require the minimum thickness of construction, which property makes it available where the headroom is limited. Often, by using a properly designed flat-slab construction, the cost of the bridge may be reduced by as much as 25 to 30 per cent of the cost of the concrete structure.

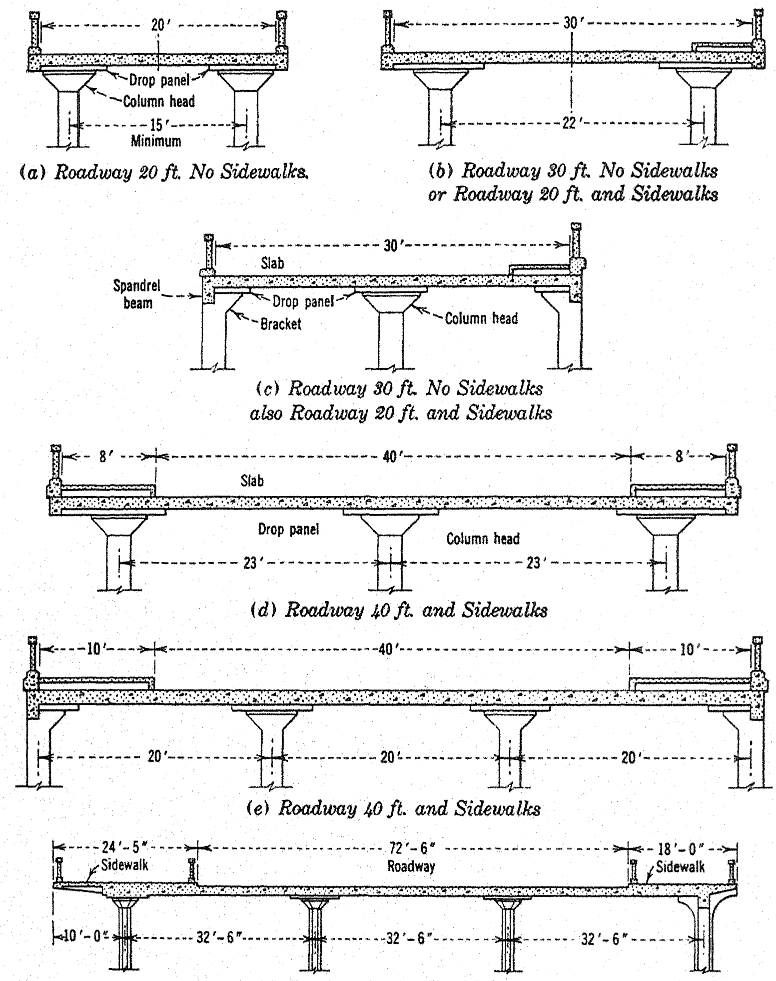
Arrangement of Columns in Flat-Slab Bridges. — In transverse direction, the spacing of columns of flat-slab bridges is governed usually by other requirements than the spacing in the longitudinal direction. For narrow bridges, the choice in lateral arrangement of columns is very limited. For a two-lane roadway, the spacing shown in Fig. 149, p. 328, offers the best solution of the problem.

For a three-lane roadway, two arrangements of columns are available as shown in Fig. 149 (*b*) and (*c*). For wider roadways, the choice of the lateral spacing of columns becomes larger; and finally for structures of considerable width the transverse spacing may depend mainly upon economy.

Examples of lateral spacings of columns in flat-slab railroad bridges are shown in Fig. 150, p. 329.

Longitudinal spacing of columns may be fixed by the topography

or by the requirements as to clear spans; as, for example, in crossings over railroad tracks or over streets where the location of columns is definitely fixed. (See Fig. 151, p. 329.) In structures of appreciable lengths, however, longitudinal spacing of columns may be governed entirely by considerations of economy.



(*a*) *Roadway 20 ft. No Sidewalks.*

(*b*) *Roadway 30 ft. No Sidewalks or Roadway 20 ft. and Sidewalks*

(*c*) *Roadway 30 ft. No Sidewalks also Roadway 20 ft. and Sidewalks*

(*d*) *Roadway 40 ft. and Sidewalks*

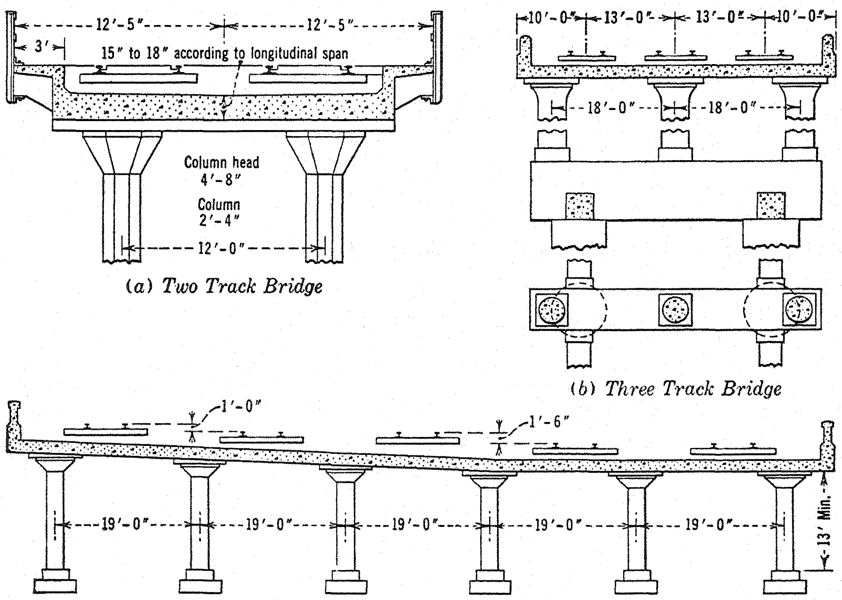
(*e*) *Roadway 40 ft. and Sidewalks*

(*f*) *Roadway 72′–6″ and Sidewalks*

FIG. 149.—Typical Cross Sections of Flat-Slab Bridges. *(See p. 327.)*

Elements of Flat-Slab Bridge. — A flat-slab bridge consists of several, or all, of the following elements: (1) Continuous slab. (2) Drop panels at columns. (3) Column heads. (4) Columns. (5) Spandrel beams. (6) Footings for columns.

Slab. — The design unit of a slab in a flat-slab structure is the panel, i.e., the portion of the continuous slab bordered on four sides by lines connecting the centers of four supporting columns. Usually



(*a*) *Two Track Bridge*

(*b*) *Three Track Bridge*

(*c*) *Five Track Shear Bridge*

FIG. 150.—Cross Sections of Flat-Slab Railroad Bridges. (*See p.* 327.)

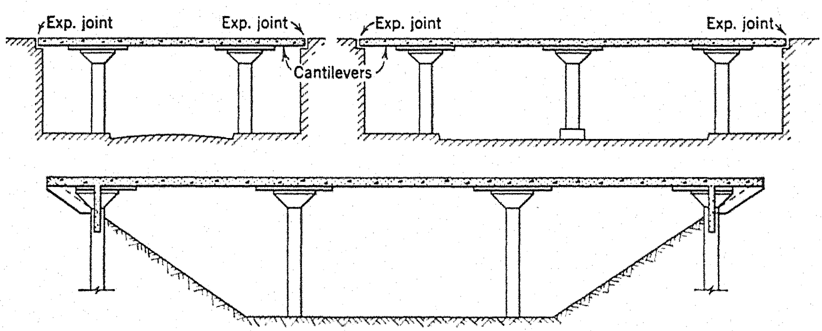


FIG. 151.—Longitudinal Sections of Flat-Slab Bridges. (*See p.* 328.)

in right-angle crossings the panels are rectangular or square; in skew structures they may be rhomboidal or rhombic. In odd parts of the structure odd-shaped panels may be necessary.

The design is most economical when the panels are square or nearly square. The thickness of the slab is usually constant throughout the structure. Occasionally, however, it may be economical to use a thicker slab in small sections of the structure where the spans are larger than the prevailing spans, rather than to base the thickness for the whole structure on these larger spans. The thickness of the slab to be used in the final design should be determined from formulas (4) and (5), p. 349, which are based upon bending moments in slabs. For preliminary estimates, and for finding the preliminary weight of slab, the following formula may be used for thickness of a flat slab extending over several panels.

Let

t = thickness of flat slab in inches.
l = largest span in feet.
w = sum of dead load and live load per square foot.

Then for a flat slab with drop panels at the columns
Thickness of Slab:

$$t = 0.02l\sqrt{w} + 1\tfrac{1}{2} \tag{1}$$

The unit load w consists of the weight of the slab and the pavement and of the equivalent unit live load, including impact, determined as explained on pp. 13 and 27.

Drop Panels at Columns. — A drop panel is a thickening of the slab at the column, used in most flat-slab bridges for reasons of economy. When necessary, drop panels may be omitted, but then the slab must be properly strengthened.

In square panels, the drop panels are square; in rectangular panels they may be either square or rectangular. The total length or width of the drop panel is usually about 0.375 of the span of the panel in that direction.

The object of the drop panels is to supply additional thickness of slab at the columns, because there the intensities of bending moments are much greater than in the center of the slab, and also greater thickness of slab may be required to take care of diagonal tension. The total thickness of construction at the column (i.e., slab plus drop panel) is governed by negative bending moments in the column strips (see p. 349) and by shearing stresses. It is considered to be good practice to limit the thickness of the drop panel to one-half of the thickness of the slab, but this should not be accepted as an inflexible rule.

Column Heads. — A column head is the flaring out at the top of the column in the shape of a truncated cone or pyramid. To be considered

as effective in strengthening the slab, the angle with the vertical of the flaring should in no place be more than 45°. For round columns, the column heads are always circular cones; for square columns they may be pyramids with square or, preferably, octagonal cross sections and sometimes cones. In end panels supported by rectangular columns rectangular column brackets are used instead of column heads.

The effective diameter of column heads to be used in computations is measured where the thickness at the edge, below the slab or the drop panel, is at least $1\frac{1}{2}$ in. Usually, the diameter varies from 0.2 to 0.25 of the largest span of the panels supported by the column. (See Fig. 157, p. 348.)

The function of the column heads is to increase the shearing resistance of the slab at the columns and, also, to reduce the effective span of the slab. The first function is self-evident. The second function is evident from formulas for bending moments in which the theoretical span is equal to $l\left(1 - \frac{2}{3}\frac{c}{l}\right)$, l being the span from center to center of the columns, and c the effective diameter of round column heads.

Columns. — Interior columns may be either round, square, or octagonal in cross section. In locations where metal forms are easily obtainable, round columns with round column heads will be found most economical.

Exterior columns in arrangements such as shown in Fig. 153, p. 337, usually have either square or rectangular cross sections.

The strength of a flat-slab construction depends upon the rigidity of the columns, and particularly of the exterior columns. In effect, a flat-slab structure is a rigid frame where the slab takes the place of the horizontal members, and the columns of the vertical members. At the exterior columns the total negative bending moment in the slab may be transferred to the column; therefore, the connection between the slab and the column should be rigid. The required rigidity and strength of the columns are determined as explained on p. 352.

Spandrel Beams. — In the design shown in Fig. 149 (*c*) and (*e*), p. 328, the outside edges of the flat slab are provided with spandrel beams, the object of which is to carry the railing and to strengthen the exterior panel of the slab. The load carried by the spandrel beams should be assumed to consist of the load directly coming upon them, and of 20 per cent of the total load on the exterior panel. The spandrels should be treated in design as continuous beams.

The beams at the edges of the slab with cantilevers shown in Fig. 149 (*a*), (*b*), and (*d*), p. 328, are not spandrel beams in the sense here used, and they carry only the loads directly resting upon them.

All beams along the edges of a flat-slab structure should be provided with temperature reinforcement consisting of continuous top and bottom bars.

STEPS IN DESIGNING FLAT-SLAB BRIDGES

In designing a flat-slab bridge, proceed as follows.

1. Select spacing of columns. The slab panels should be made as nearly square as possible. To equalize bending moments in the slab, discontinuous spans of the outside panels should be made smaller than the corresponding spans of the interior panels.

2. Decide whether to use drop panels. Select the diameter of the column head, which fixes the theoretical span.

3. Find preliminary unit dead load (see p. 330) and the equivalent uniformly distributed live load, including impact. (See pp. 13 and 27.)

4. Find bending moments in slab at the critical sections in all panels in the longitudinal and the lateral directions, separately for dead load and for live load, using appropriate coefficients from Table I or II, pp. 340 and 342. Combine the bending moments for live load and for dead load. For the largest positive bending moment in the slab, find the thickness of the slab using formula (5), p. 349; and for the largest negative bending moment, find the thickness required at the column, thereby determining the thickness of the drop. Compare the dead load based on the computed slab thickness with the assumed dead load, and, when the difference warrants it, make proper adjustments in bending moments.

5. Check shearing stresses at the edge of the column head and of the drop panel as explained on p. 349.

6. After all concrete dimensions are definitely settled, find at all critical sections the areas of steel required by bending moments. Add to these any additional steel required to take care of temperature changes. (See p. 355.) Select the system of reinforcement to be used, as given on p. 350. Decide upon the manner of bending of reinforcement in the bands. Determine the number and size of bars in each band for the positive bending moments. See whether the bent bars in the panel together with the bars carried across from the adjacent panel are sufficient to take care of the negative bending moments; if not, supply additional top reinforcement. See whether reinforcement extends far enough to take care of bending moments at the intermediate points, and, where anchorage is required, whether the bars are properly anchored to be effective.

7. Design spandrel beams, when any are used.

8. Determine the dimensions and the reinforcement of the support-

ing columns. The columns, particularly the exterior ones, must have proper rigidity, and they should be designed for direct stresses and bending moments.

9. Design footings for columns.

10. Design abutments.

Under 4 it has been assumed that bending moments in the slab are determined by formulas given in the tables on pp. 340 and 342. Where these formulas are not applicable, proceed as explained on p. 356.

BENDING MOMENTS IN FLAT SLABS

The problem of determining bending moments that are really exact in a flat slab structure is very involved. Numerous theoretical formulas have been evolved; but they are usually not only too complicated for practical use, but also the results obtained from them do not agree with the results from tests of flat-slab structures, nor with the performance of such structures in actual use.

For use in practice, therefore, comparatively simple semi-theoretical formulas have been evolved which give economical structures, amply strong to carry the design loads with more than the required factor of safety. These formulas are given in this chapter but with such modifications developed by the authors as are required to take care of the moving live loads and also to take care of the arrangements of columns not usually encountered in building construction.

Formulas for bending moments are given in Tables I and II, pp. 340 and 342, for one-span structures, two-span structures, and multi-span structures. For the last two types, formulas are based on the assumption of equal spans in all panels in the direction under consideration. However, it is accurate enough to use them also when the end spans are somewhat shorter than the interior spans, but are not less than 0.9 of the interior span length.

Where span lengths are unequal, the use of the rigid-frame method adapted by the authors to flat slab structures is recommended as explained on p. 356.

Assumptions. — The formulas are based on the following assumptions:

1. In a square or rectangular panel, there exist two sets of bending moments acting at right angles to each other. Therefore, to resist these bending moments two sets of bars placed at right angles to each other are required.

2. For continuous slabs and slabs restrained at the columns, vertical loads produce maximum negative bending moments at sections at the

supports and maximum positive bending moments at sections running through the slab center.

3. The theoretical span of the panel in each of the two directions, to be used in computing bending moments, is equal to the distance between the column centers in that direction reduced at each end by the distance from the center of the column to the centroid of one-half the column head or of the column bracket. For round column heads this distance equals $\frac{1}{3}c$, i.e., one-third of the diameter of the column head; and for rectangular column heads it is $\frac{3}{8}c$.

4. In a flat slab, either continuous or restrained at the columns, similarly as in other continuous structures, for any condition of loading in a uniformly loaded panel, the sum of the positive bending moment in the center of the panel and of the average of the negative bending moments at the two supports is equal to the maximum static bending moment, i.e., the bending moment in the center of the span when the slab is considered as simply supported at the ends. It is therefore possible to express the bending moments at all sections of the slab in terms of this static bending moment.

However, on account of the so-called "flat-slab action,"[1] the static bending moment of the loads is multiplied by the ratio $\frac{0.09}{0.125} = 0.72$ to get the modified static bending moment to be used in flat-slab design for determining tensile stresses in reinforcement or the amount of needed tensile reinforcement. This ratio is the result of investigations extending over a great number of years by American engineers engaged in the design of flat-slab structures. Thousands of structures erected and now in successful use have been designed upon this basis. When computing the thickness of slab as governed by compression stresses, however, formulas given on p. 349 should be used.

Modified Static Bending Moment to be Used in Flat-Slab Design:

Interior panels

$$M = 0.09Wl\left(1 - \frac{2}{3}\frac{c}{l}\right)^2 \text{ ft-lb.} \tag{2}$$

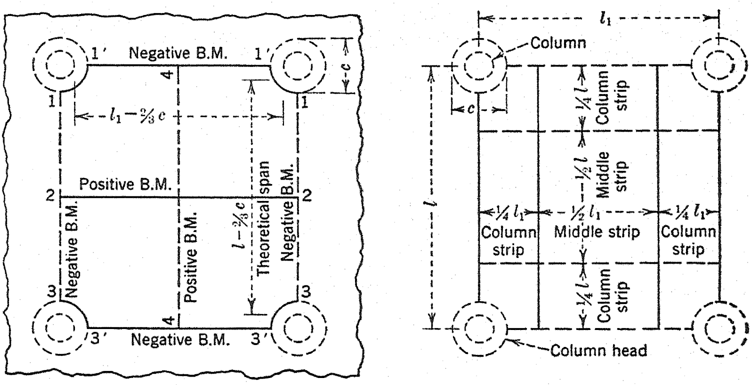
Exterior panels

$$M_e = 0.09W_e l_e\left[1 - \frac{1}{3}\left(\frac{c}{l_e} + \frac{c_1}{l_e}\right)\right]^2 \text{ ft-lb.} \tag{3}$$

[1] By flat-slab action is meant the effect of the following factors, all of which reduce the tensile stresses to be resisted by the reinforcement but increase compression in concrete. (1) The effect of the Poisson's ratio, i.e., in a slab subjected to bending in two directions at right angles to each other, the effect of deformation of a particle in one direction upon the stresses in the direction at right angles to it; (2) the arch action; and (3) beneficial effect of tensile resistance of concrete.

where $W = wll_1$ and $W_e = wl_el_1$ are total loads on panel; l and l_e are spans of panel in feet; l_1 is width of panel; c is diameter of interior round column head; c_1 is diameter of exterior round column head. For square brackets at the end columns replace $\frac{1}{3}\frac{c_1}{l_e}$ by $\frac{3}{4}\frac{b}{l}$, where b is length of bracket measured from the center of exterior column.

Critical Sections. — Critical sections for bending moments in an interior panel of a flat slab are shown in Fig. 152, p. 335. There are two sets of critical sections, one set in each direction. In rectangular and square panels the sections in one set are at right angles to the sections in the other set.



Sections of Critical Bending Moments

Design Strips

FIG. 152.—Critical Sections and Design Strips. (*See p.* 335.)

Design Strips. — For design purposes, the panel is divided into parallel strips as shown in Fig. 152, p. 335. These are as follows:

(*a*) Column strips, two per panel, one at each side of the panel, and each equal in width to one-quarter of the width of the panel.

(*b*) Middle strip, equal in width to one-half of the width of the panel.

The bending moment acting at any critical section of a panel is divided into two unequal parts, as indicated in tables on pp. 340 to 342. The larger part is assumed to be uniformly distributed over the two column strips, and the smaller part over the middle strip. The reinforcing bars used to resist these bending moments are distributed over the critical sections in the same manner.

Rhombic and Rhomboidal Panels. — When the shape of the panels is rhombic or rhomboidal, the flat slab may be treated in the same manner as for rectangular panels, but with the following modifications. The

critical sections at the columns should coincide with the outlines of the panels and with the two quadrants of the column heads. The critical sections in the center of the panels are parallel to the sides of the panels.

The span lengths should be measured along the sides of the panel and not at right angles to the critical sections.

The reinforcement for any critical section should be placed parallel to the other side of the panel, and not at right angles to the section. Although the reinforcing bars intersect the critical sections at an angle, each should be considered as fully effective at that section notwithstanding the rule on p. 350 regarding the effectiveness of bars inclined to the critical section.

Formulas in Tables I and II, pp. 340 and 342, may be used here. In the formula for the modified static bending moment, W is then equal to the total dead and live load on the panel, and l or l_c is the length of the side of the panel in the direction under consideration.

FORMULAS FOR FLAT SLABS OF EQUAL SPANS

Formulas for bending moments at critical sections of flat slabs of equal spans are given in Tables I and II, pp. 340 and 342, separately for dead load and for uniformly distributed live load. These formulas may also be used where the end spans are shorter than the interior spans, but are not less than 0.9 of the interior span length. For concentrated wheel loads, it is necessary to find the equivalent uniformly distributed loading as explained on p. 27.

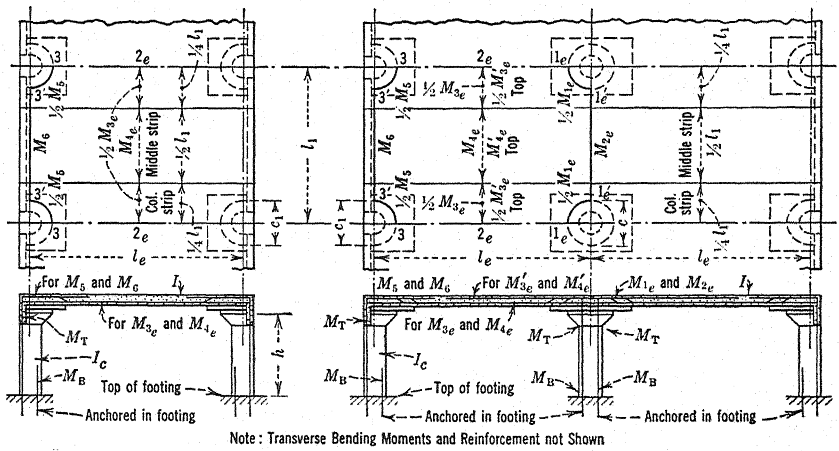
Bending moments in the tables are expressed in terms of the modified static bending moments. The formulas have been worked out by the authors by considering the flat-slab structures as one-story rigid frames, adjusting the results so as to take care of the peculiar conditions in a flat-slab frame in the manner outlined on p. 334.

Table I on p. 340 gives bending moments for flat slabs of one and two spans, respectively. Critical sections, design strips, and bending moments in the tables are indicated in Fig. 153, p. 337. The longitudinal section of the figure shows the arrangement of reinforcement to resist bending moments.

Table II gives bending moments in exterior and interior panels of flat slabs consisting of three or more spans in the direction under consideration. Critical sections, design strips, and bending moments are shown in Fig. 154, p. 337.

It should be noted that in Figs. 153 and 154 the bending moments and the reinforcement in one direction only are shown. Similar bending

moments and reinforcement should be found for the other direction, and these should be based on the span lengths and the arrangement of spans in that direction.



(*a*) *One Span Frame* (*b*) *Two Span Frame*

FIG. 153.—Bending Moments in One- and Two-Span Flat-Slab Frame. (*See p.* 336.)

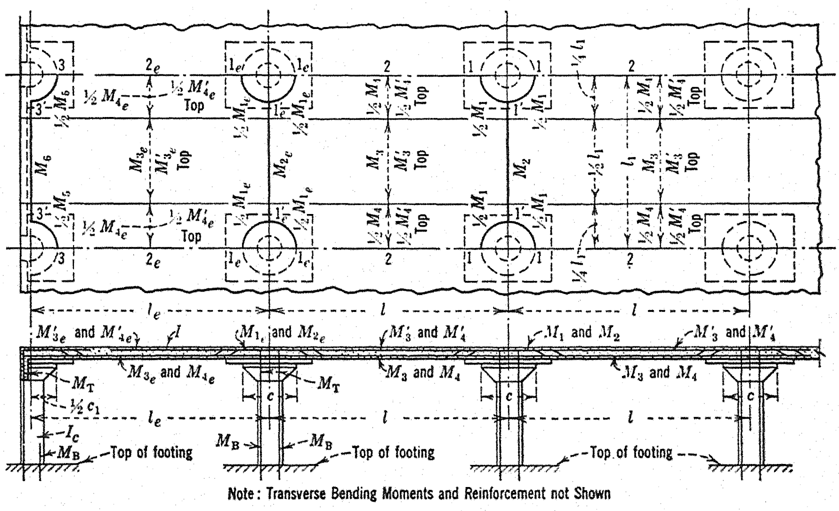


FIG. 154.—Bending Moments in Multi-Span Flat-Slab Frame. (*See p.* 336.)

Assumptions as to Rigidity of Columns. — Formulas in Tables I and II apply only when the flat slab is rigidly connected with columns fixed at the bottom to the foundations. The bending moments in slabs are

mainly affected by the relative rigidity of the exterior columns. The rigidity of the interior columns is of less influence. The following relations of the rigidity of the exterior columns to the rigidity of the slabs have been used in working out the formulas.

In the rigidity ratios, let:

I_h = moment of inertia of exterior concrete cross section of column.

h = theoretical height of exterior column, i.e., distance from top of footing to neutral axis of slab.

I = moment of inertia of concrete cross section of slab tributary to the column, at right angles to the span, found as explained on p. 352.

l = span of slab in the direction under consideration.

For Table I, three conditions are considered: (1) $\frac{I_h}{h} = 0.5\frac{I}{l}$; (2) $\frac{I_h}{h} = \frac{I}{l}$; and (3) $\frac{I_h}{h} = 2\frac{I}{l}$.

For Table II, two conditions are used: $\frac{I_h}{h} = \frac{I}{l}$ and $\frac{I_h}{h} = 2\frac{I}{l}$.

The relation of the rigidity ratios of the interior columns to those of the slab is $\frac{I_h}{h} = \frac{I}{l}$; except that where for exterior columns the ratio $\frac{I_h}{h} = 0.5\frac{I}{l}$ is used, the same ratio is used also for the interior columns.

How to Use Formulas in Tables I and II. — After the arrangement of columns in a flat slab is decided upon, and the spans of the panels are equal or nearly equal, bending moments are found by using formulas from either Table I or II. Separate sets of bending moments are computed in each of the two directions.

For example, in a flat-slab bridge consisting of five equal spans in the longitudinal direction and of two equal spans in the lateral direction, bending moments are found as follows. In the longitudinal direction, formulas are taken from Table II. The end panels are considered as exterior panels and the remaining three panels as interior panels. The following critical sections need to be considered: in the exterior panel, the critical section at the end columns, the critical section at the first interior row of columns, and the section through the center of the end panel; for all of which formulas grouped under "Exterior Panel" should be used. In the interior panel, consider the section at the interior row of columns and the section through the center of interior panel, using

formulas grouped under "Interior Panel." In all, it is necessary to find bending moments at five critical sections to determine the bending moments in the longitudinal direction.

In the lateral direction, this flat slab should be considered as a slab of two equal spans, and the bending moments should be found by means of formulas from Table I grouped under the heading "Two-Span Frame." Two critical sections for negative bending moments and one for positive bending moments need to be considered.

To use the tables, find the unit dead load and the equivalent unit live load, including impact. Compute the modified static bending moments from formulas (2) and (3), p. 334, separately for dead load and for live load. Using the appropriate formulas from the proper table, find bending moments at all critical sections, and for all strips. Add the bending moments for dead load to the appropriate bending moments for live load. Use the sums to determine the dimensions of the slab and the amounts of reinforcement as explained on pp. 348 and 350.

Allowable Variations in Distribution of Bending Moments. — The distribution of bending moments between the various critical sections, and between the various design strips, used in Tables I and II, may be varied to the following extent:

The positive bending moments in the two column strips may be decreased by $0.02M$ (or $0.02M_e$), if simultaneously the bending moments in the corresponding middle strip are increased by the same amount.

The negative bending moments in the column strips may be decreased or increased by amounts not exceeding $0.03M$ (or $0.03M_e$), if at the same time the positive bending moments are properly adjusted.

Bending Moments at Intermediate Points and Points of Inflection. — Bending moments at intermediate points of a flat slab are shown in the bending-moment diagrams in Fig. 155, p. 344. They are based upon the same assumptions as the formulas in Tables I and II. Separate curves are given for dead load and for live load. To get the combined bending moments, the curves for the dead load should be combined with the curves for live load, as explained on p. 168. The lengths of bars and the points of bending of reinforcement are governed by the combined bending-moment diagram and by the positions of the points of zero moments or points of inflection, as they are often called.

In bridge design, where live loads must be considered as moving loads, the arrangement of reinforcement is different from that used in building construction for the same spans and unit loadings. Here, the negative bending-moment reinforcement must extend farther from the columns into the span, and the positive bending moments must be carried

TABLE I

One-Span and Two-Span Flat-Slab Frames

Columns Fixed at Bottom (See p. 337)

Bending Moments in Design Strips

$$M_e = 0.09Wl_e\left[1 - \frac{1}{3}\left(\frac{c_1}{l_e} + \frac{c}{l_e}\right)\right]^2 \text{ ft-lb.} = 1.08Wl_e\left[1 - \frac{1}{3}\left(\frac{c_1}{l_e} + \frac{c}{l_e}\right)\right]^2 \text{ in-lb.}$$

$W = w_d l_1 l_e$ lb. for dead load, and $W = w_l l_1 l_e$ lb. for live load. l_1, l_e, c, and c_1 are in feet.

I_h and h are for exterior columns; I and l are for exterior panels.

Position of Critical Section	Critical Section	Strips	Notation	$\frac{I_h}{h} = 0.5\frac{I}{l}$		$\frac{I_h}{h} = \frac{I}{l}$		$\frac{I_h}{h} = 2\frac{I}{l}$	
				Dead Load w_d	Live Load w_l	Dead Load w_d	Live Load w_l	Dead Load w_d	Live Load w_l
One-Span Frame (See Fig. 155 (*a*), p. 344.)									
Slab									
Column center line	$3-3'-3'-3$	Two col. strips	M_5	$-0.28M_e$	$-0.28M_e$	$-0.37M_e$	$-0.37M_e$	$-0.45M_e$	$-0.45M_e$
		Middle strip	M_6	$-0.05M_e$	$-0.05M_e$	$-0.05M_e$	$-0.05M_e$	$-0.06M_e$	$-0.06M_e$
Panel center line	$2_e - 2_e$	Two col. strips	M_{3e}	$0.46M_e$	$0.46M_e$	$0.38M_e$	$0.38M_e$	$0.32M_e$	$0.32M_e$
		Middle strip	M_{4e}	$0.31M_e$	$0.31M_e$	$0.26M_e$	$0.26M_e$	$0.22M_e$	$0.22M_e$
Column									
Top (at bottom of column head)			M_T	$-0.33M_e$	$-0.33M_e$	$-0.42M_e$	$-0.42M_e$	$-0.51M_e$	$-0.51M_e$
Bottom			M_B	See note		See note		See note	

TABLE I (*Continued*)

Position of Critical Section	Critical Section	Strips	Notation	$\frac{I_h}{h} = 0.5\frac{I}{l}$ Dead Load w_d	$\frac{I_h}{h} = 0.5\frac{I}{l}$ Live Load w_l	$\frac{I_h}{h} = \frac{I}{l}$ Dead Load w_d	$\frac{I_h}{h} = \frac{I}{l}$ Live Load w_l	$\frac{I_h}{h} = 2\frac{I}{l}$ Dead Load w_d	$\frac{I_h}{h} = 2\frac{I}{l}$ Live Load w_l
Two-Span Frame (See Fig. **155** (*b*), p. **344.**)									
Slab									
Interior col. center line	$1_e-1'_e-1'_e-1_e$	Two col. strips	M_{1e}	$-0.74M_e$	$-0.74M_e$	$-0.70M_e$	$-0.70M_e$	$-0.64M_e$	$-0.64M_e$
		Middle strip	M_{2e}	$-0.13M_e$	$-0.13M_e$	$-0.12M_e$	$-0.12M_e$	$-0.11M_e$	$-0.11M_e$
Exterior col. center line	$3-3'-3'-3$	Two col. strips	M_5	$-0.19M_e$	$-0.22M_e$	$-0.28M_e$	$-0.31M_e$	$-0.38M_e$	$-0.40M_e$
		Middle strip	M_6	$-0.03M_e$	$-0.03M_e$	$-0.04M_e$	$-0.05M_e$	$-0.06M_e$	$-0.06M_e$
Panel center line	$2_e - 2_e$ (Positive)	Two col. strips	M_{3e}	$0.33M_e$	$0.42M_e$	$0.31M_e$	$0.36M_e$	$0.27M_e$	$0.34M_e$
		Middle strip	M_{4e}	$0.22M_e$	$0.28M_e$	$0.20M_e$	$0.24M_e$	$0.17M_e$	$0.23M_e$
	$2_e - 2_e$ (Negative)	Two col. strips	M'_{3e}		$-0.12M_e$		$-0.09M_e$		$-0.07M_e$
		Middle strip	M'_{4e}		$-0.03M_e$		$-0.02M_e$		$-0.01M_e$
End Column									
Top (at bottom of column head)			M_T	$-0.22M_e$	$-0.25M_e$	$-0.32M_e$	$-0.36M_e$	$-0.44M_e$	$-0.46M_e$
Bottom			M_B	See note		See note		See note	
Center Column									
Top (at bottom of column head)			M_T		$\pm 0.21M_e$		$\pm 0.29M_e$		$\pm 0.31M_e$
Bottom			M_B	See note		See note		See note	

NOTE: Bending moment at bottom of column is obtained by multiplying the bending moment M_T by the ratio $-\frac{h}{2h - 3a}$. (See Table II.)

TABLE II

FLAT-SLAB FRAMES. THREE OR MORE EQUAL SPANS

Columns Fixed at Bottom (See p. 337)

Bending Moments in Design Strips

$$M = 0.09Wl\left(1 - \frac{2}{3}\frac{c}{l}\right)^2 \text{ ft-lb.} = 1.08Wl\left(1 - \frac{2}{3}\frac{c}{l}\right)^2 \text{ in-lb.}$$

$$M_e = 0.09Wl_e\left[1 - \frac{1}{3}\left(\frac{c_1}{l_e} + \frac{c}{l_e}\right)\right]^2 \text{ ft-lb.} = 1.08Wl_e\left[1 - \frac{1}{3}\left(\frac{c_1}{l_e} + \frac{c}{l_e}\right)\right]^2 \text{ in-lb.}$$

$W = wl_1l$ or wl_1l_e lb.; $w = w_d$ or w_l; l, l_1, l_e, c, and c_1 are in feet.

I_h and h are for exterior columns; I and l are for exterior panels.

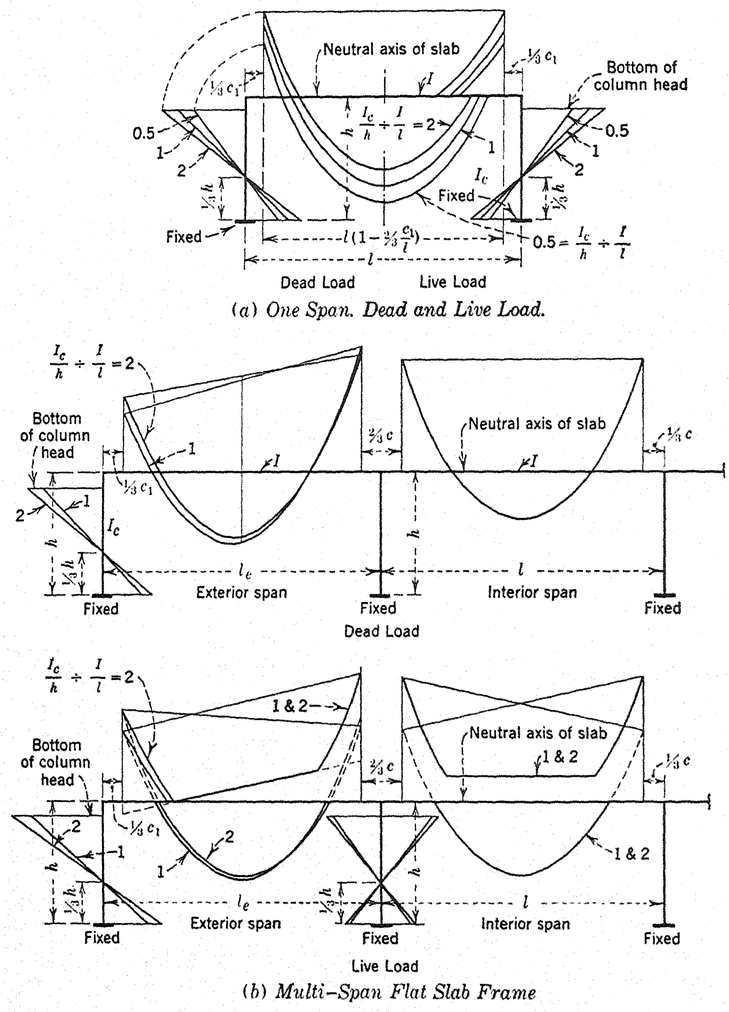
Position of Section	Section	Strips	Notation	$\frac{I_h}{h} = \frac{I}{l}$		$\frac{I_h}{h} = 2\frac{I}{l}$	
				Dead Load $w = w_d$	Live Load $w = w_l$	Dead Load $w = w_d$	Live Load $w = w_l$
Interior Panel (See Fig. 155, p. 344.)							
Slab							
Column center line	$1 - 1' - 1' - 1$	Two col. strips	M_1	$-0.54M$	$-0.64M$	$-0.54M$	$-0.64M$
		Middle strip	M_2	$-0.08M$	$-0.10M$	$-0.08M$	$-0.10M$
Panel center line	$2 - 2$ (Positive)	Two col. strips	M_3	$0.21M$	$0.33M$	$0.21M$	$0.33M$
		Middle strip	M_4	$0.14M$	$0.22M$	$0.14M$	$0.22M$
	$2 - 2$ (Negative)	Two col. strips	M_3'		$-0.14M$		$-0.14M$
		Middle strip	M_4'		$-0.02M$		$-0.02M$
Interior Columns							
Top	(at bottom of column head)		M_T		$\pm0.22M$		$\pm0.22M$
Bottom			M_B		See note		See note

TABLE II (*Continued*)

Position of Section	Section	Strips	Notation	$\frac{I_h}{h} = \frac{I}{l}$		$\frac{I_h}{h} = 2\frac{I}{l}$	
				Dead Load $w = w_d$	Live Load $w = w_l$	Dead Load $w = w_d$	Live Load $w = w_l$
Exterior Panel (See Fig. 155, p. 344.)							
Slab							
Interior col. line	$1_e - 1'_e - 1'_e - 1_e$	Two col. strips	M_{1e}	$-0.64M_e$	$-0.69M_e$	$-0.61M_e$	$-0.68M_e$
		Middle strip	M_{2e}	$-0.11M_e$	$-0.12M_e$	$-0.11M_e$	$-0.12M_e$
Exterior col. line	$3 - 3' - 3' - 3$	Two col. strips	M_5	$-0.30M_e$	$-0.36M_e$	$-0.38M_e$	$-0.46M_e$
		Middle strip	M_6	$-0.04M_e$	$-0.06M_e$	$-0.06M_e$	$-0.07M_e$
Panel center line	$2_e - 2_e$ (Positive)	Two col. strips	M_{3e}	$0.31M_e$	$0.37M_e$	$0.29M_e$	$0.33M_e$
		Middle strip	M_{4e}	$0.20M_e$	$0.24M_e$	$0.19M_e$	$0.22M_e$
	$2_e - 2_e$ (Negative)	Two col. strips	M'_{3e}		$-0.09M_e$		$-0.07M_e$
		Middle strip	M'_{4e}		$-0.02M_e$		$-0.01M_e$
Exterior Columns							
Top	(at bottom of column head)		M_T	$-0.34M_e$	$-0.42M_e$	$-0.47M_e$	$-0.53M_e$
Bottom			M_B	See note		See note	

NOTE: Bending moment at bottom of column is obtained by multiplying the bending moment M_T by the ratio $-\frac{h}{2h - 3a}$, in which h is theoretical height of column and a is distance from bottom of column head to neutral axis of slab.

nearer toward the columns than required in ordinary building construction. In bridge design it is even advisable to use some continuous bars at the top as well as at the bottom. (See example, p. 358.)



(*a*) *One Span. Dead and Live Load.*

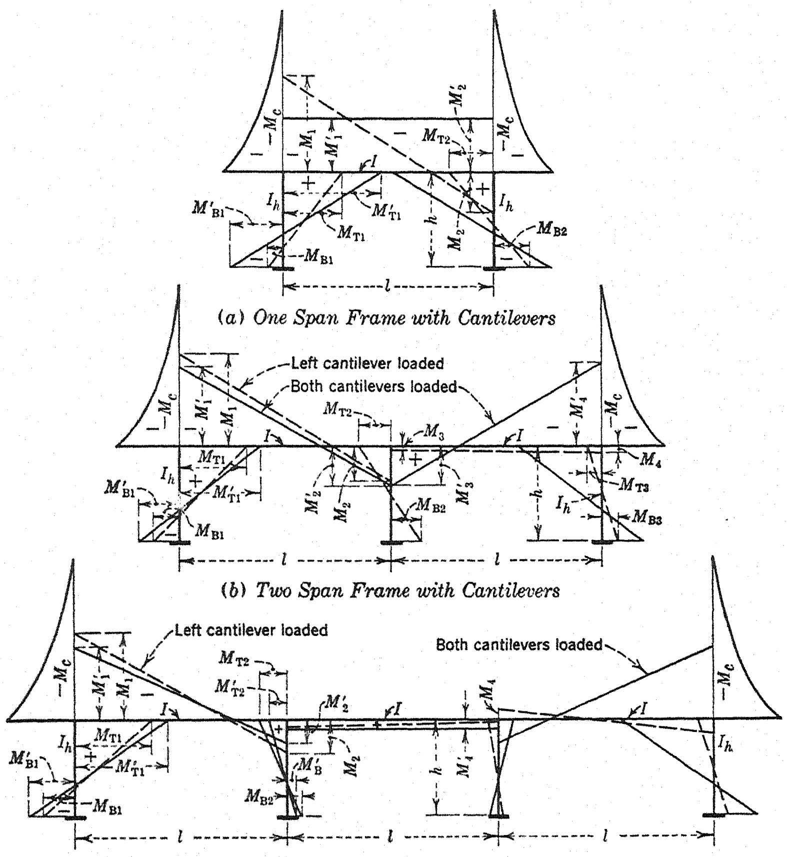
(*b*) *Multi-Span Flat Slab Frame*

FIG. 155.—Bending-Moment Diagrams. Flat-Slab Frames. (*See p.* 339.)

FLAT SLABS WITH CANTILEVERS

Very often, flat-slab bridges are provided with cantilevers in transverse directions as shown in Figs. 149 and 151, pp. 328 and 329. The following advantages are gained by such arrangements: the spacings of

columns are reduced without increasing their number; bending moments in the exterior columns are materially reduced; and the bending moments in the main spans are equalized.



(c) Three Span Frame with Cantilevers

FIG. 156.—Bending Moments in Flat-Slab Frames due to Cantilever Loads. (See p. 346.)

Longitudinal cantilevers also may be useful for the reasons stated on p. 181 for ordinary rigid frames.

Bending Moments Due to Cantilever Loads. — Formulas for bending moments in flat-slab bridges with cantilevers are given in Table III, p. 347, for structures of one, two, and multi-spans. All bending moments are expressed in terms of the maximum bending moment in the cantilever $-M_C$. The bending moments are given for the same ratios of rigidity as

are used in Tables I and II, pp. 340 and 342. They were worked out by considering the structures as rigid frames.

In the table the value M_C is positive therefore the sign given in the table determines the sign of the bending moment.

Bending-moment diagrams for cantilever loadings are shown in Fig. 156, p. 345. Solid lines in all diagrams represent bending moments for a condition when both cantilevers are loaded by symmetrical loads; dash lines show bending moments when only the left cantilever is loaded. For loads on the right cantilever, bending moments may be obtained from symmetry from the diagram for the left cantilever. For dead loads, both cantilevers should be considered as loaded simultaneously; and advantage may then be taken of the beneficial effect of the cantilever bending moments upon the bending moments in the main spans.

In Fig. 156 the bending moments for a condition when the left cantilever is loaded are designated by M_1, M_2, M_T . . .; while for symmetrical loading of cantilevers by M'_1, M'_2, M'_T . . .

For live loads, one, both, or no cantilever should be considered as loaded, depending upon which condition gives the most unfavorable results. In no case should any cantilever bending moments for live load be considered as reducing any of the bending moments due to loads on the main spans. In computing M_C for live loads, concentrated wheel loads, and not equivalent uniformly distributed loading should be used.

Multi-Span Flat Slabs with Cantilevers. — It is evident from the three cases shown in Fig. 156, p. 345, that the effect of the cantilever loads is largest in the end span adjoining the cantilevers, and is comparatively small in the second span. In a multi-span frame the effect of the cantilever loads in the third span from the end would be negligible. Therefore, for a multi-span frame the bending moments in the end spans may be taken as equal to the average of the bending moments in cases (*b*) and (*c*). In all interior spans, the same bending moments may be assumed as in the interior span of case (*c*).

Bending Moments to Be Used in Design. — The maximum bending moments at each support act in the column center or at the intersection of the column center line with the neutral axis of the slab. In design of the slab, not the maximum values should be used but the bending moments scaled from the diagrams at the edge of each column head, and for the columns those scaled at the bottom of the column heads or brackets.

Distribution of the Bending Moments over Design Strips. — The bending moments due to the cantilever loads should be considered as

TABLE III

Flat-Slab Rigid Frame with Cantilevers

Bending Moments in Frame Due to Cantilever Bending Moment $-M_C$

(For bending-moment diagram, see Fig. 156, p. 345)

$-M_C$ = maximum cantilever bending moment at column center line

$\frac{I_h}{h} : \frac{I}{l}$	Cantilever Loaded	Left End Span		Second Span		Left End Column		Second Column	
		Left End M_1	Right End M_2	Left End M_3	Right End M_4	Top M_{T1}	Bottom M_{B1}	Top M_{T2}	Bottom M_{B2}
One-Span Frame									
0.5	Left	$-0.71M_C$	$0.21M_C$			$0.29M_C$	$-0.09M_C$	$0.21M_C$	$-0.16M_C$
	Both	$-0.5M_C$	$-0.5M_C$			$0.5M_C$	$-0.25M_C$	$0.5M_C$	$-0.25M_C$
1.0	Left	$-0.6M_C$	$0.26M_C$			$0.4M_C$	$-0.09M_C$	$0.26M_C$	$-0.24M_C$
	Both	$-0.34M_C$	$-0.34M_C$			$0.66M_C$	$-0.33M_C$	$0.66M_C$	$-0.33M_C$
2.0	Left	$-0.48M_C$	$0.28M_C$			$0.52M_C$	$-0.08M_C$	$0.28M_C$	$0.32M_C$
	Both	$-0.20M_C$	$-0.20M_C$			$0.80M_C$	$-0.40M_C$	$0.80M_C$	$-0.40M_C$
Two-Span Frame									
0.5	Left	$-0.70M_C$	$0.24M_C$	$0.09M_C$	$0.04M_C$	$0.30M_C$	$-0.10M_C$	$-0.15M_C$	$0.13M_C$
	Both	$-0.66M_C$	$0.33M_C$	$0.33M_C$	$-0.66M_C$	$0.34M_C$	$-0.17M_C$	0	0
1.0	Left	$-0.56M_C$	$0.23M_C$	$0.02M_C$	$0.06M_C$	$0.44M_C$	$-0.14M_C$	$-0.23M_C$	$0.18M_C$
	Both	$-0.50M_C$	$0.25M_C$	$0.25M_C$	$-0.50M_C$	$0.50M_C$	$-0.25M_C$	0	0
2.0	Left	$-0.42M_C$	$0.18M_C$	$-0.02M_C$	$0.10M_C$	$0.58M_C$	$-0.12M_C$	$-0.20M_C$	$0.18M_C$
	Both	$-0.33M_C$	$0.16M_C$	$0.16M_C$	$-0.33M_C$	$0.67M_C$	$-0.33M_C$	0	0
Three-Span Frame									
1.0	Left	$-0.53M_C$	$0.22M_C$	$0.05M_C$	$0.01M_C$	$0.47M_C$	$-0.18M_C$	$-0.21M_C$	$0.13M_C$
	Both	$-0.43M_C$	$0.16M_C$	$0.06M_C$	$0.06M_C$	$0.57M_C$	$-0.28M_C$	$-0.10M_C$	$0.05M_C$
2.0	Left	$-0.41M_C$	$0.18M_C$	$0.02M_C$	$0.02M_C$	$0.59M_C$	$-0.18M_C$	$-0.18M_C$	$0.15M_C$
	Both	$-0.31M_C$	$0.11M_C$	$0.03M_C$	$0.03M_C$	$0.69M_C$	$-0.35M_C$	$-0.05M_C$	$0.03M_C$

distributed over the design strips of the slab in the same proportions as are recommended for the bending moments due to the loads on the slab.

Numerical Example of Determination of Bending Moments. — The use of the formulas is clearly illustrated in the numerical example on p. 358.

THICKNESS OF SLAB AND DROP PANEL

The combined bending moments for dead load and live load at all critical sections of the flat-slab structures, and in both directions, having been found, the required thicknesses of slab and drop panels are found from the following formulas. If it is desired to use the same thickness of slab throughout the structure and the same depth for all drop panels, the computed thicknesses should be based upon the largest bending moments in the structure.

The thickness of the slab is based upon the largest positive bending moments in two adjoining column strips.

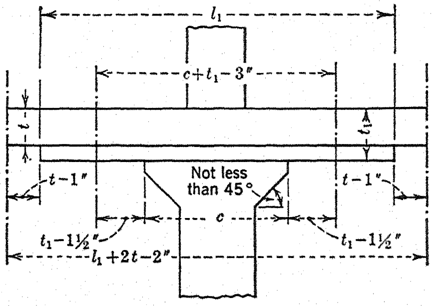


FIG. 157.—Critical Sections for Diagonal Tension. *(See p. 349.)*

The thickness of the drop panels is based upon the largest negative bending moment at the column in two adjoining column strips.

Let M_1 = largest negative bending moment in two adjoining column strips.

M_3 = largest positive bending moments in two adjoining column strips.

l = span of flat slab panel in the direction of bending moments M_1 and M_3.

c = diameter of column head.

b = width of drop panel.

b_1 = width of two adjoining column strips for which M_3 is used.

e_1 = distance of center of tensile reinforcement at support for M_1 to top of slab.

e = distance of center of tensile reinforcement in center of panel to bottom of slab.

C_1, C_2 = constants.

Then

Thickness of Slab plus Drop Panel at Column:[2]

$$t_1 = C_1\left(1 - \frac{2}{3}\frac{c}{l}\right)\sqrt{\frac{M_1}{b}} + e_1 \tag{4}$$

Thickness of Slab:[2]

$$t = C_2\sqrt{\frac{M_3}{b_1}} + e \tag{5}$$

using

$$C_1 = \sqrt{\frac{3}{jkf_c}}, \quad \text{where } f_c \text{ is allowable stress at supports}^3 \tag{6}$$

$$C_2 = \sqrt{\frac{3}{jkf_c}}, \quad \text{where } f_c \text{ is allowable stress in center} \tag{7}$$

The values of t and t_1 are in inches when M_1 and M_3 are in inch-pounds and all lengths in inches; or M_1 and M_3 are in foot-pounds and all dimensions in feet.

DIAGONAL TENSION IN FLAT SLABS

As in reinforced-concrete beams, in flat slabs shearing stresses are considered as measures of diagonal tension. In a flat slab with drop panels, there are two critical sections for diagonal tension, as shown in Fig. 157, p. 348.

The first critical section is concentric with the column, and is located on all sides from the edge of the column head at a distance equal to the effective thickness of the slab plus the drop, i.e., $(t_1 - e_1)$ inches.

The second critical section is concentric with the drop panel, and is located from its edge at a distance of $(t - e)$ inches.

Flat slabs without drop panel have only the first critical section.

[2] These are not the standard slab formulas, but modified formulas taking into account the excess of compression stresses in flat slabs over tensile stresses resisted by reinforcement, which is caused by the flat-slab action explained on p. 334.

[3] In accordance with American practice, larger allowable unit stresses are used at the supports than in the center of slab. See "Concrete, Plain and Reinforced," Vol. I, p. 282.

The unit shearing stresses at the critical sections are computed from the formula $v = \frac{V}{bjd}$, where V is the total external shear outside of the critical section, b is the perimeter of the critical section, and d is the effective depth.

The unit shearing stresses at the critical sections should not exceed the allowable unit stress of $v = 0.03\,f'_c$, where f'_c is the compression strength of concrete at 28 days. If the shearing stresses are larger, they may be reduced by increasing the diameter of the column head (or the side of drop panel), or by increasing the depth of slab.

REINFORCEMENT FOR FLAT SLAB

Cross Section of Reinforcement at Critical Sections. — Having computed the combined bending moments at all critical sections, and accepted the thicknesses of slabs and of drop panels as explained in the preceding pages, one finds the required areas of steel at all sections by means of the general formula

$$A_s = \frac{M}{jdf_s} \tag{8}$$

where M is the combined bending moment at the particular section and in the particular design strip, and d is the effective depth of slab in the strip. When M is in foot-pounds, d in feet and f_s in lb. per sq. in., A_s will be in square inches.

For flat slabs with drop panels, the effective depth for the negative bending moments in the column strips is equal to the depth of slab plus the depth of drop panel. For all other sections, depth of slab alone should be used.

The reinforcement required for each strip should be distributed over the whole width of the strip.

Effectiveness of Bars Placed at an Angle to Critical Section. — When a reinforcing bar intersects a critical section of slab at an angle, its effective area at that section is equal to the cross section of the bar multiplied by the sine of the angle of inclination. Thus a bar at right angles to a critical section is fully effective at that section because the sine is equal to 1.0, and not effective at all at a section at right angles to the first section. The effective area of a bar at 45° to a critical section is equal to 0.707 of the cross section of the bar; but the same bar is equally effective at the critical section at right angles to the first critical section.

Arrangement of Flat-Slab Reinforcement. — Flat-slab reinforcement consists either of bands of parallel bars, as in the two-way and four-way systems, or of combinations of parallel bars and rings, as in the

Smulski system. These systems are fully described and illustrated in "Concrete, Plain and Reinforced," Vol. I, pp. 358 to 369. It should be noted, however, that the lengths of bars and points of bending of reinforcement there shown are intended for flat slabs in building construction only. In bridge design, to take care of heavy moving loads, the lengths of bars and points of bending reinforcement should be found from the bending-moment diagram as explained on p. 339.

In band systems, each band, consisting of equally spaced parallel bars, coincides and is co-extensive with one of the design strips. In each band, the size of bars and their number are determined from the area required for the maximum positive bending moment in the particular strip. The use of bars of small diameter is advisable so as to keep the bond stresses within working limits. Bond stresses at the columns are likely to be large, because there the maximum stress in the bars is developed in a short distance. The spacing of bars should be neither too close nor too wide.

Of the bars in a band, some are bent up at one or both ends, and extended over the support into the adjoining panel where they may serve as negative bending-moment reinforcement. The straight bottom bars (or straight ends of bent bars) are extended beyond the line of zero positive bending moments as determined by the bending-moment diagram for positive bending moments a distance not less than $\frac{1}{20}$ of the span length, nor less than 20 bar diameters. Some of the bottom bars should be extended to the edge of the panel.

The negative bending-moment reinforcement at any critical section is made up of the bent portions of the bars coming from the two panels on both sides of the section, and of straight top bars added to make up any deficiency. To serve as negative bending-moment reinforcement in the adjoining span, the bent bars must extend beyond the line of zero negative bending moments in that span a distance not less than $\frac{1}{20}$ of the span length nor less than 20 bar diameters. Some top bars should extend the whole length of the structure to take care of any possible negative bending moments in the central portions of the panels due to moving loads, and also of temperature stresses as explained on p. 354. It should be noted that the position of points of zero moments for negative bending moments does not coincide with that for the positive bending moments, because each of them is determined for a different loading condition.

In the end panels, the negative bending-moment reinforcement at the critical section passing through the end columns must be extended beyond the section and anchored in the columns or in the spandrels so as to be able to resist full working stresses at that critical section. Since

full bending moments are transferred from the slab into the end column, the joint between the slab and the column must be properly reinforced either by corner reinforcement extending from the column into the slab, or by lapping of the slab reinforcement with the column bars.

COLUMNS

A flat-slab structure is in effect a rigid frame, and the bending moments in the slab depend not only upon the spans and the loadings of the slab but also upon the relative rigidity of the columns and of the slabs. The columns, therefore, must not only be strong enough to keep the stresses within working limits, but also they must have the rigidity anticipated in the formulas.

Rigidity of Columns. — Formulas given in Tables I and II, pp. 340 to 342, and in Table III, p. 347, are based upon a definite relation between the rigidity ratio of the columns $\frac{I_h}{h}$ and the rigidity ratio of the slab $\frac{I}{l}$. The assumptions are explained on p. 338. In designing the columns, it is necessary to provide the proper rigidity, which is determined by the ratio $m = \frac{I_h}{h} : \frac{I}{l}$.

In the rigidity ratio, the moments of inertia of the column and of the slab are based upon concrete dimensions only, without considering the reinforcement. The moment of inertia of the column is based upon its cross section; the corresponding moment of inertia of the slab is based upon a width of slab equal to the width tributary to the column, and upon a thickness of slab equal to the actual slab thickness plus one-third the depth of the drop panel. When the widths of panels on both sides of the column under consideration are equal, the tributary width is equal to the panel width. When the structure is provided with a cantilever along one side of the column, the tributary width for the first row of columns is equal to the width of one-half of the panel plus the length of the cantilever.

To get proper rigidity for the end columns, the following relation must exist between the dimensions of the column and those of the slab.

Let l = span of square or rectangular end panel.

l_1 = width of slab tributary to one column.

h = theoretical height of column, from top of footing to neutral axis of slab.

t_2 = thickness of slab plus one-third of depth of drop panel.

d = thickness of rectangular column, or diameter of round column.

b = width of rectangular column.

I_h = moment of inertia of column.

I = moment of inertia of tributary width of flat slab, based on thickness of slab t_2.

$m = \frac{I_h}{h} : \frac{I}{l} = \left\{ \text{ratio of rigidity of column to that of slab anticipated in computations.} \right.$

Dimensions of Columns Determined by Rigidity:

$$d = \sqrt[3]{m \frac{l_1}{l} \frac{h}{b}}\, t_2 \qquad \text{rectangular columns} \tag{9}$$

$$d = \sqrt[4]{m \frac{l_1}{l} \frac{h}{t_2}}\, t_2 \qquad \text{square columns} \tag{10}$$

$$d = 1.14 \sqrt[4]{m \frac{l_1}{l} \frac{h}{t_2}}\, t_2 \qquad \text{round columns.} \tag{11}$$

Dimensions l, l_1, h, and t_2 under the radical sign must be in the same units. Then the values of d will be in the same units as the value t_2 outside the radical.

Design of Columns. — Columns for flat-slab bridges should be designed for direct compressions due to the reactions of the loads, and for bending moments. The resulting stresses may be considered as fiber stresses, and the same allowable unit stresses may be used as in the design of beams.

The effect of bending moments upon the columns is particularly large in exterior columns of structures without cantilevers. Where cantilevers are used, it is permissible to reduce the bending moments in the column, due to the loads on the main spans, by the amount of the bending moments produced there by the cantilever dead loads. It is obvious that full bending moments from the slab are transferred to the end column: the bending moments in the column strips directly, and the bending moments in the middle strips by torsion in the spandrels.

Bending moments in columns are found from the Tables I and II, and for slabs with cantilevers from Table III.

In interior columns of a flat slab of equal spans, bending moments are produced only by live loads placed unsymmetrically in respect to the column under consideration. Where panels are unequal, bending moments in interior columns are produced by dead load as well as by live load.

FOUNDATIONS FOR FLAT-SLAB BRIDGES

Foundations for columns of a flat-slab bridge may consist of independent footings, one for each column; or each transverse row of columns may be placed on a combined footing. The footings may rest directly on the ground, or they may be supported on piles or caissons.

The methods of design of footings for flat-slab columns are the same as for vertical members of other rigid-frame structures. They are discussed in detail on p. 436.

EFFECT OF TEMPERATURE CHANGES

Flat-slab bridges, owing to the uniformity of their cross sections and the even distribution of reinforcement, are well adapted to take care of the effect of temperature changes and shrinkage. Many structures of considerable length built without any special provision for temperature changes, after many years of service show no bad effect attributable to the action of temperature changes.

As an example may be mentioned the viaduct, carrying railroad tracks in the Buffalo terminal of the D. L. & W. R.R., which is 810 ft. long and 154 ft. wide, and was built without expansion joints. It consists of 27-ft. square panels, and has a slab 2 ft. thick.[4] The same road built many other flat-slab bridges up to 500 ft. in length without expansion joints.

The adaptability of flat-slab structures to temperature changes is undoubtedly due to the fact that contraction caused by shrinkage and by the fall of temperature produces in the slab a great number of minute incipient cracks well distributed throughout the structure, each panel taking care of its own contraction without transferring the changes in length to the adjoining panels. The only effect then is a small increase in tensile stresses in the slab reinforcement. The rise of temperature closes these cracks and increases somewhat the compression stresses without causing any appreciable movement of the column heads.

There is no uniformity in practice as far as taking care of the effects of temperature changes in flat-slab bridges is concerned. Some engineers provide expansion joints only where the structure changes its width appreciably. To take care of additional tension produced by contraction, they provide small additional amounts of continuous bars, and also short bars at construction joints.

Other engineers space expansion joints from 150 to 300 ft. apart. In still other cases flat-slab structures are considered as rigid frames, and

[4] See A. B. Cohen, " Reinforced-concrete Flat-slab Railway Bridges," *Proceedings of American Concrete Institute*, 1918, pp. 325, 328, and 335.

the effect of temperature is determined as explained in connection with rigid frames. An example of this method is the double-deck street in Chicago shown in Fig. 161, p. 371. This structure is 135 ft. wide, and is divided into longitudinal sections by expansion joints spaced about 150 ft. apart. Each section was considered in design as a rigid frame.

The authors are in favor of using expansion joints in structures more than 250 ft. long, and also where the structure undergoes appreciable changes in width. They also recommend, to take care of tension stresses produced by temperature changes, that temperature reinforcement should be added to the reinforcement required by the loading, the amount of which should vary in interior panels from 0.1 to 0.2 per cent of the cross section of the slab, depending upon the length of the structure between the expansion joints. In end panels, use twice the amount of temperature reinforcement recommended for the interior panels. The temperature reinforcement should consist of bars of small diameters placed both near the top and near the bottom of the slab. Additional bars should be used along the edges of the structure. At construction joints, special short bars should be placed near the top of the slab to prevent opening of the joints. In exterior columns, special negative bending-moment reinforcement should be added at the top of the column and positive reinforcement at its bottom to take care of the effect of temperature changes. No special provision is required in interior columns.

ABUTMENTS FOR FLAT-SLAB BRIDGES

Abutments Separate from Main Structure. — In many cases abutments are built separate from the flat-slab structure, serving solely to retain the embankment. The flat-slab structure is then free to expand and contract without interference from the abutments. In Fig. 151 (*a*) and (*b*), p. 329, the flat slab is provided in the longitudinal direction with cantilevers; and there are positive expansion joints between the abutments and the slab.

In the railroad bridge[5] built by the D. L. & W. R.R., the flat-slab structure at the ends is supported on end columns which are built separately from the abutment wall and are placed in recesses in the wall. The details are clearly shown in the figure.

Abutments Forming Part of Flat-Slab Structure. — The embankment may be retained by a wall connected with the end columns of the flat-slab bridge. The structure as a whole is then subjected to bending moments due to the earth pressures acting upon the walls.

[5] See A. B. Cohen, " Reinforced-concrete Flat-slab Railway Bridges," *Proceedings of American Concrete Institute*, Vol. XIV, 1918, p. 331.

In design of the structure, the wall may be considered as a slab subjected to earth pressures, and supported by the columns; or horizontal beams may be introduced between the columns, and the wall considered as a slab supported along four sides. (See also p. 323.) The earth pressure produces bending moments in the columns as well as in the horizontal slab in the same manner as explained on p. 247 in connection with rigid frames.

Lost Abutments. — In many cases, a design shown in Fig. 151 (*c*) may be used in which only shallow apron walls are needed to retain the earth pressures. At the sides, wing walls may be used, rigidly attached to the aprons. This is also discussed on p. 422.

FLAT-SLAB BRIDGES CONSIDERED AS RIGID FRAMES

The formulas for bending moments given in Tables I to III, pp. 340, 342, and 347, are for flat-slab structures of equal spans only; and they were worked out for definite relations between the rigidity of the columns and those of the slabs. Where the conditions are appreciably different from those assumed in the tables, flat-slab structures may be designed by considering them as series of longitudinal and lateral rigid frames.

Procedure. — To use for flat-slab structures the rigid-frame methods of computation given in Chapters XI to XIII, it is necessary to make the following assumptions:

1. The flat-slab bridge is replaced by a series of longitudinal and lateral frames. Each frame in the longitudinal direction consists of one longitudinal row of columns, which form the vertical members of the frame, and of a longitudinal strip of slab tributary to that row of columns, which acts as the horizontal member of the frame. Ordinarily, the tributary width of the slab is equal to one-half the width of the panel on each side of the row of columns. If the row of columns carries a cantilever slab along one side, the whole cantilever should be considered as the tributary slab on that side. These longitudinal frames are designed for the total dead and live loads coming upon the tributary width of slab.

In the lateral direction, the flat slab is considered as a series of lateral frames, each frame consisting of one lateral row of columns and of the slab tributary to that row. These lateral frames must be designed to carry the total dead and live load coming upon the lateral tributary width. It should be noted that, in flat-slab design, the total load is used in design twice: once in the longitudinal direction and the second time in the lateral direction.[6]

[6] In designing slabs reinforced in two directions and supported by beams, only a fraction of the total load is used in designing the slab in each direction.

2. The theoretical span in each panel of the horizontal member is taken as $l\left(1 - \frac{2}{3}\frac{c}{l}\right)$, as explained on p. 334.

3. The theoretical height of column is taken as the vertical distance from the top of the footing to the underside of the drop panel.

4. Moments of inertia of the slab and of the columns are computed as explained on p. 352. They are based on concrete sections only.

5. With the assumptions just made, bending moments in the frame are computed in the same manner as for ordinary rigid frames, using formulas and methods given either in Chapter XI or in Chapter XIII, depending upon the number of spans.

6. After the bending moments in the rigid frames are computed, they must be modified to take care of the peculiarities which distinguish flat-slab frames from ordinary rigid frames.

(*a*) To allow for the " flat-slab action " in the manner explained on p. 334 the final bending moments in the rigid frame, M_1 and $M_{max.}$, should be multiplied by the ratio $\frac{0.09}{0.125} = 0.72$ which is the ratio used in flat-slab formulas (2) and (3), p. 334.

(*b*) The modified negative bending moments at the columns ($0.72M_1$) and the modified maximum positive bending moments in the spans ($0.72M_{max.}$) should be distributed between the design strips in the proportions given in the following table, using notations:

M_1 = combined negative bending moment at the column in rigid frame;

$M_{max.}$ = combined maximum positive bending moment in a span of rigid frame.

Distribution of Bending Moments between Design Strips

Sign of Bending Moments	Strips	Bending Moments
Negative	2 column strips	$0.835\,(0.72M_1) = 0.6M_1$
	Middle strip	$0.125\,(0.72M_1) = 0.09M_1$
Positive	2 column strips	$0.69\,(0.72M_{max.}) = 0.5M_{max.}$
	Middle strip	$0.49\,(0.72M_{max.}) = 0.35M_{max.}$

In the table, it is assumed that the tributary width of the slab for the frame is one panel wide, and therefore consists of two column strips, and of one-half of a middle strip on each side, totaling one middle strip.

When the slab tributary to the row of columns consists of other arrangements of strips, the ratios in the table must be properly adjusted. For instance, in the arrangement shown in Fig. 156 (*a*), p. 345, the tributary slab consists on one side of one column strip and of one-half of a middle strip; and on the other side only of the cantilever slab which may be classed as a column strip. In this case, the column strips predominate and should resist a larger proportion of the total bending moments.

It should be noted that in the table the sum of the negative bending moments in the two column strips and in the middle strip is less than the total modified negative bending moment $0.72M_1$; while the sum of the positive bending moments in these strips is larger than the total modified positive bending moment $0.72M_{\text{max.}}$. This is so because the distribution of the bending moments has been adjusted to take care of the difference between the conditions in an ordinary rigid frame and in a flat-slab frame.

The values M_1 and $M_{\text{max.}}$ are computed for a rigid frame composed of horizontal girders and of columns. In a flat-slab rigid frame, the place of a compact girder is taken by a slab composed of two different types of strips each of which shows different behavior; the relation between the positive bending moments and the negative bending moments in the column strips is different from the same relation in the middle strip, as is evident from Tables I and II, pp. 340 and 342. In the column strips, the negative bending moment is very much larger than the maximum positive bending moment; in the middle strip, the maximum positive bending moment is larger than the corresponding negative bending moment. The distribution just suggested in the table has been selected so as to maintain the same relations between the bending moments in the various strips as is used in Tables I and II.

NUMERICAL EXAMPLE

The design formulas and the suggestions given in the preceding pages are illustrated by the following numerical example. Bending moments in this example are computed using formulas from Table I and II, pp. 340 and 342.

Example. — Design a flat-slab structure for the following conditions. Length of structure, out to out, 175 ft.; width of roadway, 40 ft.; no sidewalks, but a wide curb; loading, 20-ton trucks as shown in Fig. 23, p. 60; weight of pavement, 50 lb. per sq. ft.; lost abutment to be used.

Unit stresses same as in example on p. 60.

Solution. — In the longitudinal direction, the spans are arranged so that the three interior panels are 36.0 ft. long, and the exterior panels 32.0 ft. long, making the length of exterior panels about 0.9 of the length of interior panels. (See Fig. 158,

p. 359.) In the lateral direction the spacing of columns is 30 ft., and the slab is cantilevered out on both sides. The cross sections are shown in Fig. 159, p. 360.

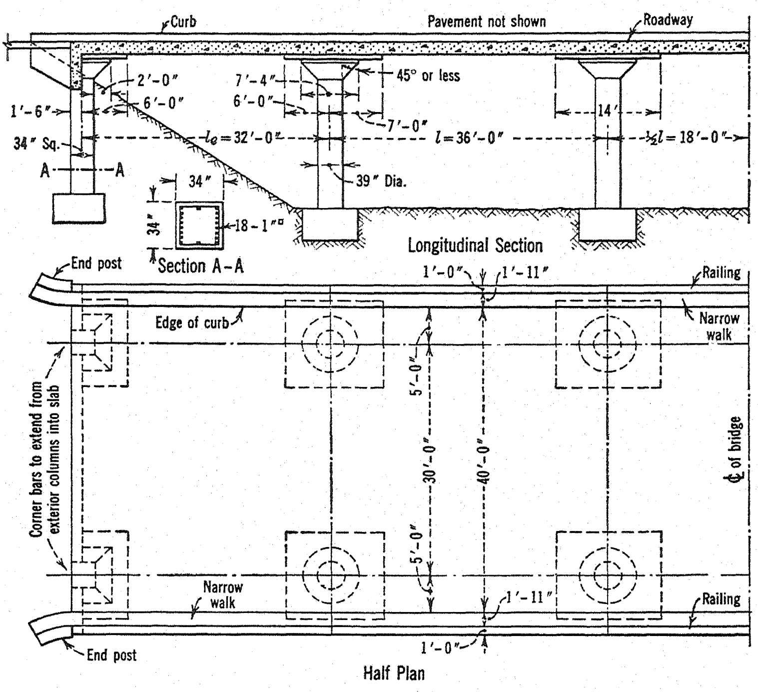


FIG. 158.—Example. General Dimensions of Flat-Slab Bridge. (*See p.* 358.)

Accept a round column head with a diameter of c = 7 ft. 4 in. which gives the following values for the different span lengths:

$$l = 36 \text{ ft.};\quad c = 7 \text{ ft. } 4 \text{ in.};\quad \frac{c}{l} = \frac{7.33}{36.0} = 0.204;$$

$$\left(1 - \frac{2}{3}\frac{c}{l}\right) = 0.864;\quad \left(1 - \frac{2}{3}\frac{c}{l}\right)^2 = 0.745$$

$$l_1 = 30 \text{ ft.};\quad c = 7 \text{ ft. } 4 \text{ in.};\quad \frac{c}{l} = \frac{7.33}{30.0} = 0.245;$$

$$\left(1 - \frac{2}{3}\frac{c}{l}\right) = 0.836;\quad \left(1 - \frac{2}{3}\frac{c}{l}\right)^2 = 0.70$$

$$l_e = 32 \text{ ft.};\quad c = 7 \text{ ft. } 4 \text{ in.};\quad \frac{c}{l} = \frac{7.33}{32.0} = 0.229;$$

$$\left(1 - \frac{2}{3}\frac{c}{l}\right) = 0.847;\quad \left(1 - \frac{2}{3}\frac{c}{l}\right)^2 = 0.72$$

Dead Load. — Assuming a thickness of slab of 18 in., the total dead load for slab and pavement is $w = 275$ lb. per sq. ft.

Flat-slab static bending moments for dead load, using formulas (2) and (3), p. 334, in which

$$W = 30 \times 36 \times 275.0 = 297\,000 \text{ lb., and } W_e = 30 \times 32 \times 275.0 = 264\,000 \text{ lb.}$$

Longitudinal

$$M_d = 0.09 \times 297\,000 \times 36.0 \times 0.745 = 717\,000 \text{ ft-lb.}$$

$$M_{de} = 0.09 \times 264\,000 \times 32.0 \times 0.72 = 547\,000 \text{ ft-lb.}$$

Transverse

$$M_e = 0.09 \times 297\,000 \times 30.0 \times 0.70 = 561\,000 \text{ ft-lb.}$$

These values are used to find the bending moments given in the tables on pp. 340 and 342 for the various critical sections of the slab.

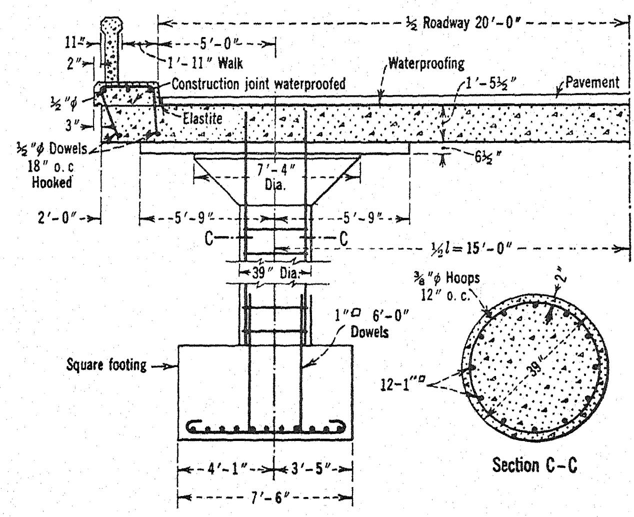


FIG. 159.—Example. Cross Section through Flat-Slab Bridge. (*See p.* 359.)

Live Loads. — The concentrated wheel loads are replaced by an equivalent uniformly distributed loading. In the longitudinal direction, the most unfavorable equivalent loading is obtained by computing the maximum bending moment of the truck loadings for a span equal to the simply supported longitudinal span of the panel; equating this bending moment to a formula for maximum bending moment for uniformly distributed loading; and solving the equation for w.

Longitudinal Equivalent Unit Live Load. — Using the formula from the table on p. 20, item 2, the bending moment for concentrated loads including 22% for impact is

$$M_{L+I} = \frac{1}{4}\left(1 - \frac{2.8}{36.0}\right)^2 \times (40\,000 \times 1.22) \times 36.0 = 373\,300 \text{ ft-lb. per lane of traffic.}$$

For equivalent uniformly distributed loading, $\frac{1}{8}$ $(10 \times w_l) \times 36.0^2 = 373\,300$.
Therefore equivalent unit live load is $w_l = \dfrac{373\,300 \times 8}{10 \times 36^2} = 230$ lb. per sq. ft.

This value is used as equivalent loading in the longitudinal direction of the interior panels. Actually a much smaller value would be sufficient because the concentrated loads are spread over the slab both longitudinally and laterally, and, therefore, the static bending moment for concentrated loads is smaller than that used above.

Similarly in exterior panels in the longitudinal direction the equivalent loading is

$$w_{le} = \frac{326\,000 \times 8}{10 \times 32^2} = 265 \text{ lb. per sq. ft.}$$

It should be noted that, to get a fully loaded panel, the loading must consist of three lines of heavy trucks traveling in the same direction at full speed, all of them being simultaneously in the most unfavorable positions. The likelihood of such unfavorable loadings is small, so that a structure designed for such a condition is able to carry safely occasional trucks much heavier than the design trucks. If desired, the unit live load may be reduced as explained on p. 11.

Lateral Equivalent Unit Live Load. — In the lateral direction it is impossible to load a panel so as to get the most unfavorable loading simultaneously for all strips. For any one of the lateral strips the most unfavorable position of the loading is when the rear axles of three lines of trucks are placed side by side along the center line of the strip. This load may be safely considered as distributed over an area equal to the width of the strip multiplied by the width of three traffic lanes. Actually, since at the same time the adjacent strips would have practically no live load, the concentrated loads would be distributed longitudinally over the whole panel but with varying intensities. The load on the rear axle of a truck, including impact, is $32\,000 \times 1.22 = 39\,000$ lb., and for a width of strip of 18 ft. the equivalent loading is

$$w_l = \frac{39\,000}{10 \times 18.0} = 217 \text{ lb. per sq. ft. Use 220 lb. per sq. ft.}$$

Static Flat-Slab Bending Moment for Live Load. — Using the equivalent loadings just computed, and formulas (2) and (3), p. 334, the modified static bending moments for live load are:

Longitudinal

$$M_l = 0.09 \times (30 \times 36 \times 230.0) \times 36.0 \times 0.745 = 600\,000 \text{ ft-lb.}$$
$$M_{le} = 0.09 \times (30 \times 32 \times 265.0) \times 32.0 \times 0.72 = 530\,000 \text{ ft-lb.}$$

Lateral

$$M_e = 0.09 \times (30 \times 36 \times 220.0) \times 30.0 \times 0.70 = 450\,000 \text{ ft-lb.}$$

Bending Moments in Flat Slab in Longitudinal Direction. — Accept a ratio of rigidity of column to slab $\dfrac{I_h}{h} \div \dfrac{I}{l} = 1.0$. Refer to Table II, p. 342. The corresponding bending moments for two longitudinal column strips and a middle strip are tabulated in the table on p. 362, which gives bending moments for dead load, live load, and combined bending moments; and finally depths of slab and areas of steel for each strip.

LONGITUDINAL DIRECTION

Critical Section	Strip	Dead Load: Ratio*	Dead Load: Bending Moment, ft-kips	Live Load: Ratio*	Live Load: Bending Moment, ft-kips	Combined Bending Moment, ft-kips	d, in.	A_s, sq. in.
Interior Span								
		M_d = 717.0 per panel		M_l = 600.0 per panel				
Column center line	2 col. strips	−0.54	−387.0	−0.64	−384.0	−771.0	22.0	31.1 Top
	Middle strip	−0.08	− 57.4	−0.10	− 60.0	−117.4	15.75	6.5 "
Panel center	2 col. strips	0.21	151.0	0.33	198.0	349.0	15.75	19.4 Bottom
	Middle strip	0.14	101.0	0.22	132.0	233.0	15.75	13.0 "
Exterior Span								
		M_{de} = 547.0 per panel		M_{le} = 530.0 per panel				
Int. column center line	2 col. strips	−0.64	−350.0	−0.69	−366.0	−716.0	22.0	28.9 Top
	Middle strip	−0.11	− 60.0	−0.12	− 64.0	−124.0	15.75	6.9 "
Ext. column center line	2 col. strips	−0.30	−164.0	−0.36	−191.0	−355.0	22.0	14.3 "
	Middle strip	−0.04	− 22.0	−0.06	− 32.0	− 54.0	15.75	3.0 "
Panel center	2 col. strips	0.31	170.0	0.37	196.0	366.0	15.75	20.3 Bottom
	Middle strip	0.20	109.0	0.24	127.0	236.0	15.75	13.1 "
Interior Column (See note below)								
				M_{l1} = 416.0 per column				
	Top			±0.22	±91.5	±91.5		
	Bottom			±0.15	±62.4	±62.4		

LONGITUDINAL DIRECTION (*Continued*)

Critical Section	Strip	Dead Load		Live Load		Combined Bending Moment, ft-kips		
		Ratio*	Bending Moment, ft-kips	Ratio*	Bending Moment, ft-kips			
Exterior Column (See note below)								
		$M_{de1} = 457.0$ per column		$M_{le1} = 364.0$ per column				
	Top	−0.34	−155.0	−0.42	−153.0	−308.0		
	Bottom	0.25	114.0	0.30	109.0	223.0		

NOTE: Each longitudinal row of columns supports on one side one row of panels, and on the other side a strip of cantilever slab. To get the total static bending moment upon which to base bending moments in columns in the longitudinal direction, it is necessary to use for the load W in formula (2), p. 334, the load on one-half of the panel plus the load on the cantilever strip.

For interior span, only live load needs to be considered

$$M_{l1} = 0.09 \times (20 \times 36 \times 230 + 1.9 \times 36 \times 100\ddagger) \times 36.0 \times 0.745 = 416\,000 \text{ ft-lb.}$$

For exterior column, both live and dead loads are considered

$$M_{de1} = 0.09 \times (22.75 \times 275 + 650\dagger) \times 32 \times 32 \times 0.72 = 457\,000 \text{ ft-lb.}$$

$$M_{le1} = 0.09 \times (20 \times 32 \times 265.0 + 1.9 \times 32 \times 100\ddagger) \times 32 \times 0.72 = 364\,000 \text{ ft-lb.}$$

For interior column: $h = 21.5$ ft., $a = 3.75$ ft., and $\dfrac{h}{2h - 3a} = \dfrac{21.5}{31.75} = 0.68.$

For exterior column: $h = 18.5$ ft., $a = 3.75$ ft., and $\dfrac{h}{2h - 3a} = 0.72.$

* From Table II on p. 342 for $\dfrac{I_h}{h} = \dfrac{I}{l}$.

† Weight of curb and railing. ‡ Live load on walk.

Thickness of Slab and Drop Panel. — Thicknesses of slab and drop panels are found from formulas (4) and (5), p. 349. The largest positive bending moment acts in the center of two column strips in the exterior panel, and the thickness of the slab is computed for this bending moment. The largest negative bending moment acts at the column of the interior panels, and for this value the thickness of slab plus drop is computed.

Thickness of slab in center of panel. For $f_c = 800$, $f_s = 16\,000$, $k = 0.429$, $j = 0.857$, and $b_1 = \frac{l_1}{2} = 15.0$ ft.

$$C_2 = \sqrt{\frac{3}{0.857 \times 0.429 \times 800}} = 0.101$$

and

$$t = 0.101 \times \sqrt{\frac{366\,000}{15.0}} + 1.75 = 17.5 \text{ in.}$$

Thickness of drop panel. Accept width of drop panel as 11 ft. 6 in., and its length as 14 ft. Then for $f_c = 900$, $f_s = 16\,000$, $k = 0.458$, $j = 0.847$, and $b = 11.5$ ft.

$$C_1 = \sqrt{\frac{3}{0.847 \times 0.458 \times 900}} = 0.093$$

and

$$t_1 = 0.093 \times 0.864 \times \sqrt{\frac{771\,000.0}{11.5}} + 2.0 = 22.7 \text{ (larger depth is accepted)}$$

Use

$t = 17\frac{1}{2}$ in. $\quad d = 17.5 - 1.75 = 15.75$ in.

$t_1 = 24.0$ in. $\quad d_1 = 24.0 - 2.0 = 22.0$ in. $\quad$ Drop $24.0 - 17.5 = 6.5$ in.

Using these thicknesses in the table, the required areas of steel in each strip are found from the general formula $A_s = \frac{M}{jdf_s}$, where M must be changed to inch-pounds when d is in inches.

Bending Moments in Lateral Direction

Bending moments in the lateral direction are found by considering the slab as a one-span frame with cantilevers.

The ratio of rigidity of the columns to that of the slab in the lateral direction must now be computed. In the longitudinal direction, this ratio has been accepted as $m = 1.0$. In the lateral direction, this ratio is different, because, while the columns remain the same as in the previous case, the width of the slab and its span are different. The rigidity of the slab in the lateral direction due to its increased width and the reduced span is 1.9 times as large as the rigidity of the slab in the longitudinal direction. The ratio of rigidity of column to slab in lateral direction is therefore 1.9 times smaller than in longitudinal direction. It is approximately $m = 0.5$, and formulas for this ratio are used in determining bending moments.

Bending Moments for Loads on Main Span. — For loads on the main span, bending moments are found by means of formulas in Table I, p. 340. The flat-slab static bending moments in the lateral direction have been previously computed,

and they are for dead load $M_e = 561\,000$ ft-lb., and for live load $M_e = 450\,000$ ft-lb. The results for the various strips are tabulated on p. 367.

Bending Moments for Cantilever Loads. — For bending moments due to cantilever loads, use formulas in Table III, p. 347.

For dead load, the cantilever bending moment is computed in the usual manner.

Total cantilever load $P_d = 96\,900$ lb.

Cantilever bending moment at the edge of the panel $M_{cd} = -423\,700$ ft-lb.

Bending moment in the cantilever, at the theoretical edge of the column head (i.e., at a distance of $\frac{1}{3}c$ from panel edge), $M = -200\,300$ ft-lb.

In the main span, for dead load on both cantilevers, the bending moments are negative and uniform throughout the span; they are equal to $0.5M_{cd} = -211\,900$ ft-lb., and are distributed so that the two column strips resist $-180\,000$ ft-lb. and the middle strip $-31\,900$ ft-lb.

At the top of the column a positive bending moment acts, equal to $-0.5M_{cd} = 211\,900$ ft-lb., and at the bottom, $-105\,950$ ft-lb. From proportions the bending moment at the bottom of the column head is 157 000 ft-lb.

Live Loads on Cantilevers. — Live loads on one cantilever consists of the uniformly distributed loading on the narrow walk, and of one longitudinal line of trucks placed so that the centers of the outside wheels are 1 ft. 6 in. from the edge of the curb.[7] The inside line of the wheels of these trucks is within the main span, but to simplify computations the small effect of these loads upon bending moments in the main span is disregarded.

For the most unfavorable position of the trucks, they are placed so that the rear axle of one truck is on the column center line, and the spans on both sides of the column are fully loaded. The loading will consist of two trucks and one rear axle of the third truck as shown in Fig. 23, p. 60. The reaction of the loads on the cantilever strip in respect to the column under consideration is

$$\left[16\,000 \times \left(1 + \frac{2 \times 3}{36}\right) + 4\,000 \times \frac{22 + 17}{36}\right] \times 1.22 = 28\,100 \text{ lb.},$$

including 22% for impact. The moment arm for these concentrated loads is $5.0 - 1.5 = 3.5$ ft.

The cantilever bending moments for live loads are:

$$\begin{aligned} \text{Loads on walk: } -36 \times 1.91 \times 100 \times \left(5.0 + \frac{1.91}{2}\right) &= -\ \ 40\,800 \\ \text{Trucks: } -28\,100 \times 3.5 &= -\ \ \underline{98\,400} \\ M_c &= -139\,200 \text{ ft-lb.} \end{aligned}$$

As evident from Table III, p. 347, and Fig. 156, p. 345, the cantilever bending moment produces in the main span the following bending moments. At the column in the main span the largest bending moments are produced when only one cantilever is loaded. Then, from Table III on p. 347, $M_1 = -0.71M_c = 98\,800$ ft-lb., and $M_2 = 0.21M_c = 29\,200$ ft-lb. At the theoretical edge of the column head, the bending moment obtained from proportions is $M = 91\,000$ ft-lb., of which 79 000 ft-lb. is resisted by the column strips, and 12 000 ft-lb. by the middle strip. See summary.

[7] As explained on p. 28, it is not permissible to use equivalent uniformly distributed loading to find the cantilever bending moments for wheel loads.

Most unfavorable results in the center of the panel are obtained when both cantilevers are loaded simultaneously, in which case the total bending moment is $-0.5M_c = -69\,600$ ft-lb., of which the column strips resist $-60\,000$ ft-lb. and the middle strip $-9\,600$ ft-lb.

In the columns the most unfavorable loading is when both cantilevers are loaded simultaneously, in which case bending moments at the top and bottom are: $M_T = 0.5M_c = 69\,600$ ft-lb., and $M_B = -0.25M_c = -34\,800$ ft-lb., and the bending moment at the bottom of the column head by proportions is $M = 54\,000$ ft-lb.

Summary of Bending Moments in Lateral Direction. — Bending moments in the lateral direction at all design strips and in the columns are given in the following summary. Bending moments are given separately for dead load and for live load. Combined bending moments are also given as well as the areas of reinforcement.

Since the bending moments in the lateral direction are smaller than in the longitudinal direction, the lateral reinforcement is placed on the top of the longitudinal reinforcement. The depths of slab used in computing the areas of lateral reinforcement are therefore 1.0 in. smaller than those used for the longitudinal reinforcement.

REINFORCEMENT OF SLABS

Reinforcement in Longitudinal Direction. — The areas of reinforcement for the different design strips in the structure are given in the table on p. 362. To this reinforcement, it is necessary to add the temperature reinforcement, which, in this case, for the interior panels, is taken as equal to 0.1 per cent of the cross section of the slab. One-half of this steel is placed at the bottom, and the other half near the top of the slab.

Temperature reinforcement for interior panels amounts to $0.001 \times 12 \times 17.5 = 0.21$ sq. in. per lineal foot of slab, of which 0.105 sq. in. are placed near the bottom and the same amount near the top. In the end panels $\frac{3}{4}$-in. round bars 18 in. on centers are used near the top of the slab to take care of possible negative bending moments due to fall of temperature. (See p. 354.)

In cross section, the bridge consists of one complete panel and of two cantilever strips. The reinforcement required in the panel may be obtained directly from the summary on p. 362. At each column, there is on one side one column strip, and on the other side the cantilever strip 7 ft. 9 in. wide. To find the area of reinforcement required for the cantilever strip, the average area per lineal foot in the panel should be found and then multiplied by the width of the cantilever strip. In this case, however, since the cantilever strip is almost as wide as one column strip, the same area of reinforcement is used there as in the column strip.

Column Strips. — To simplify the placing of steel, the same area of steel is used for the positive bending-moment reinforcement in the interior panels as in the exterior panels. In the two column strips from the table, $A_s = 20.3$ sq. in. To this add the temperature reinforcement equal to $15.25 \times 0.105 = 1.6$ sq. in., making a total of $A_s = 20.3 + 1.6 = 21.9$ sq. in. Use twenty-eight 1-in. round bars, for which $A_s = 28 \times 0.785 = 21.98$ sq. in.

Bend each bar at one end only, and extend the bent part into the adjoining panel. Arrange bars so that straight and bent ends alternate.

At the supports, the required area of reinforcement for two column strips from the table on p. 362 is $A_s = 31.1$ sq. in. To this add the top temperature reinforcement, making a total $A_s = 31.1 + 1.6 = 32.7$ sq. in.

The bent bars in the panel and those from the adjoining panel supply an area of steel 21.98 sq. in. The balance $32.7 - 21.98 = 10.72$ sq. in. must consist of straight

Section	Strip	Loads on Main Span				Loads on Cantilevers		Total, ft-kips	d, in.	A_s, sq. in.
		Dead Loads		Live Loads		Dead Load	Live Load			
		Formula	Bending Moment, ft-kips	Formula	Bending Moment, ft-kips	Bending Moment,† ft-kips	Bending Moment,† ft-kips			
		$M_e = 561.0$		$M_e = 450.0$						
Column center line	Column	$-0.28M_e$	−157.1	$-0.28M_e$	−126.0	−180.0	−79.0	−542.1	21.0	22.8 Top
	Middle	$-0.05M_e$	− 28.0	$-0.05M_e$	− 22.5	− 31.9	−12.0	− 94.4	15.75	5.0 Top
Panel center line	Column	$0.46M_e$	258.0	$0.46M_e$	207.0	−180.0	−60.0	+285.0 −111.0	14.75	17.0 Bottom −6.8 Top
	Middle	$0.31M_e$	174.0	$0.31M_e$	139.5	− 31.9	− 9.6	+281.6	14.75	16.7 Bottom
Column	Top*	$-0.33M_e$	−185.0	$-0.33M_e$	−148.5	157.0	54.0	−176.5 +118.5		
	Bottom	$0.22M_e$	123.0	$0.22M_e$	99.0	−105.9	−34.8	+116.0 − 79.2		

* At bottom of column head. † For computations see p. 365.

top bars. Therefore add fifteen 1-in. round bars at each interior column, the area of which is $15 \times 0.785 = 11.8$ sq. in.

To supply temperature reinforcement in the interior panels at the top of the slab in the central portion of the span, and to take care of possible negative bending moments due to unbalanced live loads, $\frac{5}{8}$-in. round bars, spaced 1 ft. 6 in. on centers, are used. In end panels thirty $\frac{3}{4}$-in. round top bars are used to take care of temperature stresses. All these bars must lap the main negative reinforcement.

The negative reinforcement of the column strips in the exterior panel at the exterior columns from the table on p. 362 is $A_s = 14.3$ sq. in. Add to this the temperature reinforcement, making a total $A_s = 14.3 + 3.2 = 17.5$ sq. in. The bent bars supply an area of $\frac{21.98}{2} = 11.0$ sq. in. The balance will be supplied by nine 1-in round short bars hooked at the column ends.

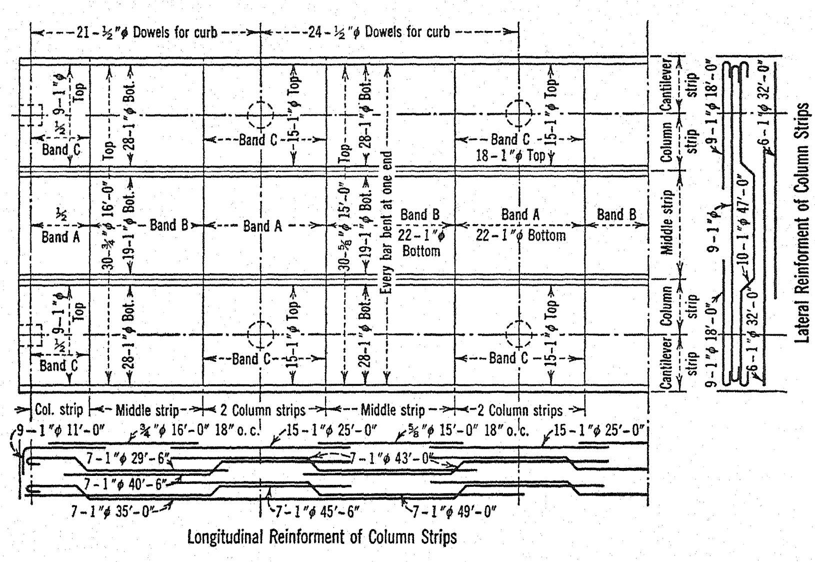


FIG. 160.—Example. Reinforcement of Flat-Slab Bridge. *(See p. 368.)*

Middle Strips. — In the middle strip, the required area of positive bending-moment reinforcement from the table on p. 362 is $A_s = 13.1$ sq. in., and, with temperature steel, $A_s = 13.1 + 1.6 = 14.7$ sq. in.

Use nineteen 1-in. round bars with an area of $19 \times 0.785 = 14.9$ sq. in. Bend and arrange the bars in the same manner as in the column strips.

The bent bars are sufficient to take care of the negative bending moments in the middle strips.

It should be noted, as shown in Fig. 160, p. 368, that, of the twenty-eight bars forming the positive reinforcement of the column strips, seven bars at each end extend to the edge of the panel.

Lateral Reinforcement. — The required amount of reinforcement in the lateral direction is given in the last column of the summary on p. 367. No temperature steel is needed in the lateral direction.

In the two column strips, the area of steel required by the positive bending moments is $A_s = 17.0$ sq. in.

Use twenty-two 1-in. round bars with an area of $A_s = 22 \times 0.785 = 17.3$ sq. in.

Bend every bar at one end and arrange the bars as shown in Fig. 160, p. 368.

To supply the required area of negative bending-moment reinforcement at the columns, $A_s = 22.8$ sq. in., add to the bent bars eighteen 1-in. round top bars. The total area then is $A_s = 8.65 + 18 \times 0.785 = 22.8$ sq. in.

Of these eighteen top bars, nine are short and nine extend the whole width of the slab and supply the needed negative reinforcement in the center of the span.

To supply the required area of $A_s = 16.7$ in the middle strip, use twenty-two 1-in. round bars 46 ft. long with an area of $A_s = 17.3$ sq. in. Bend bars at one end.

Columns

Concrete Dimensions of Columns. — As is evident from p. 361, an assumption has been made that the ratio of rigidities in the longitudinal direction is $\frac{I_h}{h} \div \frac{I}{l} = 1.0.$ The dimensions of columns must now be selected so as to realize the assumed ratio. Formulas (10) and (11) p. 353 will be used to get the concrete dimensions of columns.

The value t_2 in the formulas is equal to $t_2 = 17.5 + \frac{6.5}{3} = 19.7$ in. or 1.63 ft.

The theoretical heights of columns are: for interior columns $h = 18.5$ ft., and for exterior columns $h = 21.5$ ft.

The tributary width of slab for each column is $l_1 = \frac{30.0}{2} + 7.75 = 22.75$ ft.

Diameter of round interior columns, required for rigidity, $l = 36.0$ ft.

$$d = 1.14 \sqrt[4]{1.0 \times \frac{22.75}{36.0} \times \frac{21.5}{1.63}} \times 1.63 = 3.15 \text{ ft. or } 37.8 \text{ in.} \quad \text{Use 39 in.}$$

Side of square exterior column, required for rigidity, $l = l_e = 32.0$ ft.

$$d = \sqrt[4]{1.0 \times \frac{22.75}{32.0} \times \frac{18.5}{1.63}} \times 1.63 = 2.8 \text{ ft. or } 33.6 \text{ in.} \quad \text{Use 34.0 in.}$$

The columns must be reinforced with at least 1 per cent of vertical reinforcement. Stresses in columns will now be investigated for this minimum amount of reinforcement.

Interior Columns. — The columns are subjected to direct pressures and to bending moments. The direct pressures are the vertical reactions for dead and live loads.

Vertical reactions for dead loads computed as for simply supported slabs are $P_d = 280\,000$ lb.

Vertical reactions for live loads are found as follows: The main span is loaded by three lines of trucks, and there is no live load on the cantilevers. This is the most unfavorable loading for the column because it produces in it the largest bending moments in the lateral direction. To get the largest reaction for such loading the rear axle of one truck in each line is placed on the line through the centers of the columns, and the panels on both sides of this line are fully loaded. The loading is described on p. 358. The reaction per column, including impact, is

$$P_l = 1.5 \times \left[32\,000 \times \left(1 + \frac{2 \times 3}{36}\right) + 8\,000 \times \frac{27 + 17}{36}\right] \times 1.22 = 86\,200 \text{ lb.}$$

The total column load is

$$P = P_d + P_l = 366\,200 \text{ lb.}$$

For this loading, the lateral bending moment in the column, taken from the summary on p. 367, is $M_T = -176\,500$ ft-lb. The longitudinal bending moment is zero.

Diameter of column $d = 39$ in.; area of steel for $p = 0.01$ is $A_s = 0.01 \times 1\,194 = 11.9$ sq. in.

$$e = -\frac{176\,500}{366\,200} = -0.48 \text{ ft.} \quad \text{and} \quad \frac{e}{d} = -\frac{0.48 \times 12}{39} = -0.148.$$

From appropriate tables, not given in this treatise, for the above ratio of eccentricity, and for 1 per cent of vertical steel, the maximum compression stresses are equal to about twice the average compression stresses. Only small tension is developed in the section. The unit stresses are

$$f_c = \frac{366\,200}{1\,194.0} \times 2 = 612 \text{ lb. per sq. in.}$$

Since this stress is in the nature of fiber stresses, it is satisfactory. The column investigated for double live load is also satisfactory; therefore, the following dimensions are accepted as final (see section in Fig. 159, p. 360):

Use 39-in. round columns, with twelve 1-in. square bars and $\frac{3}{8}$-in. hoops spaced 12 in. on centers.

Exterior Columns. — The dimensions as determined by the required rigidity are: Side of square column, $d = 34$ in.; area of concrete section $A = 1\,156$ sq. in.; area of steel section based on the minimum percentage, $p = 0.01$, is $A_s = 11.56$ sq. in. The design is shown in Fig. 158, p. 359. The bars at the sides of the section are not included in the effective area of reinforcement. The reinforcement consists of eighteen 1-in. square bars, of which twelve bars are considered as effective in resisting bending moments, and four bars as temperature reinforcement.

The most unfavorable condition of loading, as far as the exterior columns are concerned, is when the panel is fully loaded, which gives not only the largest reactions, but also the largest bending moments in the columns. No lateral bending moment will be considered because the columns are well braced laterally by the heavy cross beam at the top and by the fill in which they are embedded.

The loads on exterior columns are

Dead load	150 000
Live load, including impact,	85 000
Total	235 000 lb.

From the summary on p. 362, the bending moment in the column in the longitudinal direction is $M = -308\,000$ ft-lb. (To change M to in-lb. multiply it by 12.)

$$\text{Therefore} \quad e = -\frac{308\,000}{235\,000} = -1.31 \text{ ft.}, \quad \frac{e}{d} = -\frac{1.31 \times 12}{34} = -0.46.$$

In this case, part of the concrete section is in tension. Use tables for determining constants in " Concrete, Plain and Reinforced," Vol. II, p. 658, with the notation

there given. For $h = 34$ in., $d = 34 - 2.5 = 31.5$ in., $b = 34$ in., $\frac{e}{d} = \frac{1.31 \times 12}{31.5} = 0.5$; $p = \frac{11.56}{34 \times 31.5} = 0.011$, the constants are $k = 0.61$ and $C_a = 0.16$.

Therefore maximum compression stresses are, using M in inch-pounds,

$$f_c = \frac{308\,000 \times 12}{34 \times 31.5^2 \times 0.16} = 685.0 \text{ lb. per sq. in.}$$

and maximum tensile stresses

$$f_s = -685.0 \times 15 \times \frac{1 - 0.60}{0.60} = -6850.0 \text{ lb. per sq. in.}$$

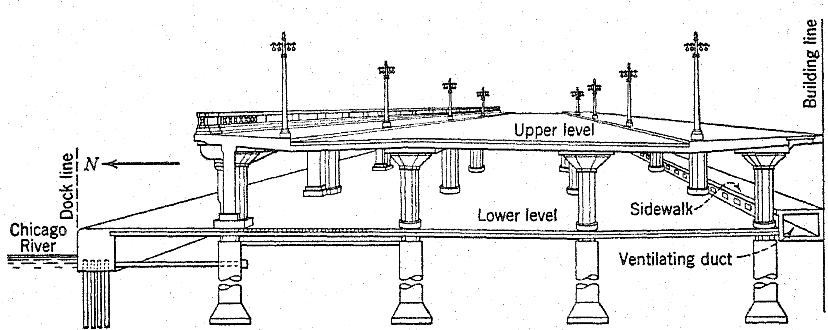
Since the maximum compression stresses are in the nature of fiber stresses, they are satisfactory. The section investigated in a similar manner for twice the live load is also satisfactory.

The bending moments at the bottom of the column are smaller than at the top; therefore, the column does not need to be investigated there.

To transfer bending moments from the slab to the exterior columns, corner reinforcement must be provided lapping the slab and column tensile reinforcement.

EXAMPLES FROM PRACTICE

Double-Deck Roadway, South Water Street, Chicago. — An interesting example of the use of flat-slab construction for bridges is furnished by the double-deck roadway for South Water Street in Chicago. The



From Engineering News-Record.

FIG. 161.—Perspective Cross Section of South Water Street, Chicago. *(See p. 371.)*

project is more than 3 000 ft. in length, and is of flat-slab construction with the exception of odd panels which are of beam and slab design, and of a few long spans where steel girders are used.

The structure is 114 ft. wide, and consists in transverse direction of three panels with cantilevers on both sides as shown in Fig. 161, above. Longitudinally the structure is divided by vertical expansion joints into five span long sections. The span lengths in general are about 32 ft. 6 in. in both directions. At each lateral expansion joint double columns

are used at each panel point, one-half of the double column being placed at each side of the joint. The slab is strengthened along the expansion joint by a beam of a depth somewhat less than that of the drop panel.

The columns rest on piers carried to hardpan through 40 ft. of clay, and the pressure on foundation is limited to 7 tons per square foot. Where the piers straddle existing or future tunnels or sewers, they are braced at the top by girders placed below the roadway at the lower level.

In designing, each section between the expansion joints was considered as a series of rigid frames. In addition to the stresses due to the loading, the frame was analyzed for the effect of temperature changes, using a total range of temperature of 60°. The reinforcement of the slab was arranged in four directions.[8]

Approaches for Hackensack River Crossing. — The approaches for the Hackensack River crossing[9] carrying the D. L. & W. R.R. are of flat-slab design with the exception of long spans.

Longitudinally the columns are spaced 22 ft. 6 in. on centers. The width of the flat-slab approach, which carries three tracks spaced 13 ft. on centers, is 45 ft. overall. The transverse spacing of columns is 18 ft. on centers, and the slab is cantilevered on both sides, the overhang amounting in each case to 5 ft. The slab was designed for Cooper's loading E 65. The thickness of the slab is 1 ft. 10 in., and of the 8 ft. 6 in. square drop panel, 10 in. Reinforcement is arranged in four directions. In addition to the bending-moment reinforcement, longitudinal temperature reinforcement was provided, placed in the top of the slab.

On account of their unusual length, from 36 to 41 ft., the columns were made 3 ft. 8 in. in diameter, and reinforced by longitudinal bars and spiral reinforcement. All columns, with the exception of four rows in the western approach, rest on footings supported by piles. The last rows in the western approach rest on caisson piles 8 ft. in diameter extended to rock. Two caisson piles were used per row of columns, and the loads from the columns are transmitted to them by means of 9 ft. 6 in. deep girders, braced by longitudinal struts extending between them.

[8] For further description and illustrations see: "Construction Methods on Double-Deck Street, Chicago," *Engineering News-Record*, Oct. 15, 1925, p. 662.

[9] See "Lackawanna Makes Costly Line Changes to Reduce Bridge Delays," by M. Hirschthal, *Engineering News-Record*, May 17, 1928, p. 779.

CHAPTER XV

MISCELLANEOUS GENERAL DETAILS

General details applicable to all types of reinforced-concrete bridges are treated in this chapter. They cover:

Effect of temperature changes and shrinkage.
Expansion joints.
Bearings.
Waterproofing and drainage.
Railings.
Details pertaining to arrangement and placing of reinforcement.
Wearing surface.
Surface treatment of concrete.

Each of these subjects is treated under a separate heading.

EFFECT OF TEMPERATURE CHANGES AND SHRINKAGE

Variation of Temperature in Concrete Structures. — The extent of temperature changes to be provided for in the design of a reinforced-concrete structure depends upon the climatic conditions in the locality in which the structure is to be built. However, the range of temperature changes affecting the structure is not necessarily the range of the temperature changes of the air. Since concrete is a poor conductor of heat, there is usually an appreciable difference between the extreme high or low temperature of the air and the corresponding extremes in the body of the concrete structure. The extent of this difference depends upon the character of the structure; the mass composing the members of the structure; duration of the extremes of temperature; and finally upon whether or not the members are exposed to the direct action of the sun. It is obvious, for example, that the range of temperature changes in the railings and in other similarly exposed members would be much larger than in heavy girders under the bridge deck; and still larger than in an arch rib covered by fill.

The fact that the temperature in a concrete member may not be uniform throughout its mass complicates the exact determination of the effect of temperature changes. Not only simple expansion or contraction, but also warping and bending of the members, may be involved.

Reinforced concrete, however, is so well adapted to resist such secondary stresses, and normal factors of safety are so conservative, that

their treatment is seldom needed. A thorough analysis of this subject, therefore, is not included in this treatise.

For practical purposes, it is sufficient to assume uniform temperature changes throughout the body of the structure. In the latitude of New York City, an assumption of a change of temperature of $\pm 30°$ F. is considered as sufficient for ordinary concrete structures. For concrete railings this value should be increased by 50 per cent.

Effect of Temperature Changes. — In properly designed statically determinate structures, changes of temperature produce either expansion or contraction of their members. When one end of any member is free to move longitudinally, change in temperature lengthens or shortens the member, but does not produce any stresses. When both ends of the member are restrained from longitudinal movement, either by the supports or by other members, temperature changes produce stresses in the member and reactions upon the restraining members or supports. The forces generated by the temperature changes may be found from the following formulas.

The effect of temperature changes is largest for structures built either in extremely hot or cold weather, as then the effective variation is larger than when the structures are built in average temperatures.

Let α = coefficient of expansion per degree Fahrenheit (av. α = 0.000 005 5).

E = modulus of elasticity of concrete (av. E = 2 000 000 lb. per sq. in.).

t = effective variation of temperature in degrees Fahrenheit.

l = length of member.

A = cross section area of member in square inches.

Δ = change in length of member.

Then

Change in Length of Member Due to Temperature Change:

$$\Delta = \alpha t l \tag{1}$$

Δ is in the same units as the length l.

Stresses in Member When Ends are Restrained. — When the member is restrained from expansion or contraction, the rise of temperature produces compression in the member, and the fall of temperature tension. The unit stresses are given by the following formula.

Unit Stress in Member Due to Temperature Changes, Ends Restrained:

$$f = \alpha t E \quad \text{lb. per sq. in.} \tag{2}$$

Reaction Produced by Expansion or Contraction. — When a member is restrained from expansion or contraction, it exerts at both ends upon the restraining members reactions which are compression for the rise of

temperature, and tension for the fall of temperature. The magnitude of the reaction may be found from the following formula.

Reaction upon Restraining Member Due to Temperature Changes:

$$H = \alpha t E A \quad \text{lb.} \tag{3}$$

The force due to expansion or contraction exerted upon the restraining member or abutment is independent of the length of the restrained member, but the extent of the movement necessary to relieve the pressures is proportional to this length.

An idea of the magnitude of the reactions may be gained from the following: A 12-in.-thick slab for a change of 30° F. exerts upon the abutment a force equal to $H = 0.000\,005\,5 \times 30 \times 2\,000\,000 \times 144 = 47\,500$ lb. per lin ft. of width of slab.

When expansion and contraction are not properly provided for, changes of temperature may have the following effects upon the abutments or supports.

1. If the abutment or support is heavy and heavily reinforced, the force created by temperature changes may compress or lengthen the member, thereby producing in the member compression or tension stresses. In an extreme case, contraction may cause cracks in the member.

2. The force may bend or tip the abutment or pier.

3. It may tear or rupture parts of the abutment; or it may break or push out of line the restraining parts of the abutment. Ruptured edges are often the result of ineffective expansion bearings. Broken parapets are often caused when the expansion joint is not sufficiently wide.

Effect of Temperature Changes upon Rigid Frames. — In rigid frames, expansion and contraction of the various members composing the frame produce bending moments throughout the frame. Formulas for determining bending moments due to temperature changes are given for each type of structure in Chapters XII to XIV.

Effect of Shrinkage of Concrete. — Shrinkage to be considered in design of a reinforced-concrete structure is that which takes place after the concrete has set. The extent of shrinkage varies with the nature of the concrete. It is customary, however, to consider the effect of shrinkage as equivalent to the effect of a fall of temperature of 15° F.

EXPANSION JOINTS

Expansion joints are used to a large extent in reinforced-concrete structures; they serve the following purposes:

1. To permit free expansion and contraction due to temperature changes and shrinkage.

2. To separate, from each other, parts of the structure which differ

appreciably in dimensions, and, therefore, are expected to behave differently under the influence of the loads as well as under the influence of temperature changes.

3. To separate parts of the structure which serve different static purposes. Thus, in an arch bridge, expansion joints are provided to separate the arch ring from the spandrel walls, so as to permit free deformations of the arch ribs. Also, the deck of an arch bridge is separated from the deck of approaches. Another example is the vertical expansion joints separating an abutment from its wing walls.

4. To separate parts of a structure built at different times, in which case they should extend through the foundations.

5. To change a multi-span structure into statically determinate parts, as in the case of the cantilever bridges described in Chapter VII, p. 130.

The separation between the ends of two adjoining girders, or between the end of a girder and the parapet wall of the abutment, consists of vertical expansion joints of proper width. These may be filled with plastic material to protect them from moisture; or the open joint may be covered at the top and the sides as shown in Figs. 57, p. 133, and 62, p. 145.

Expansion joints between ends of simply supported girders are shown in Fig. 36, p. 90.

Spacing of Expansion Joints. — In a structure consisting of one-span simply supported girders, a vertical expansion joint is provided at the expansion end of each girder.

In continuous girder bridges of two or three spans, expansion joints are required only at the ends. For a long structure, consisting of a larger number of spans, the structure should be separated by vertical expansion joints into sections not more than 250 ft. long.

Flat-slab structures up to 350 ft. in length may be built without expansion joints. Longer structures should be properly divided into separate sections by vertical expansion joints. These may consist of double columns resting on a common foundation, one on each side of the expansion joint. In another arrangement, at each side of the joint the flat slab is provided with a cantilever of a length equal to about one-fifth of the span. This requires that the spacing of columns in the panel in which the expansion joint is located should be only about two-fifths of the regular column spacing in other panels.

BRIDGE BEARINGS

In American practice, reinforced-concrete girders usually rest upon concrete abutments or piers built of concrete of practically the same strength. The bearing area of the girders is therefore sufficient to trans-

mit the loads to the support without exceeding the allowable stresses. There would be no need of special bearings, were it not for the necessity of providing for the movements at the bearings described under separate headings.

In European practice, abutments and piers are often built of masonry, brickwork, or lean concrete. To transmit the pressures from the girder to the supports without exceeding the allowable stresses in the weaker material composing the supports, it is necessary to cover the bridge seat with a properly designed reinforced-concrete distributing slab.

Movements at Bearings. — At the bearings of a bridge one or several of the following three movements may have to be provided for:

1. Longitudinal movement due to expansion and contraction.
2. Rotating movement due to the deflection of the girders.
3. Vertical movements due to the deflection or settlement of the structure supporting the girder.

Longitudinal Movement. — To take care of expansion or contraction, a simply supported girder span requires at one end a bearing at which longitudinal movements can take place. With efficient expansion bearings, practically no stresses are developed in the girder due to temperature changes.

In continuous girders, all bearings, with the exception of one, should be expansion bearings to take care of expansion and contraction. A girder of two spans should have a fixed bearing at the center support and expansion bearings at the abutments.

In a bridge consisting of three spans, it may be permissible to use fixed bearings at both interior piers (instead of only at one pier), and to provide expansion bearings at the abutments only. In such case expansion and contraction of the center span are taken care of by slight longitudinal movements of the tops of the piers.

In a bridge consisting of four spans, the bearing at the center pier should be fixed, and the other bearings should be expansion bearings. Occasionally, when the first and last piers are slender and elastic, it may be possible to provide expansion bearings only at the abutments. The expansion and contraction of one span at each side of the center is then taken care of by movements of the tops of these slender piers.

Rotating Movements at Supports. — A simply supported girder under load deflects, the bottom assuming the shape of a concave curve. The bearings at both ends must then rotate to assume a position tangent to the deflection curve.

When the girder is provided with hinged bearings, the bottom adjusts

itself very readily to the deflection curve. When, on the other hand, the girder rests upon a horizontal unyielding surface, the bottom can adjust itself to the deflection curve only by rotating about the edge of the support, thereby transmitting the full reaction to the edge, which may cause crushing of concrete along the edge. To prevent injury to the bridge seat, it is advisable to make proper provisions at the support for the rotating movements.

Vertical Movement at Supports. — In ordinary cases, vertical movements at the supports may be caused by settlement of foundations or by vertical expansion and contraction of the supports. Also, where girders are supported by flexible structures, as is the deck of an arch bridge, the deflection of the supporting structure may cause vertical movements of the ends of the girders. A properly designed expansion joint should take this possibility into consideration.

BEARINGS FOR SLAB BRIDGES

For slab bridges and girder bridges of small spans, usually no special expansion bearings are provided. Often, the slabs are doweled to the abutment, as explained more fully on p. 416.

To take care of the rotating movement, each bearing area should be beveled or rounded at the edge. Also several layers of roofing felt should be placed between the slab and the abutment.

When it is desired to take care of the horizontal movement of the slab at the support, a two-plate bearing, shown in Fig. 162, p. 379, may be used. This design was used in a deck of an arch bridge.

Friction and danger of rust are materially reduced in the design shown in Fig. 163, p. 379, where the bottom plate of the bearing is provided with a $\frac{1}{4}$-in. bronze bearing plate which is polished at the top and covered with graphite grease before placing.

These bearings take care only of horizontal movements, and are effective only when the bearing areas are perfectly level and in full contact at all points.

BEARINGS FOR GIRDERS

Simply supported girders as well as continuous girders should be provided with fixed and with expansion bearings arranged as explained on p. 377. Fixed bearings should have provision for rotating movements, whereas the expansion bearings should be free to move longitudinally and also be free to rotate.

Roller Bearings. — The most effective expansion bearing is the roller bearing shown in Fig. 63, p. 147, which has perfect provision both for longitudinal and for rotating movements. Usually two rollers per

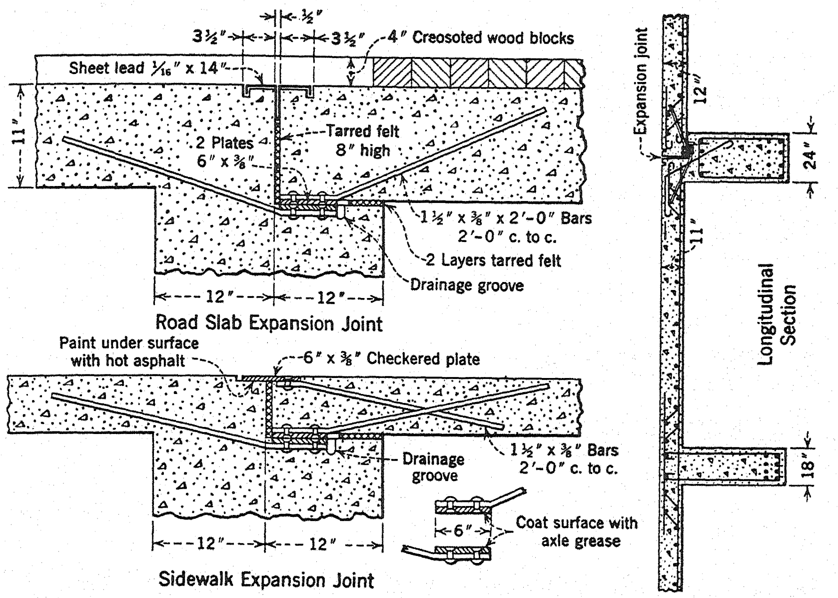


FIG. 162.—Expansion Joint and Bearing for Deck Slab of Arch Bridge. (*See p.* 378.)

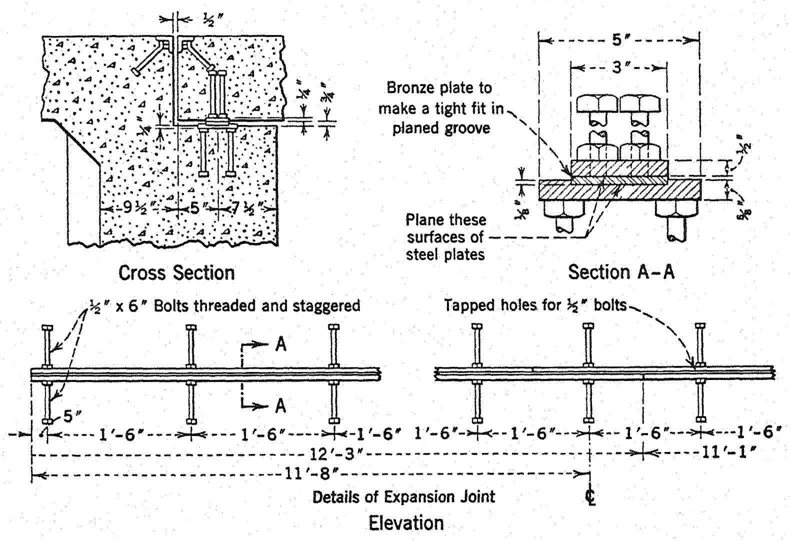


FIG. 163.—Expansion Joint and Bearing, Truckee River Bridge. (*See p.* 378.)

bearing are sufficient to take care of the reactions. The design of the roller bearing is the same as for steel bridges.

Roller bearings are expensive and therefore can be used only for long span girders.

Steel Rocker Bearings. — A somewhat less effective expansion bearing is a steel rocker bearing consisting of two horizontal steel plates with a cast-steel rocker placed between them. Such a bearing is shown in Fig. 36, p. 90.

The dimensions of the plates depend upon the reaction of the girder and the allowable unit bearing stress in concrete. The thickness of each plate is computed by considering it as supported along the center line by the rocker and loaded by the reaction of the concrete. When necessary, the plate may be stiffened by ribs or angles which are then imbedded in the concrete. The diameter of the rocker depends upon the reaction per unit length of the rocker.

The rocker should be turned to an exact diameter, and the contact surfaces should be polished to reduce friction. To prevent lateral displacement of the rocker, the plates and the rocker, when assembled, are provided with loose-fitting plugs extending from the plates into the rocker. After erection, the exposed areas of the bearing, with the exception of the areas of contact, are painted to prevent corrosion.

The rocker is placed either partly or wholly in a pocket especially prepared for it in the abutment or pier. This pocket, after erection of the rocker, is usually filled with asphalt to protect the bearing from moisture. The disadvantage of this is that asphalt may harden and thus reduce the efficiency of the bearing by preventing free movement of the rocker.

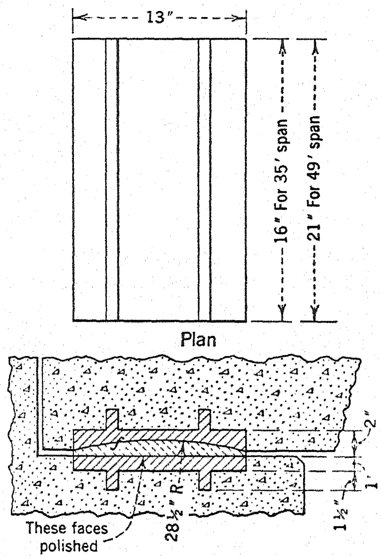
Sliding Shoe. — A fairly effective bearing is shown in Fig. 57, p. 133. It consists of three parts, one of which is attached to the support, the second is attached to the underside of the girder, and the third is placed between the first two parts. The surfaces of contact of the bottom part and the intermediate part are polished, to make possible a sliding motion. The top surface of the intermediate part and the bottom surface of the top part are cylindrical, thus permitting a rotating movement of the end of the girder due to deflection. The two parts are kept in alignment by a loose-fitting plug.

For another design of a sliding shoe see the bearing shown in Fig. 47, p. 116.

Three-Plate Bearing. — In the expansion bearing shown in Fig. 164, p. 381, developed by the Highway Department of the State of Washington, longitudinal and rotating movements are made possible by three plates of phosphor bronze. The intermediate plate is loose, and it is

provided with a polished horizontal sliding surface at the bottom; at the top the convex surface of the intermediate plate fits into a convex surface of the top plate. Both surfaces are polished, and they take care of the rotating movement of the girder end.

Two-Plate Bearings with Intermediate Lead Sheet. — In this type of design, a thin lead sheet is placed between two steel plates. The steel plates are attached to the bottom of the girder and to the top of the bridge seat, respectively. The lead sheet is either placed loose between the two plates, or it is fastened to the bottom plate by copper rivets. Longitudinal motion in this bearing is produced by sliding of the upper plate upon the lead sheet. Rotating movements are produced when the top bearing plate of a loaded and deflected girder exerts a larger pressure at its inside edge than at the outside edge. The lead sheet, being plastic, assumes the shape of a wedge with the top tangent to the deflection curve.



From Engineering News-Record.

FIG. 164.—Three-plate Expansion Bearing, Tokul Creek Bridge. *(See p. 380.)*

The dimensions and the thickness of the two steel plates are determined by the allowable bearing stresses on concrete. The transverse dimension of the lead sheet is the same as that of the two bearing plates, but longitudinally with the bridge the dimension is usually from one-third to one-half of the dimension of the steel plates. The thickness of the lead sheet depends upon the desired magnitude of the rotating movement. For small spans, the thickness may be as small as $\frac{1}{8}$ in. For large rotating movements, the thickness of the lead sheet may be as great as 1 in.

The use of two plate bearing with a lead sheet is shown in Fig. 62, p. 146.

REINFORCED-CONCRETE ROCKER BEARINGS

In recent years, in continental Europe reinforced-concrete rocker bearings have been used to considerable extent for girder bridges of long spans. They are cheaper than steel roller bearings, and when properly

designed they are fully as effective. The cost of maintenance of such bearings is negligible in comparison with that of the steel bearings.

Early Designs of Rockers. — In the early designs of reinforced-concrete rockers, the top and the bottom of each rocker were provided with metal plates having cylindrical surfaces of contact; the rocker was placed upon a metal plate to distribute the pressures to the concrete of the pier; and finally a metal plate was placed between the top of the rocker and the bottom of the girder.

Modern Design of Rocker Bearings. — Modern designs of rockers are cheaper and equally effective. The metal plates have been eliminated, and the bearing consists of three parts: a rocker block and two strips of lead sheet of proper thickness. A typical design is shown in Fig. 165, p. 382.

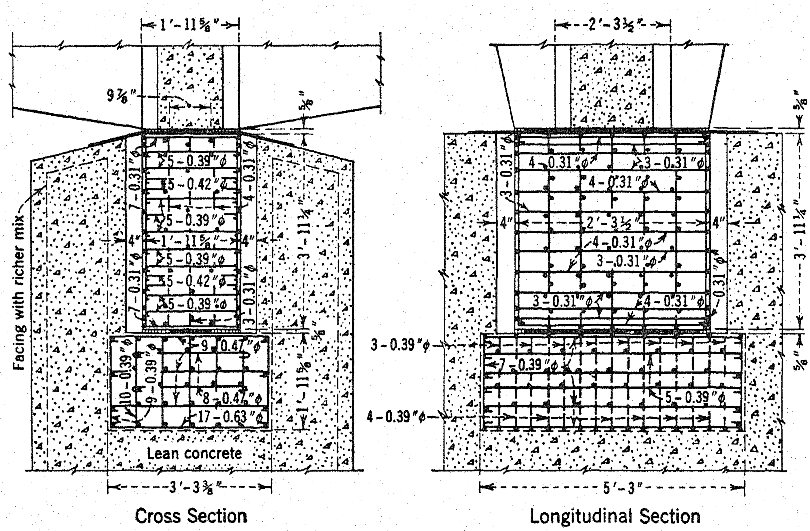


FIG. 165.—Reinforced Concrete Rocker Bearing, Lindau Bridge. (*See p.* 382.)

The concrete rocker block is rectangular in cross section, and is from 2 to 4 ft. high, depending upon the span of the girder. It is built in place, of very rich concrete, and it is reinforced with vertical bars and hoops. Both the top and the bottom of the rocker block are level.

A lead sheet is placed at the bottom between the rocker block and the pier, and another at the top between the rocker block and the bottom of the girder. The transverse dimension of the lead sheet is substantially the same as that of the rocker block. The dimension in the longitudinal direction of the bridge, however, is much smaller than the dimension of the rocker, usually not more than one-half of the rocker dimension.

The balance of the surface of the rocker block is covered with a proper elastic filler. In Europe, treated cork sheets are used for the purpose.

When the pier is built of leaner concrete than the girder, a distributing block of rich concrete is supplied upon which the bottom lead sheet is placed, and which distributes the pressures from the lead sheet to a sufficiently large area of the pier.

The rotating and longitudinal motion of the rocker are obtained in the following manner: To rotate, the girder compresses the lead sheet along the inside edge more than along the outside edge, and the underside of the girder assumes the desired inclination.

To move longitudinally, the girder causes the rocker block to rotate upon the lead sheets. Thus, to expand, the lower lead sheet is compressed along its outside edge which makes the block tilt outside. At the same time, the upper layer of lead is compressed along the inside edge. When the angle of inclination of the wedge formed by the lower lead layer is β, and the height of the rocker block is h inches, the longitudinal movement of the end of the girder is equal to $h \sin \beta$. From this it follows that, the higher the rocker block, the more effective is its action, because a smaller angular compression of lead is required to get the desired longitudinal motion.

The rocker is usually placed within a well provided for it in the pier. Sufficient clearance is provided between the walls of the well and the faces of the rocker block to permit the desired motions. The well should be properly protected from moisture and also properly drained.

In the bearing, the area of contact of the lead is determined by dividing the load to be carried by the allowable unit bearing pressure on concrete. Since the total area of the block is much larger than the actual area of contact, the allowable bearing pressure is appreciably larger than the maximum allowable fiber stress. It is safe to accept the allowable bearing stress as equal to the fiber stress multiplied by the third root of the ratio of the area of the block to the area of contact. Thus for an area of contact equal to one-third of the total area, the bearing stress may be 45 per cent larger than the allowable fiber stress.

The amount of horizontal reinforcement to be used in the rocker block longitudinally with the bridge may be determined by assuming that the concentrated reaction develops in the block a tension equal in magnitude to one-third of the total reaction acting on the rocker. This tension should be resisted by hoops or spirals spaced throughout the height of the block and uniformly distributed along the block. Only the prongs in the direction of the bridge are considered as effective in resisting this tension.

Columns Hinged at Top and Bottom. — Rocker effect also may be obtained by providing the supporting columns on one side of the span with hinges at top and bottom. The hinges may consist of crossed diagonal bars extending from the column into the girder at the top and into the foundation at the bottom. Lead plates should be placed on top and bottom of the columns.

DRAINAGE

It is of great importance for the durability of the structure to provide proper drainage, i.e., means for a speedy removal from the structure of atmospheric moisture.

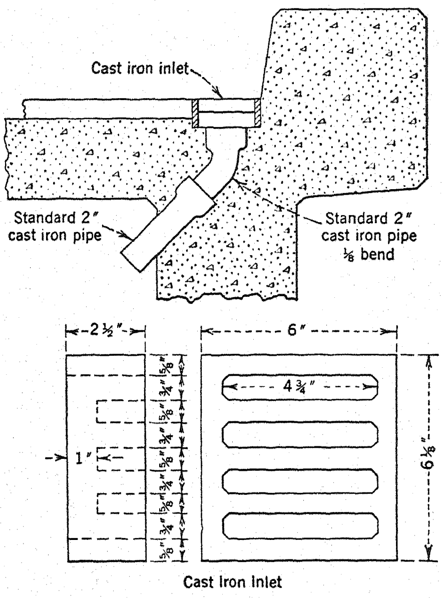


FIG. 166.—Drain Details, Trestle across Savannah Delta. (*See p.* 385.)

In small bridges, proper drainage may be obtained by sloping the top of the bridge on both ends toward the abutments. The water is then carried behind the abutments where proper provision for its disposal should be made.

In larger structures, provision for drainage should consist of drain openings through the slab placed at frequent intervals. The water may be emptied through the drain openings directly into the air; but where the bridge spans a street, the water is carried by downspouts to the abutments or piers where it is brought down to the sewers or to the road drains.

To facilitate drainage, the roadway is provided with a crown. The required transverse slope of the roadway depends upon the kind of pavement. For asphalt pavements, a slope of 1 in. in 6 ft. is sufficient; for macadam pavements, 1 in. in 2 ft. may be required for proper drainage. The sidewalk slab should slope toward the gutters at a rate of 1 in. in 5 ft. Water should never be allowed to run off the bridge along its sides.

Drain outlets should be arranged with due regard to expansion joints. It is important to prevent water from dripping through expansion joints. For this purpose, they should be protected by dikes, and also the top of the slab should slope away from the joints toward the drain outlets.

The arrangement and design of drain outlets are shown in Fig. 166, p. 384, Fig. 167, p. 385, and Fig. 168, p. 386.

To protect outside surfaces from dripping water from the deck, proper drips should be provided as shown in Fig. 167, below.

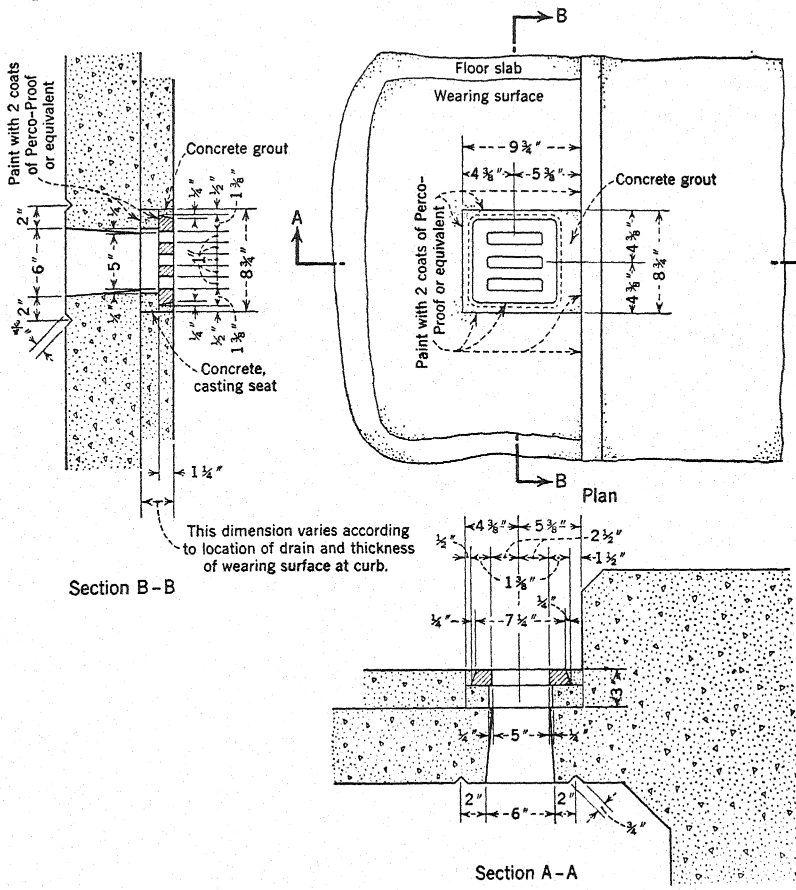


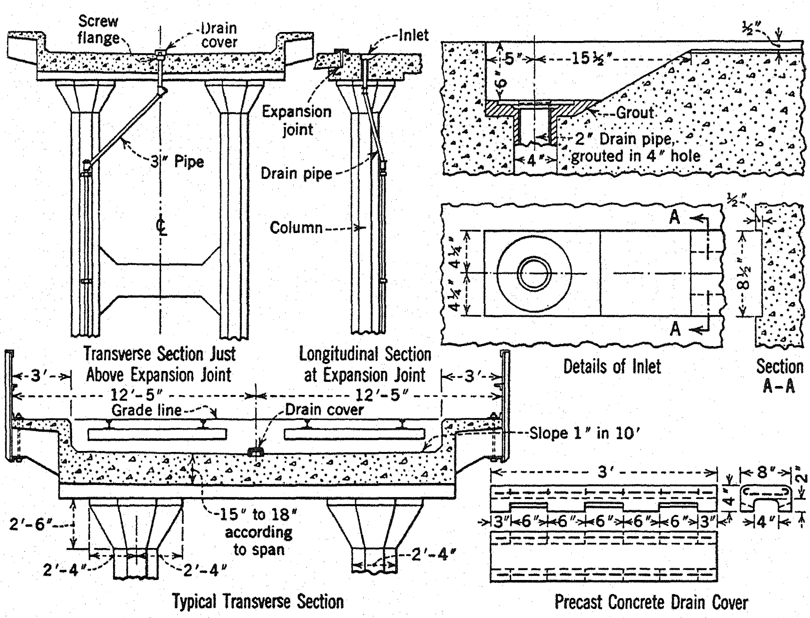
FIG. 167.—Drain Standards, Michigan Highway Department. (*See p.* 385.)

WATERPROOFING

Purpose of Waterproofing. — Waterproofing is intended to prevent rain water from permeating through any part of the structure. Even when concrete is substantially impervious, water may find its way through contraction cracks or through construction joints. Sometimes, also, porous places occur in otherwise good concrete. Prolonged seepage of water may affect the durability of the structure. Seeping water washes away free lime; it tends to destroy the cement film around the

reinforcement, thus exposing it to rust; and finally, in conjunction with the action of frost, it may cause disintegration of concrete.

In designing a bridge, therefore, proper attention should be given to waterproofing, and such means should be used as are warranted by the nature of the structure and by the climatic conditions to which it will be exposed. Plans should show clearly the desired means of waterproofing, including such details as flushing and arrangements at expansion joints and at drains. The specifications should cover fully the materials to be used for waterproofing as well as the method of their application.



From Engineering News-Record.

FIG. 168.—Drainage Details, Flat-Slab Railroad Bridge. (*See p.* 385.)

Methods of Waterproofing. — The importance of waterproofing is coming to be recognized more and more, but at present there is no uniformity in practice as far as this feature of design is concerned. Actually, it would not be possible to formulate rules applicable to all structures and to all localities.

It is clear that the necessity for waterproofing is much less in a country with small rainfall and no snow or ice, and where it is possible to drain the bridge deck in comparatively short time, than in a country with prolonged rainfalls and where even the best means of drainage may become temporarily ineffective on account of ice and snow. Also, the waterproofing requirements are different in structures with a solid deck

of concrete or asphalt pavement where the rain water drains rapidly before it has a chance to soak in; and in structures with macadam pavement, railroad ballast, or other permeable pavements where water may accumulate in larger quantities and remain on the structure a longer time.

The cost of proper waterproofing is comparatively small, and its ratio to the cost of the structure decreases with the increase of the unit cost of the structure.

Waterproofing as used in practice may be divided into three general groups: (1) integral waterproofing; (2) surface treatment; (3) membrane waterproofing.

Integral Waterproofing. — Dense concrete which is practically impermeable may be obtained without admixtures by proper selection and grading of aggregates, use of a sufficient amount of cement in the concrete mixture, sufficiently long mixing, control of water content, and proper care in placing and spading of concrete. Obviously the forms should be tight so as to prevent the escape of cement. Sufficient tamping and spading give the structure a proper skin surface which improves its watertightness. The use of vibrators may be of advantage.

Various types of integral waterproofing are on the market designed to improve watertightness of concrete by the addition to the concrete mixture of compounds which are claimed to increase the density of concrete either by chemical or physical action. Some of these have merits.

Reliance upon dense concrete or integral waterproofing is sufficient only where it is possible to guard against cracks due to contraction and shrinkage, or to opening of construction joints.

Surface Method. — In the surface method, the object is to cover the surface of the concrete and to fill the pores and voids with water-repelling material. The waterproofing may consist of a coat of cement mortar properly bonded to the structure, or of a coating of asphaltic compounds applied in the same manner as paint. Several coats of the asphalt are usually required to get a proper thickness of the protective cover.

The surface method is effective only where it is possible to prevent cracking of the surface. Once cracks are developed in the protective cover, the effect of the waterproofing is practically destroyed.

Surface treatment is recommended only for the inside surfaces of the abutments and retaining walls. For decks of bridges, this method is not reliable.

Membrane Waterproofing. — Membrane waterproofing is the most effective method of protecting the decks of bridges from the influence of water. It consists of several layers of membrane imbedded in and protected by coats of asphaltic compounds which act as binders for the membrane.

The membrane is usually a fabric saturated in asphalt. To serve the function for which it is intended, the fabric, after saturation, must be flexible within the range of temperature from 0 to 250° F., and it should bend around a $\frac{3}{4}$-in. diameter without cracking. It should have a tearing resistance between 40 and 50 lb. per in., and should stretch not less than 10 per cent in either direction without tearing. For membrane, a cotton fabric, or a wool felt is customarily used. A very effective but also very expensive membrane consists of lead sheets about 0.08 in. thick.

The binder, which is the coating applied on the top of the concrete and between the layers of the membrane, actually performs the main function of waterproofing, while the membrane keeps the binder intact and prevents it from cracking. To be effective, the binder must be elastic at all temperatures to which the structure may be exposed. It must not be brittle at low temperatures, nor flow at high temperatures. It must adhere readily to the concrete surface and to the membrane. It should have lasting qualities and not be affected by any chemical action to which it may be subjected. Finally, it should be easy to apply. The best material for the binder is asphalt. Coal-tar pitch products are also used for the purpose, but they are not considered a desirable material for the binder.

Effective waterproofing consists of three coats of the membrane and four coats of the binder applied hot. Often, a priming coat consisting of diluted asphalt is first applied on the concrete surface; and upon this, when dry, is applied the first coat of the binder. The first ply of the membrane is then placed and is covered by a coat of the binder. This performance is repeated until all plies of the membrane are in place, and then the last coat of the binder is applied.

The success of the waterproofing depends not only upon the materials but also in a very large measure upon the workmanship in placing the membranes and applying the coats of binder. It is very important to prepare properly the concrete surface to be waterproofed. It should be floated to a smooth surface, and all projecting aggregates must be removed because they are likely to pierce or injure the membrane. Before applying the priming coat or the first coat the surface should be cleaned and all dust removed. Waterproofing should not be applied in wet or misty weather; neither should it be applied on wet or moist surfaces. The membrane should be placed without folds and creases. A proper method of lapping the membrane should be worked out beforehand, and it should be strictly adhered to during construction.

As mentioned before, to get satisfactory results the details should be carefully worked out and shown on the plans. It must be remembered

that waterproofing does not take the place of drainage, since, with improper drainage, even the best waterproofing method may become ineffective. Very important details of waterproofing are the flushing of the membrane and the arrangements at expansion joints and at the drains. It should be remembered that the surface of waterproofing must be unbroken, because any break in the waterproofing either by design or by accident might admit water under the waterproofing and thus destroy its effect. The expansion joints must be properly protected to prevent water from seeping to the underside of the structure or to the abutments or piers. All pockets such as those used for rocker bearings should be protected from water.

Several examples of provisions at the expansion joints are shown in Figs. 162, p. 379; 62, p. 145; and 185, p. 432. The object there is to supply waterproofing without interference with free expansion and contraction of the structure. This may be accomplished by overlapping of the membrane, each of which is pasted to the structure at one side of the joint but is free at the other side. Copper and lead sheets are also often used to cover expansion joints.

Flushing of waterproofing should receive proper attention. In railroad structures, the waterproofing membrane is flushed along the parapet as shown in Fig. 70, p. 160. In highway bridges, it is often difficult to terminate the membrane elsewhere than on the face of the curb. Where possible, this should be avoided.

Protection of Waterproofing. — Before placing the pavement or the ballast, the waterproofing of the deck should be protected by one of the following methods: (1) By a thin layer of cement mortar, 1 : 3 mix, placed on the top of the waterproofing. This layer is often reinforced with wire netting. (2) By a layer of common brick or asphalt bricks on the top of the waterproofing laid with open joints which are then filled with asphalt.

Examples of Waterproofing. — Examples of waterproofing are shown in Fig. 16, p. 41; Fig. 69, p. 158; Fig. 70, p. 160; and Fig. 175, p. 407.

The arrangement at expansion joints is illustrated in Fig. 57, p. 133; Fig. 62, p. 145; and Fig. 185, p. 432.

The method of flushing the membrane is shown in Fig. 70, p. 160.

RAILINGS

Railings for reinforced-concrete structures may be of iron, steel, wood, or reinforced concrete. In America, reinforced-concrete railings are most common. Pipe railings are used occasionally, and less important structures are sometimes provided with timber railings. In Europe,

iron railings are quite common, but even there important structures are provided with reinforced-concrete railings.

Strength of Railings. — Railings must be fastened securely to the structure, and they must be strong enough to resist the impact exerted by a skidding automobile. They must be at least strong enough to resist a horizontal pressure of 170 lb. per lin. ft. exerted at the top of the railing. Experiments show that such pressure may be exerted by a crowd of people leaning upon the railing.[1]

Steel and Iron Railing. — Iron railing such as is used in Europe is shown in Fig. 70, p. 160. Details of connection of the railing to the structure are shown in Fig. 147, p. 324.

A design of pipe railing in which the posts and rails are of steel pipe is shown in Fig. 69, p. 158. In Fig. 169, p. 390, is shown a combination of steel pipe railings with concrete posts. Both these designs are used occasionally in America.

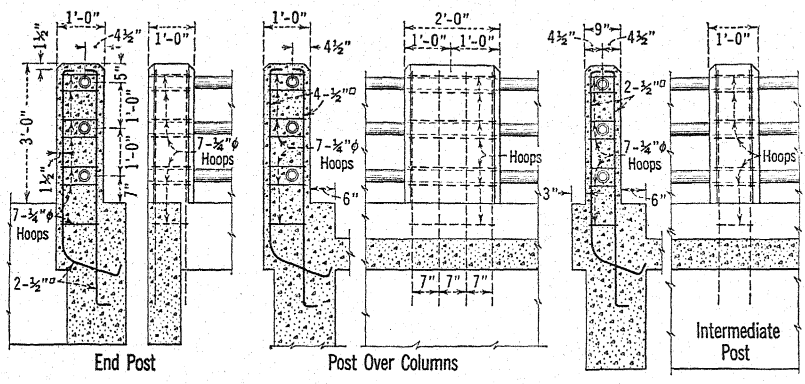


FIG. 169.—Pipe Railing, Concrete Posts. *(See p. 390.)*

Timber Railings. — In the simplest arrangement, timber railings consist of heavy posts about 8 by 8 in. in cross section spaced about 6 ft. on centers, and of one rail 6 by 6 in. in cross section fastened to the posts by carriage bolts so that its top surface is about 1 ft. 9 in. above the top of the curb. The posts are dapped $2\frac{1}{2}$ in. to receive the rail, and they are placed and grouted in holes provided for them in the curbs of the structure. Steel loops of $\frac{1}{2}$-in. bars are usually placed around each of the holes to prevent cracking of the concrete. The holes are painted inside with asphalt for waterproofing. The posts and rails may be built of Douglas fir. The timber railings should be painted.

[1] See "Strength of Concrete Bridge Rail and Pressure of Crowd," *Engineering News-Record*, Jan. 29, 1925, p. 200.

Reinforced-Concrete Railings. — Reinforced-concrete railings are durable, give pleasing effects, and when properly designed and constructed require little maintenance. A good idea of the appearance of a concrete railing may be had from Fig. 170, p. 391, illustrating the San Gabriel River bridge in California.

Railings are subjected to appreciable changes of temperature, much larger than the rest of the structure. Therefore, it is very essential to make proper provisions for expansion and contraction. In simply supported structures, expansion joints in the railings should be provided

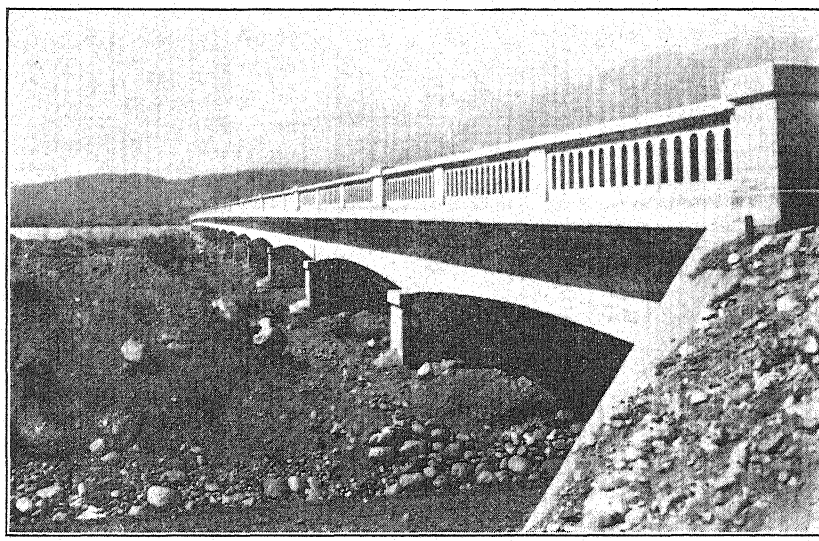


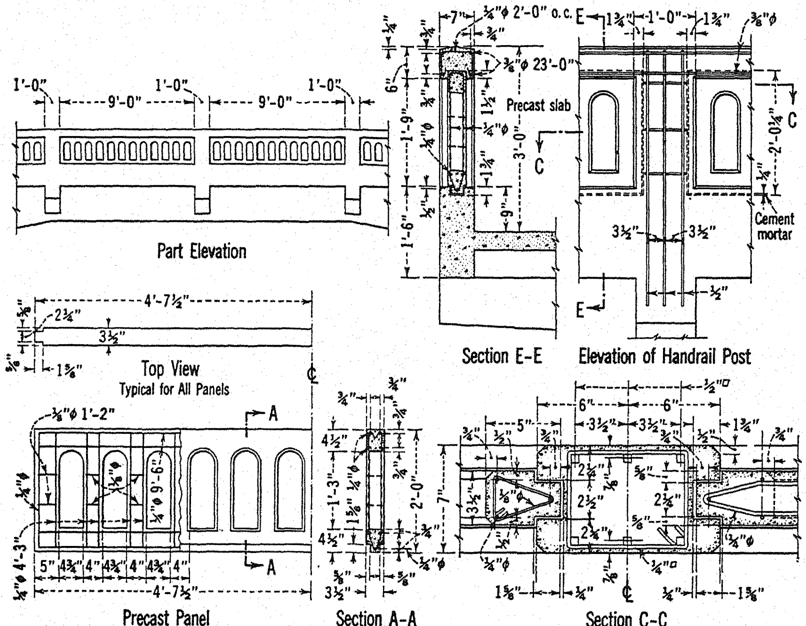
FIG. 170.—View of Bridge with Reinforced Concrete Railings. (*See p.* 391.)

not only at the expansion joints in the deck, but also at one or more intermediate points. In continuous structures, closely spaced expansion joints in the railing should be used throughout the structure. In some designs, sliding expansion joints are used. In recent years railings have been built with closely spaced vertical expansion joints, which entirely separate from each other the adjoining sections of the railing.

Reinforced-concrete railings may be divided into three general groups: (*a*) solid panel type; (*b*) spindle or baluster type; (*c*) rail type. With the exception of the cases where the railing forms an integral part of the bridge, as in Fig. 62, where the facia girder forms the railing, it is built separately and after the structural concrete has hardened, the forms have been removed, and the deflection under the dead load has taken place.

Solid Panel Type of Railing. — This type of reinforced-concrete railing consists of vertical posts securely fastened by dowels to the structure, horizontal rails and solid panels filling the space between the posts and the railing. The panels may be precast, and the posts and rails built in place.

A variation of the solid panel railing is shown in Fig. 171, p. 392, where the precast panels are provided with openings.



Engineering Department, C. M. St. P. & P. R. R.

FIG. 171.—Railing with Precast Panels. Bridge at Monroe, Wis. (*See p.* 392.)

Spindle and Baluster Types. — In the spindle type, the railing consists of a base, pilasters, spindles and rails. The spindles are usually precast, while the base, the pilasters, and the posts are built in place.

In the standard railing of the Michigan State Highway Department, shown in Fig. 172, p. 393, the spindles are rectangular in cross section. Each spindle is reinforced with one $\frac{1}{2}$-in. round bar placed in the center which projects at the top 3 in. into the rail, and $9\frac{1}{2}$ in. at the bottom into the base. The bottom of the bar is threaded and provided with a nut which is used for adjustment during erection.

The base of the railing is reinforced with four $\frac{1}{2}$-in. longitudinal bars and is tied to the structure by $\frac{1}{2}$-in. round dowels spaced about 18 in. on

centers. The pilasters are reinforced by $\frac{3}{4}$-in. round bars extending from the structure into the pilasters. Finally the rails are provided with six $\frac{1}{2}$-in. round horizontal bars. Between the pilasters and the rails, vertical construction joints are provided which are painted with white lead and allowed to dry before casting the rail.

An example of railings with precast balusters is shown in Fig. 173, p. 394. This railing was used in the Savannah River bridge in Georgia described on p. 89.

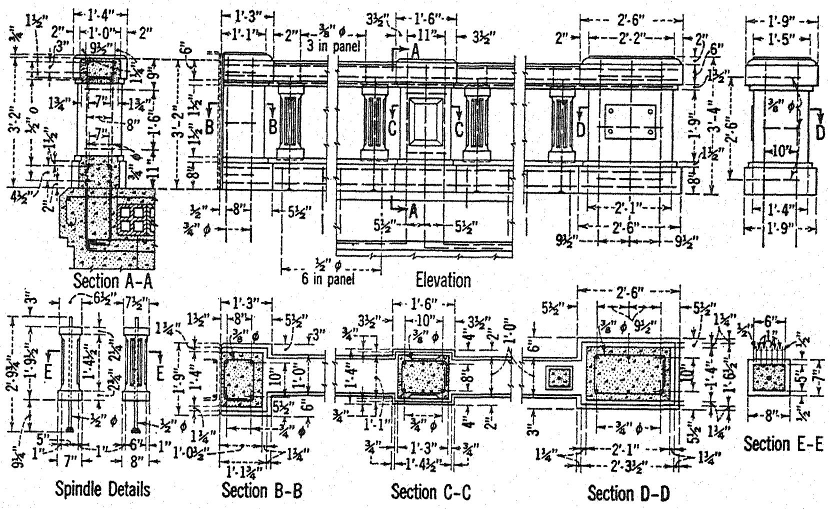


FIG. 172.—Railing Standards. Michigan Highway Department. (*See p.* 392.)

Two-Rail Type. — In this type, the railing consists of posts, a top rail, and an intermediate rail. The design shown in Fig. 174, p. 395, was developed by the Bureau of Highways and Bridges of the City of Seattle, Washington.[2]

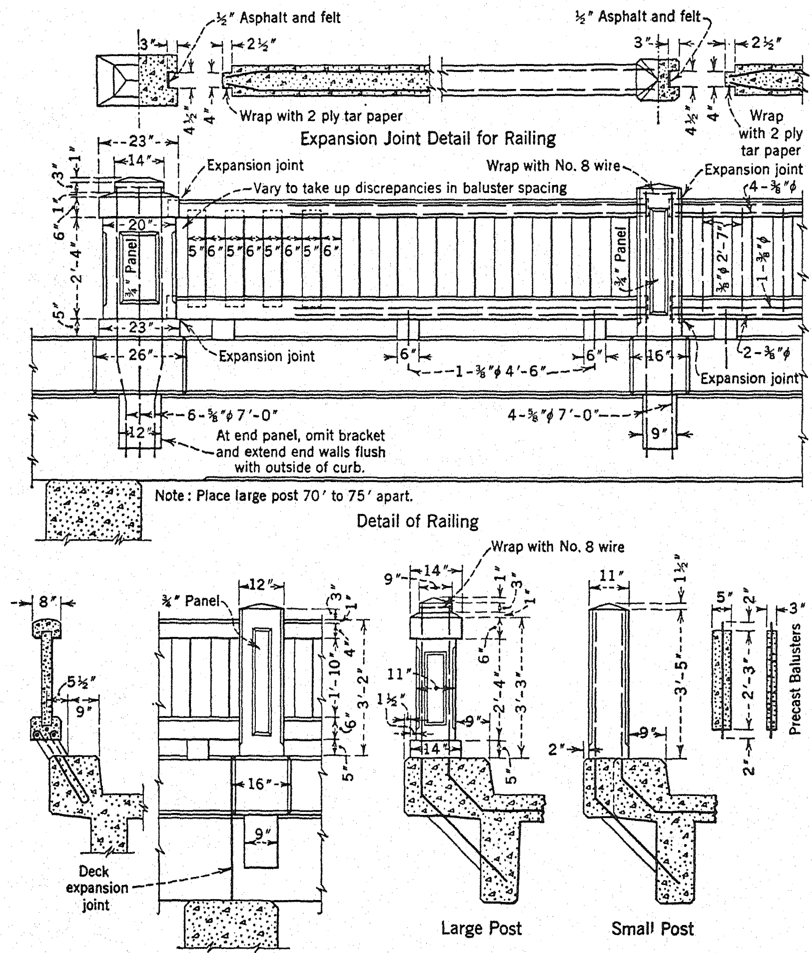
REINFORCEMENT DETAILS

In this section, information is given relative to the proper protection of reinforcing bars; minimum spacing of parallel bars in concrete members; spacing of parallel layers of bars; requirements as to proper splicing and anchoring of bars; and finally recommendations as to bending of bars in beams and girders.

Considerations Affecting Thickness of Protective Cover and of Spacing of Bars. — The minimum spacing of parallel bars, and the minimum thickness of the protective cover, are affected by theoretical and by practical considerations.

[2] See *Engineering News-Record,* June 13, 1918, p. 1133.

Reinforcing bars are brought into action by the bond between the steel and concrete. It is self-evident that the thickness of concrete in which the bar is imbedded must be sufficient on all sides to prevent splitting or shearing off of the concrete between and below the bars.



S. B. Slack, Engineer.

FIG. 173.—Railing for Bridge over Savannah River. *(See p. 393.)*

Tests show that a clear spacing between parallel bars equal to one diameter of the bar and a protective cover of the same thickness are more than sufficient to insure cooperation between steel and concrete. These dimensions, however, are usually insufficient for the following practical reasons: (1) This protective cover may not be sufficient to protect the

concrete from external influences. (2) The spacing of parallel bars would not be sufficiently large to allow free flow of concrete under and around the bars.

The values determined by practical consideration are discussed under separate headings.

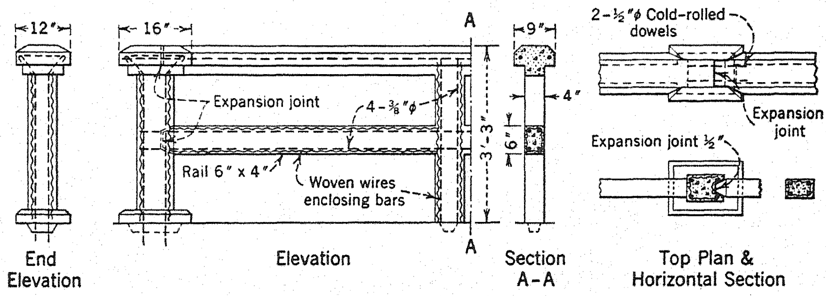


FIG. 174.—Two-Rail Type of Railing, City of Seattle. (*See p.* 393.)

Protective Covering for Reinforcement. — The thickness of the protective covering of reinforcement, i.e., the distance from the outside face of the reinforcement to the face of concrete, must be sufficient to protect effectively the steel from the effect of the external agencies acting upon the structure. In bridge design, the possible harmful external agencies are: moisture and frost; gases from locomotives; in maritime localities, salt air; in smelting regions, the gases from the smelting furnaces; and finally in some localities, alkali. In substructures, the effect of water and of the salts, acids and other impurities which it may contain in solution comes into consideration.

All these agencies must be guarded against because they may contribute toward rusting of the reinforcement; and rust is one of the most harmful destructive agents as far as concrete is concerned. When steel imbedded in concrete rusts, it expands and exerts against the protective cover a force which is often sufficient to crack the concrete cover. Progressive rusting of steel then follows, which aided by moisture and frost may be sufficient to disintegrate the concrete. The effect of rust may be harmful even when it affects only nails imbedded in concrete, exposed ends of chairs and spacers, etc. Even when rust has not sufficient force to crack the concrete cover, it is harmful because its stains discolor the concrete surface very badly.

The efficiency of concrete cover in protecting reinforcement depends not only upon its thickness but also upon the density of concrete. A thin cover of dense concrete is more effective than a thick cover of porous concrete.

It is recommended that, before selecting the thickness of the protective cover to be used in any particular case, all conditions should be properly considered. It is obvious that a thicker cover is required for a low structure spanning a river, where the underside is constantly moist, than would be necessary in the immediate vicinity for a viaduct across a valley or street. In sections with very small rainfall and no frost, appreciably smaller thickness of the protective cover may be used than in a country with large rainfall and possible low temperatures. For a structure across tracks, larger protective cover is required if the particular structure is at, or near, a station or freight terminal, and therefore is exposed for considerable lengths of time to the action of gases escaping from the standing locomotives, than for a structure on the line where only the gases from passing locomotives need to be considered.

In the United States, larger thicknesses of the protective cover of concrete are generally used than is customary in Europe. This may be partly attributed to the greater care taken in Europe in procuring dense concrete and hard concrete surfaces.

The thickness of the protective covering for the steel, where no fire protection is involved, as recommended by the American Joint Committee on Standard Specifications for Concrete and Reinforced Concrete in 1938, is a minimum of 1 in. of concrete in members above ground such as beams, girders, slabs, and columns. When the members are below ground, as footings, base walls, or other supports, the minimum thickness of the protective concrete is not less than 3 in. to guard against penetration of water from the soil. Where no water can penetrate, as in the case of earth-imbedded members entirely protected from the weather and from ground water, a minimum of 2 in. is permissible.

In beams, girders, and columns, stirrups and hoops are usually permitted within the protective cover, provided that their diameter does not exceed $\frac{1}{2}$ in. The authors recommend, however, especially in structures exposed to the weather, such as bridges, a minimum of 1 in. outside of all steel reinforcement, while the principal reinforcement in small beams should have a minimum of $1\frac{1}{2}$ in. protection and in girders and columns at least 2 in.

In European practice, thicknesses in slabs as low as 0.4 in. (10 mm.) and in girders and columns from 0.8 to 1.0 in. are often used. For structures subjected to the effect of locomotive gases a protective cover of 1.4 in. is considered ample. Inspection of structures which have been in use considerable lengths of time seems to indicate that the above thicknesses are sufficient.

Means of Keeping Bars above Forms. — Positive means should be used to insure the required thickness of the protective cover. Working

drawings used in the field should show clearly in all cases the required distance from the face of the lowest layer of bars to the formwork, and also from the outside face of the end bar to the side of the form. Bottom reinforcement should be supported on concrete blocks. Steel chairs are not satisfactory in bridge construction because of the danger from rust of the exposed ends.

Top bars of the girders may be kept in position by short pieces of steel angles placed at right angles to the reinforcement and resting on concrete blocks on the slab forms. For heavy and complicated reinforcement, it may be advisable to use a framework built up of steel sections.

Column bars may be kept from the face of the formwork by means of concrete rings which are slipped over and tied to the vertical reinforcement. Spiral reinforcement should be kept in place by means of mechanical spacers which fix the pitch of the spiral.

Stirrups should be securely wired at one end to the tensile reinforcement and at the other end to special bars often called pencil bars. Outside legs of stirrups should be kept a proper distance away from the sides of the formwork. They should be prevented from bending during construction. Misplaced stirrups are very often a source of spalling of concrete and rusting.

Spacing of Parallel Bars in a Layer and Spacing of Layers of Bars. — In American practice, the spacing of the bars is made sufficient to permit free flow of concrete under and around the bars without the necessity of undue spading and tamping of concrete. The following rule is recommended:

The spacing of parallel bars should not be less than 1 in. nor less than $1\frac{1}{2}$ times the largest aggregate used for concrete. Neither should the spacing be less than $1\frac{1}{2}$ times the diameter for round bars and twice the side for square bars. Where possible, mechanical spacers should be used to insure proper spacing of bars.

Superimposed layers of parallel bars should have a clear spacing of not less than one diameter of the heaviest round bar or the side of the heaviest square bar in the layer, but it must not be less than 1 in. It is preferable to limit the number of layers of bars to three. Where a larger number of layers is unavoidable, the concrete should be placed with greater care to get proper imbedment for the reinforcement. The layers should be separated by positive means such as bolsters or pieces of reinforcing bars of proper diameter placed across the reinforcement and spaced not more than 5 ft. on centers. Each bar should be securely fastened to the spacer by annealed wire.

A design with several layers of bars is improved by bending up at least 60 per cent of the reinforcement at points where this is permitted by

bending moments. The bent bars are then anchored in the compression zone of the girder, which insures the development of tensile stresses in the bar even if for any reason the protective cover should become injured.

Superimposed layers of bars at an angle to each other, such as two layers of slab reinforcement in a slab reinforced in two directions, should be placed directly one on the top of the other. The bars should touch at the intersections and be tied together by annealed wire.

Splicing of Bars. — When it is necessary to splice reinforcing bars, the following general rules should be followed.

Splicing of bars at the points of maximum stress should be avoided. Splices should be staggered so as to splice only a small part of the total effective reinforcement at any one section.

Splicing may be accomplished by the following means: simple lapping; lapping with hooks; screw connections; and welding.

Bars of comparatively small diameter such as are used in slabs and walls may be safely spliced by straight laps. A simple lap of 40 diameters of the bar to be lapped is sufficient to develop in a bar a working unit stress of 16 000 lb. per sq. in. with the required factor of safety. For different unit stresses, the length of the lap should be changed proportionally. The lapped bars should be wired with annealed wire.

Bars of larger diameter, $\frac{7}{8}$ in. and over, used for main reinforcement of girder bridges, preferably should be provided in addition to the lap with hooks at the ends to be spliced. The small additional cost of the hooks is justified by the importance of the reinforcement for the safety of the structure.

Heavy bars, $1\frac{1}{4}$ in. in diameter and over, should not be spliced by lapping. An effective tension splice may be obtained by a screw connection in which the ends of the bar to be spliced are threaded and connected by a sleeve nut. The effective cross section of the sleeve nut should be equal to the cross section of the spliced bar. The length of the sleeve engaging each end of the bar should be sufficient to develop full strength of the bar. Since the threads in the bar reduce its effective area at the splice, the deficiency should be compensated by additional reinforcement at the splice. If, however, it is desired to provide full cross section of the bar at the splice, the ends should be thickened (upset) before threading.

If reliable and experienced welders are available, bars may be spliced by welding. The main objection to welding is that the effectiveness of the splice depends mainly upon workmanship. The question of cost should also be considered.

The advantage of the last two methods of splicing bars is that they prevent crowding of bars such as is inevitable where splicing is done by lapping.

Hooks. — For main reinforcement of beams and girders, semicircular hooks should be used at the ends. The inside diameter of the hook should be equal to 5 diameters of the bar, and the length of the hook should be 3 diameters longer than one-half of the circumference of the circle. The hook should be backed with concrete of a thickness equal to at least 4 diameters of the bar. In a simply supported girder, the hooks provided at the ends of the bottom reinforcement may be backed with concrete 2 diameters thick.

Bending of Main Reinforcement in Girders. — The points of bending of the tension reinforcement in girders should be determined from bending-moment diagrams. When the bent portion of a bar is extended to the top of a girder and then the bar is carried to the support, both the bottom and the diagonal tension reinforcement may be considered as sufficiently anchored.

When the bent bar is intended to serve as negative bending reinforcement at the supports, it must be properly extended into the adjacent span.

When the bent bar is not extended at the top to the support, it must be properly anchored in the compression zone to be effective as diagonal tension reinforcement. For bars of small diameter, it may be sufficient to extend the bar to the top and there to provide a semicircular hook. For heavier bars, $\frac{7}{8}$ in. and larger, a straight horizontal portion should be provided at the end equal in length to 15 diameters of the bar, and followed by a semicircular hook.

When the bent portion of a bar is not intended to be used as diagonal tension reinforcement, the anchorage at the end should consist of a bent portion not less than 20 diameters long and a semicircular hook. This anchorage is required to develop tension stresses in the bar.

Under no circumstances should tension reinforcement in a girder be reduced within the region subjected to tension stresses by simply stopping short the excessive bars.

WEARING SURFACES FOR DECKS OF CONCRETE BRIDGES

Since the roadway of reinforced-concrete bridge decks is subjected to wear, the concrete slab of the deck should be protected by a suitable wearing surface or pavement. The pavement serves the following additional purposes: it reduces the effect upon the slab of the impact action of the moving loads; it assists in distributing the concentrated loads over the slab; and finally it protects the concrete from the direct action of the rays of the sun, thus reducing the range of temperature changes for the structural concrete.

Concrete Wearing Surface. — The cheapest wearing surface consists of a granolithic finish of the slab, not less than $1\frac{1}{2}$ in. thick, properly bonded to the structural slab. The finish may be built monolithic with the slab, i.e., applied before the concrete of the slab has hardened; or it may be applied after the concrete slab has set in which case it must be properly bonded. Even when built monolithic, the finish should not be considered in computations as a part of the structural slab, since it is subject to wear.

A separate concrete pavement may also be built on the top of the structural slab. The thickness of such a pavement should be not less than 4 in. at the center, which, to provide a crown for the roadway, may be reduced to not less than 3 in. at the curbs. Although a plain pavement may give satisfactory service, better results are obtained by reinforcing it with wire netting. It is best to waterproof the structural slab at the top by coating it with asphalt before placing the concrete pavement. This not only waterproofs the slab, but also makes the replacement of the wearing surface easier. The pavement should be provided with expansion joints along the curbs, at the expansion joints of the structure, and, when necessary, with additional intermediate joints.

Bituminous Pavements. — Bituminous pavements are often used for reinforced-concrete bridges especially if the highway is paved with this material.

Bituminous pavement may consist of a thin bituminous coat, about 1 in. thick, of graded aggregates with a tar or asphalt binder. To insure its adhesion to the concrete slab, a priming coat is usually necessary. When properly laid, such pavements give satisfactory service for light loads.

For heavier traffic, the bituminous pavement should be at least 3 in. thick; and it may be either of the hot mixed kind, or of the natural or artificial rock asphalt type.

Block Pavements. — The bridge pavement may consist of paving bricks, asphalt blocks, or wood blocks. To give satisfaction, the blocks should be properly placed. Wood blocks have often been found too slippery in rainy weather. For wood block pavement see pp. 92 and 379.

For heavy trucks, granite and basalt block pavements are most appropriate. Such pavements are used more extensively in Europe than in America. The pavement shown in Fig. 66, p. 155, consists of 6-in.-thick granite blocks placed upon a 1-in.-thick sand cushion. See also Fig. 62, p. 145.

Macadam Pavements. — If the pavement of the roadway is of macadam, it may be extended also over the bridge. Macadam pavement is usually placed in three layers aggregating at an average 7 in. It

is important to place the pavement with unusual care because the macadam on the bridge gets out of order much more easily than elsewhere on the road.

TREATMENT OF EXPOSED SURFACES OF CONCRETE BRIDGE

The treatment of the exposed surfaces of a concrete bridge depends upon the effect it is desired to produce, and also upon the available funds for the purpose. It is obvious that the appearance of a structure erected in ordinary surroundings or in open country requires less attention than that of a structure built in an attractive location. (See also " Concrete, Plain and Reinforced," Vol. 1, p. 732.)

In most structures, exposed concrete surfaces are retained, and only in rare cases are they faced with brick or stone, natural or artificial. Facing of concrete surfaces is expensive, and the additional cost is warranted only under exceptional circumstances.

The imperfections of the concrete surfaces as they appear after the forms are removed should always be reduced to a minimum. Frequently the surfaces are subjected to special treatment described in the succeeding paragraphs under proper headings.

General Requirements. — To insure satisfactory results, the exposed concrete surfaces should be even, with as few form marks as possible, and without projecting fins or other imperfections. For this reason, the formwork must be rigid and tight. It is obvious that, where the forms bulge or sag, satisfactory results from the standpoint of appearance are impossible.

It is preferable to use planks planed on the inside, and with tongue and grooved edges. In recent years, wide sheets of pressed wood fiber and laminated or plywood specially treated for resistance to the moisture from concrete have been introduced in America for formwork. The plywood is usually five ply and about $\frac{5}{8}$ in. thick; and the sheets are wide enough to form the whole width of a girder or column form, which results in perfectly smooth surfaces without any form marks. Since these sheets are thin, they require special supporting framework to prevent bulging.

Concrete should be deposited with particular care to avoid air holes, voids, and honeycomb. A smooth, hard surface may be obtained by spading against the forms, forcing all coarse aggregates from the surfaces and working the mortar against the forms. Proper use of vibrators may be found useful.

Forms should be removed with special care so as not to damage the corners and edges. Forms should be designed so as to be easily remov-

able. All edges and corners should be chamfered by nailing proper fillets in the forms.

Irrespective of the treatment to be given to the concrete surfaces, it is necessary to perform the following work immediately after the removal of the forms. All protruding tie wires must be cut down. Any damage to the concrete due to the removal of formwork or cutting down of the wires should be repaired, and any voids and bolt holes should be filled with concrete of the same mix as was used in the structure.

Simple Surface Finish. — In many cases, satisfactory results may be obtained without any additional treatment of the exposed concrete surfaces aside from that mentioned in the preceding paragraph.

Surface Treatment by Rubbing. — One of the simplest effective treatments of exposed concrete surfaces is rubbing; this consists of two operations.

The first operation takes place directly after the sides of the form have been removed, usually 24 to 48 hours after the concrete is poured. All repairing and pointing is done as previously explained. After the pointing has set, the whole surface is thoroughly rubbed so as to remove all form marks. In some cases, as a part of the first operation 1 : 3 mix cement mortar is rubbed into the surface, which produces a hard, well-sealed surface.

The second operation is performed after the structure is completed. It consists of wetting the surface with a brush and rubbing it with carborundum stone. This operation removes the cement film and, by forming a surface of smooth texture of uniform color, gives life to the structure. The work may be done by hand or by specially designed finishing machines.

Brush Finish. — In this finish, the object is to expose the concrete aggregates by removing the cement film by brushes. The work should be performed when the concrete is neither too old nor too green. If too old, the work of removing the film is very much harder and the effect is not successful. With too green concrete, on the other hand, brushing may dislodge the aggregates.

Bush Hammer Finish. — An attractive, but expensive, finish is obtained by bush hammering, which exposes the coarse aggregates. This work should not be performed until after the concrete has hardened and is able to withstand the hard treatment.

Exposing Aggregates by Chemical Treatment. — The aggregates on the surface may be exposed by treating the surface of green concrete with chemical preparations which retard the setting of the cement near the surface, and make easy the removal of the surface cement film. The chemical preparations may be applied upon the form in contact with the

surface to be treated before the concrete is poured. Then the chemicals are most effective because they act upon the concrete before it has set. Concrete surfaces may also be treated with the chemicals after the forms are removed, but their effectiveness decreases with the age of the concrete, so that appreciably stronger solutions are required to get the desired result.

Special Aggregates and Facing with Special Concretes. — The appearance of a structure may be appreciably improved by the use of selected aggregates which give the concrete the desired texture and color. When these aggregates are appreciably more expensive than the ordinary aggregates, they are used only as a facing of the concrete, 3 to 4 in. thick, the bulk of the structure being built of ordinary concrete. The same method is used when an abutment or pier built of lean concrete is faced with concrete of structural strength, as is often done in Europe. See piers for Lindau Bridge, Fig. 62, p. 145.

The facing coat is poured at the same time with the rest of the concrete as follows. At a proper distance from the form, a metal sheet is placed which separates the ordinary concrete from the concrete of the facing. Both concretes are poured at substantially the same time; and after the concrete has stiffened somewhat, but before it has set, the metal sheet is lifted gradually and the two concretes are allowed to bond.

CHAPTER XVI

ABUTMENTS AND PIERS

This chapter is devoted to abutments, piers, and other vertical supports for bridges. The forces acting upon them are discussed, methods of design are outlined, and examples from practice are given.

In this treatment, abutments are divided into gravity abutments and reinforced-concrete abutments. The simple types of reinforced-concrete abutments are fully treated; also several more complicated designs are described.

For the purpose of discussion the piers are divided into river piers, and vertical supports located where the effect of the flow of water does not need to be considered. Also, supports for pile and pier trestles are discussed and illustrated.

ABUTMENTS

The purpose of the abutments is twofold: (1) to transmit to the foundations the reactions of the superstructure; (2) to finish up the bridge and to retain the embankments. Abutments for reinforced-concrete structures may be built of plain or reinforced concrete, stone masonry, or brickwork. The selection of the material to be used depends in most cases upon the cost, which, in turn, depends upon local conditions. It is obvious that, where stone is plentiful and wages of masons are comparatively low, stone masonry may be cheaper than concrete either plain or reinforced. Thus, in many parts of Europe, masonry abutments are used to a large extent. In the United States, however, plain and reinforced concrete are the most common materials for abutments of permanent structures. Abutments for monumental structures are often built of concrete and faced with stone.

In this section are considered only abutments which are independent of the superstructure. So far as the design of abutments is concerned, it is of comparatively minor importance whether the superstructure to be supported is of reinforced concrete or of structural steel. The design of the superstructure affects only the width of the bridge seat and the design of the parapet wall.

Width of Abutment and Arrangement of Slopes. — The width of the abutment is determined by the width of the graded roadway, i.e., of the

paved roadway and of the shoulders. This width is often greater than the width of the bridge.

In abutments with wing walls, the length of the wings depends upon the arrangement of the slopes of the embankment. Ordinarily a slope of $1\frac{1}{2}:1$ is used, which means $1\frac{1}{2}$ ft. horizontal for 1 ft. vertical. If it is desired to use a steeper slope, the face of the embankment must be strengthened in an effective manner. See Fig. 147, p. 324.

The arrangement of slopes at an abutment for a pile trestle is shown in Fig. 187, p. 434.

Arrangement of slopes for buried abutments is shown in Fig. 72, p. 162. Attention is called to the concrete crib retaining wall varying in height from 2 to 5 ft. which is here used to reduce the lengths of the slopes.

Forces Acting upon Abutments. — Abutments must be designed to resist the following forces:

1. Vertical reactions of the superstructure, i.e., the live and dead loads transferred from the bridge to the abutment. Impact usually does not need to be considered in designing abutments.
2. Weight of the abutment and of the fill resting upon it.
3. Pressure of the earth behind the abutment, which also should include the pressure due to the live load transmitted by the fill to the abutment.
4. Horizontal traction force, usually one- to two-tenths of the live load.
5. Horizontal components of vertical loadings for bridges on grade.
6. Horizontal forces due to expansion and contraction of the superstructure.
7. Centrifugal forces for bridges on curves.

The forces acting on the abutment should be combined so as to give the most unfavorable results. The location of the resultant should be found; and the stresses produced by this resultant in the body of the abutment must not exceed the allowable unit stresses for the materials of which the abutment is built. Also, the dimensions of the base, or the number and disposition of piles, if used, must be sufficient to keep the maximum unit pressures on the ground, or the maximum unit pressure on the piles, within working limits.

Traction forces need to be considered only in railroad bridges. Where one end of the superstructure is fixed and the other is provided with an expansion bearing, full traction force should be considered as applied at the fixed end, while at the other end only a sufficiently large force should be assumed to overcome friction at the expansion bearing. In two-track structures, the traction forces for both tracks should be

assumed as acting simultaneously in the same direction. The most unfavorable effect upon the abutment is produced by traction forces acting in the same direction as the earth pressure.

If the bridge is provided with efficient expansion bearings, the effect upon the abutment of temperature changes does not need to be considered.

When the superstructure is provided with expansion bearings, the abutment must be considered in design as a cantilever free at the top and fixed at the bottom.

Where the top of the abutment is anchored to the superstructure, and the superstructure is also anchored at the other end to the support having sufficient horizontal stability, the main wall of the abutment in resisting earth pressures may be considered as supported at the top and bottom. At the top, the wall is freely supported by the superstructure; at the bottom it may be either fixed or freely supported.

ABUTMENTS OF GRAVITY AND SEMI-GRAVITY SECTIONS

The distinguishing feature of the gravity and the semi-gravity type of abutment is that the earth pressures acting upon them are resisted mainly by the weight of the materials of which the abutments are composed.

The component parts of an abutment of either of these two types are: a front wall which supports the superstructure, and two wing walls which retain the embankment at the sides. According to the angle of deflection of the wing walls in respect to the front wall, the abutment may be divided into wing abutments and U-abutments.

In wing abutments, the wing walls may be placed on line with the face of the front wall, or they may be deflected at any desired angle. The angle of deflection may be governed by economy, in which case the most economical angle, so far as the materials in the wings are concerned, is about 35°; or it may depend upon topographical conditions. Usually, the angles of deflection of both wings are the same. In many cases, however, it is necessary to use unsymmetrical arrangements. In U-abutments, the wing walls are deflected at 90°. A design of a semi-gravity section is shown in Fig. 175, p. 407.

Cross Section of Abutment. — In cross section, the front wall of an abutment of either of the two types consists of a parapet wall located back of the bridge seat; the bridge seat; the stem of the wall; and the foundation slab. Since the width of the wall is smallest at the bridge seat and increases toward the bottom, the back of the wall is either sloped or stepped, while the front of the wall may be only slightly battered for the sake of appearance.

The foundation slab projects beyond the faces of the stem either on one or on both sides. The purpose of the projections is not only to enlarge the base but also to shift the center of the base so as to reduce the eccentricity of the resultant, so far as the foundation is concerned. This is more fully explained below under "Design of Gravity Abutments."

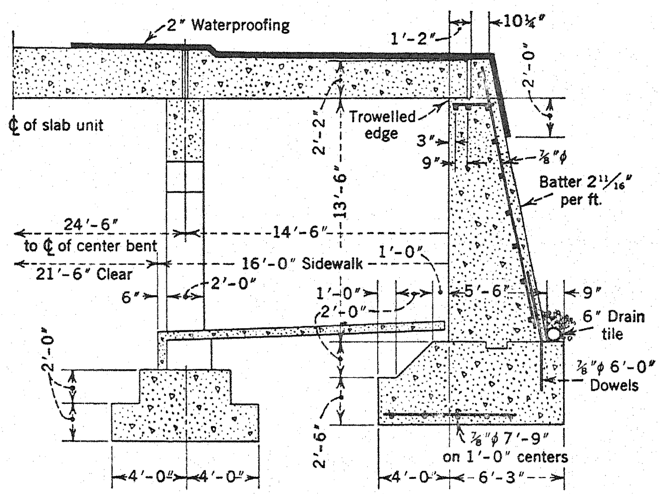


FIG. 175.—Vertical Supports. Kensington Track Elevation in Chicago. (*See p.* 406.)

Vertical Expansion Joints between Front Wall and Wing Walls. — The front wall and the wing walls serve different purposes and resist forces of appreciably different magnitudes. When the wing walls are connected with the front wall, the dimensions of the foundation base for each part of the structure must be carefully proportioned so that in case of settlement the abutment will settle uniformly as a unit. Unequal settlements of the different parts may cause cracks at the junctures of the wing walls with the front wall. Owing to uncertainties of the magnitude of the earth pressures, proper proportioning of foundations is difficult and uncertain.

To simplify the design of foundations, the wing walls are often separated from the front wall by vertical expansion joints which extend either to the top of the foundation slab or even through the foundation slab. The various parts of the abutment may be kept in alignment by vertical keys. The expansion joints may be filled with asphalt or with premolded elastic strips.

Design of Gravity Abutments. — The following method of design of an abutment may be used when the wing walls are separated from the

front wall by vertical expansion joints, or when the wing walls are on a line with the front wall. In both cases the front wall and the wing walls are designed separately, each for the forces acting upon it.

In designing the front wall, the dimensions at the top are determined first. These are governed by the thickness of the parapet wall and by the width of the bridge seat. The dimensions of the stem at different heights are then assumed. For railroad bridges, the width of the stem of a gravity abutment at any section may be assumed at about 0.45 of the height of the abutment above that section; for highway bridges, this fraction may be reduced to 0.4. For semi-gravity designs, the width of the section may be reduced to any desired degree up to a point where the semi-gravity design changes to a reinforced-concrete section. Finally the base of the foundation is tentatively chosen.

For the assumed dimensions, the weight of the abutment is computed. Lines of pressure are then drawn for the dead load, the horizontal earth pressures, and several assumptions as to the reactions of the superstructure.

For gravity abutments, the lines of pressure must remain within the middle third of all horizontal sections of the abutment. For semi-gravity sections, any tension at the back of the wall must be provided for by upright bars placed near the back face and anchored in the foundation slab.

The line of pressure is then extended to the bottom of the foundation slab, and the pressures upon the foundation are computed, taking into account the magnitude and the eccentricity of the resultant. When the computed unit pressures are not satisfactory, the dimensions of the base may be adjusted so as to increase the width and also to reduce the eccentricity of the resultant pressure.

When the abutment rests on fairly unyielding ground, it is sufficient for the most unfavorable resultant to be just within the middle third of the base. When, however, the abutment is on compressible ground, the resultant for the most unfavorable combination of loadings should be as near the center of the base as possible to prevent tipping of the abutment. This is particularly important for high abutments where even a slight angular movement at the base may mean an appreciable movement at the top.

When the abutment rests on piles, and the resultant is eccentric, the pressure on piles may be equalized by proper arrangement of piles, such as using a closer spacing of piles in the front row, and spacing the rows in the front closer than in the back.

The foundation slab is almost always extended beyond the faces of

the stem of the abutment. The projections should be treated as cantilevers loaded by the reaction of the foundation. When the projections are of plain concrete, the tensile stresses should be kept within working limits for that material. (See "Concrete, Plain and Reinforced," Vol. I, p. 844.) Often, to reduce the thickness of the projections, they are reinforced by bars placed in the bottom and extended sufficiently into the abutment to develop the full strength of the reinforcement at the edge of the stem.

Wing walls, when considered as acting independently of the main wall, may be designed in the same manner as retaining walls of the same height, and resisting the same earth pressures.

Design of Abutment with Monolithic Wing Walls. — When wing walls which are deflected in respect to the front wall are built monolithic with the front wall, the whole abutment must be considered as a unit. The stability of the front wall is then favorably affected by the wing walls. The larger the angle of deflection, the greater is the effect of the wing walls.

Examples of Gravity Abutment. — An example of a gravity abutment with wing walls is shown in Fig. 66, p. 155. U-abutments of gravity design are shown in Figs. 63, p. 147, and 70, p. 160.

REINFORCED-CONCRETE ABUTMENTS

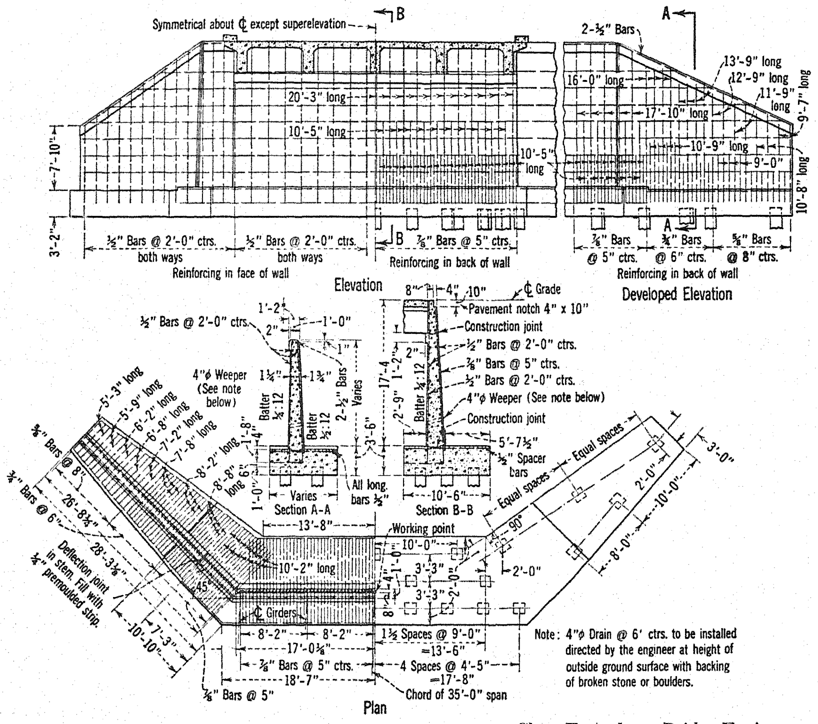
General Description of Reinforced-Concrete Abutments. — In general, reinforced-concrete abutments consist of combinations of vertical and horizontal members properly tied together. In designing such abutments, it is necessary to investigate the stability of the abutment as a whole, and also to determine the dimensions and the amount of reinforcement for each member composing the abutment. Each member should be made strong enough to resist bending moments and shears produced by forces acting upon it. Thus, vertical walls should be designed for the horizontal pressures, while the horizontal foundation slabs should be designed for the upward reactions of the foundation or the downward weights of earth and concrete.

The simplest reinforced-concrete abutments are of the T-type. Each abutment consists of a horizontal foundation slab supporting vertical wall slabs arranged so that any vertical section resembles an inverted letter T. Abutments with counterforts have a somewhat more complicated arrangement of component parts, as in addition to vertical walls and foundation slabs they are provided with counterforts extending between the walls and the base slabs. Their design is discussed on p. 418.

T-ABUTMENTS. FRONT WALL FREE AT TOP

The most common reinforced-concrete abutments are those of the T-type in which the front wall at the top is free to move horizontally. The component parts are: (1) vertical wall, and (2) horizontal base slab.

Vertical Wall. Front Wall Separated from Wing Walls. — When, in a T-abutment, the front wall is separated from wing walls by vertical expansion joints, each part acts independently of the other parts. The



Chas. E. Andrew, Bridge Engineer.

FIG. 176. Details of Abutment. Bridge across Castiac Creek, California. (*See p.* 410.)

abutment shown in Fig. 176, p. 410, used in a bridge across Castiac Creek in Los Angeles County, California, may serve as an example of this arrangement.

In an abutment of this type, the vertical slabs act as vertical cantilevers fixed at the bottom by the base slab and free at the top.

The front wall is subjected to the direct pressures due to its dead load and due to the vertical reactions of the superstructure, and to bend-

ing moments due to the earth pressures and to the traction forces, while the wing walls are subjected only to earth pressures.

At the top, the thickness of the front wall is governed by the width required for the bridge seat and the parapet, if one is used. At the bottom, the thickness of the wall is governed by bending moments; and since it is always appreciably greater than the thickness at the top, the back of the wall is sloped, while the front is provided only with a small batter for the sake of appearance. To avoid noticeable deflection of the top of the wall, slender walls should not be used.

The main reinforcement of the wall consists of vertical bars which resist bending moments and are placed near the back of the wall. The bars must be anchored in the foundation slab so as to be able to resist full stresses at the bottom of the wall. The vertical bars are held in place during erection by horizontal distributing bars. In addition to this reinforcement, vertical and horizontal bars are often placed near the face of the wall to prevent cracks due to temperature changes and shrinkage.

Ordinarily, a construction joint is formed between the base slab and the vertical slab. To facilitate erection, vertical dowels are then imbedded in the horizontal slab of the same size and number as required for the vertical wall reinforcement. The dowels must be fully anchored in the base and provided with a lap with the vertical bars of sufficient length to develop their strength by bond.

The bridge seat is usually reinforced by horizontal longitudinal bars. Also, short cross bars are often used directly under the bearings.

Wing walls are designed in the same manner as ordinary retaining walls of the T-type, and are reinforced in substantially the same manner. Retaining walls are treated in "Concrete, Plain and Reinforced," Vol. I, p. 832.

Vertical Slab. Front Wall and Wing Walls Monolithic. — When the front wall of the abutment is built monolithic with the wing walls, the parts composing the abutment cannot act independently of one another, but each part exerts a restraining effect upon the other parts. The kind and the degree of restraint depend upon the arrangement of the wing walls in respect to the front wall; and, in any particular case, the condition is somewhere between the following two extremes.

In the first extreme, the wing walls are at right angles to the front wall, and their effect upon the front wall is similar to that of the counterforts in an abutment with counterforts. The second extreme condition is when the wing walls are on a line with the front wall, in which case their effect upon the front wall is negligible.

With the wing walls at right angles, the first extreme, the front wall

is not only fixed at the bottom to the foundation, like the wall discussed under the previous heading, but, also, it is restrained at both sides by the wing walls. When subjected to horizontal earth pressures, the front wall is not free to deflect like a free vertical cantilever, and the top of the wall, instead of moving bodily forward, assumes the shape of a reverse curve. The pressure acting upon the wall is resisted in part by the wall acting as a vertical cantilever, and the balance of the pressure is resisted by the wall acting as a slab spanning between, and restrained by, the wing walls.

A rational theoretical solution of the problem of determining bending moments in a vertical slab under these conditions would be very complicated; usually, in practice the matter is simplified as will be explained later.

In the second extreme, where the wing walls are on a line with the front wall, all parts deflect in the same manner. The wing walls do not affect the front wall to any extent, therefore each part may be designed separately in the same manner as if the wing walls were separated by a vertical expansion joint.

When the wing walls are deflected in respect to the front wall at an angle smaller than 90°, their effect upon them decreases with the decrease in the angle of deflection.

Design of Vertical Wall. Wing Walls Monolithic. — Abutments in which the wing walls are monolithic with the front wall are often designed in the same manner as if the parts were separated, except that some horizontal reinforcement is added at the junctures of the wings with the wall. Such solution of the problem is neither logical nor economical. Much of the vertical reinforcement is wasted, while at the same time large horizontal bending moments are not provided for.

The following simple solution of the problem is recommended. It applies strictly where the angle of deflection between the wings and the front wall is 90°. For other angles of deflection the amount of horizontal reinforcement should be reduced and that of the vertical reinforcement increased.

The wall should be reinforced with horizontal and vertical reinforcement determined from the bending moments given in the table on p. 413. The disposition of the reinforcement is fully discussed under proper subheadings.

Bending Moments. — The table on p. 413 gives bending moments in the wall due to earth pressures when the wing walls are monolithic with the front wall and the angle of deflection is 90°, for several ratios of the horizontal distance between the wing walls, l to the height of the wall h. The following bending moments are given: (1) vertical bending moments at the bottom of the wall; (2) horizontal bending moments

at the wing walls; and (3) horizontal bending moments in the center of the horizontal span of the wall.

Let $M_{\text{max.}}$ = maximum bending moment in center of horizontal span of wall.
M = bending moment at support of horizontal span of wall.
M_S = bending moment, considering all earth pressure as applied on simply supported horizontal span.
M_B = vertical bending moment at bottom of wall.
M_C = total cantilever bending moment of wall.
l = horizontal span of front wall between wing walls.
h = height of front wall.

Ratio Span to Height	Bending Moments in Horizontal Span	Bending Moments in Cantilevers
$\frac{l}{h} = 1.0$	$M_{\text{max.}} = -M = 0.58M_S$	$M_B = 0.5M_C$
2.0	$M_{\text{max.}} = -M = 0.40M_S$	$M_B = 0.6M_C$
3.0	$M_{\text{max.}} = -M = 0.27M_S$	$M_B = 0.8M_C$

To use this table for determining bending moments, proceed as follows: Compute the earth pressures acting upon the upper one-half of the wall. Compute the bending moment M_S, considering this whole pressure as acting on a simply supported horizontal slab. Using constants from the table for proper ratio $\frac{l}{h}$, compute the bending moments, $M_{\text{max.}}$ and $-M$. For these bending moments, find the required amount of horizontal reinforcement and distribute it uniformly over the upper one-half of the wall. For the lower one-half of the wall, consider the bending moments as varying in intensity from the maximum at the upper one-half of the wall, to zero at the bottom of the wall.

The vertical bending moment at the bottom of the wall is found by computing the maximum cantilever bending moment in the wall, considering it as a free cantilever, and by multiplying it by the constant obtained from the table for the proper ratio of $\frac{l}{h}$.

The vertical reinforcement required by this bending moment should be spaced uniformly in the middle third of the span of the wall. At both ends, the amount of reinforcement per foot of width should be varied from this maximum to zero at the wing walls.

Horizontal Bars. — Horizontal bars should be placed near the outside face of the wall in the central portion of the span to resist bending mo-

ment $M_{max.}$, and near the inside face to resist bending moments $-M$. Either separate sets of bars should be used for the positive and negative bending moments; or part of the central reinforcement may be bent and carried near the opposite face to the supports, where it should be anchored in the wing wall.

Vertical Bars. — Vertical bars should be arranged in the same manner as in a wall which is separated from wing walls.

Design of Wing Walls When Monolithic with Front Wall. — It is obvious that the wing walls when built monolithic with the front wall not only restrain the front wall, but are in turn restrained by it. They are subjected to a composite deflection: they deflect as vertical cantilevers fixed at the bottom; and also they deflect as horizontal cantilevers fixed at the juncture of the walls. Part of the earth pressure is resisted by one action and the remainder by the other action. They must therefore be reinforced accordingly. Horizontal bars must extend from the front wall into the wing wall to bind the parts and to insure their co-operation.

Parapet Wall. T-type Abutment. — The parapet wall is the part of the abutment located above and back of the bridge seat, and its function is to prevent the fill from spreading upon the bridge seat. For abutments carrying steel girders and trusses, parapet walls are indispensable. For reinforced-concrete superstructure, parapets are often omitted, particularly when the ends resting upon the abutment are provided with fixed bearings. (See Fig. 176, p. 410.)

When expansion bearings for the superstructure are used at the abutment, it is preferable to use parapets so as to permit free expansion of the superstructure. The expansion joints provided between the ends of the superstructure and the parapet wall may be filled with elastic material; or water stops may be provided at the top. See also the discussion on p. 376.

Foundation Slab. — In designing the foundation slab, the following requirements must be fulfilled: (1) the maximum unit pressure upon the soil or upon the piles must be kept within working limits; (2) the thickness of the slab and the amount of reinforcement must be sufficient to keep the unit stresses within working limits.

The unit pressures upon the foundation are found by determining the magnitude and the position of the resultant of all the forces acting upon the abutment. To prevent uplift, the resultant should be kept within the middle third of the base.

Where the wing walls are separated from the main wall by vertical expansion joints, the foundation slab for each part is designed independently of the other sections.

Where the wing walls form an integral part of the front wall, the design of the foundation slab is affected by the degree of restraining effect of the wing walls upon the front wall; and this is dependent upon the angle between the face of the wall and the face of the wings.

When the wing walls are on line with the front wall, the walls do not exert any appreciable restraint upon each other. Therefore the foundation for each part is designed in the same manner as if the walls were separated.

When the wing walls are at right angles to the main wall, and the ratio of the horizontal span to the height is small, the foundation for the front wall needs to be designed only for the vertical reactions acting upon the wall and for the small bending moments transferred from the wall at the bottom to the foundation. In designing the foundation for the wing walls, however, it is necessary to take into consideration the horizontal earth pressures transferred from the front wall to the wing walls.

Where the ratio of the span of the front wall to its height is large, or the angle of deflection of the wing walls is appreciably less than 90°, the stabilizing effect of the wing walls upon the main wall is rather indefinite. For this reason, it is very often neglected in designing the dimensions of the base of the abutment. This gives much larger dimensions of the base than required.

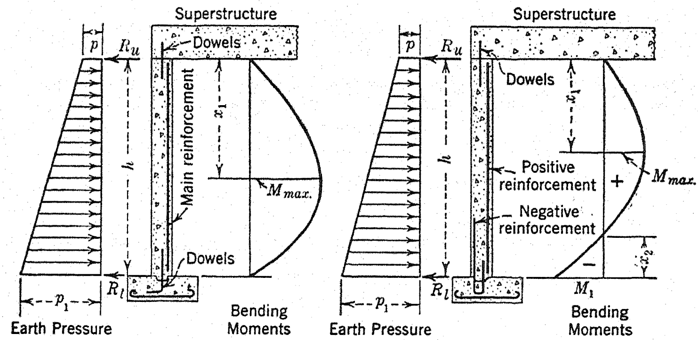
The following method yields more economical results. First, the dimensions of the base are determined, considering the front wall as a free cantilever not restrained by the wing walls. Then the dimensions of the base are found for a condition when the front wall is considered as fully restrained by the wing walls. Finally intermediate dimensions are accepted depending upon the expected degree of restraining effect of the wing walls upon the front wall.

In determining the thicknesses of the foundation slab and the amount of reinforcement, the foundation slab is considered as consisting of two cantilevers. One of the cantilevers is in front of the wall, and is loaded by the upward reaction of the soil. Its main reinforcement consists of bars placed at the bottom of the slab at right angles to the wall and extending beyond the face of the wall a sufficient distance for full anchorage. The other cantilever is in the back of the wall, and it must be made strong enough to carry the weight of the fill placed upon it. Its main reinforcement is placed near the top of the slab, also at right angles to the wall, and should be extended into the front cantilever a sufficient distance for full anchorage. Some distributing bars, parallel with the wall, should be placed in both cantilevers. Special reinforcement should be added where the foundation slab for the front wall joins the foundation slab for the wing walls.

T-ABUTMENTS. SUPERSTRUCTURE ANCHORED AT BOTH ENDS

For concrete bridges of short spans, the superstructure is often anchored to the supports at both ends of each span. Advantage may be taken of this in designing the abutments.

Vertical Main Wall. — A vertical main wall of the abutments to which the superstructure is anchored, in resisting horizontal earth pressures, may be considered as a slab supported at the top by the superstructure, and freely supported or fixed at the bottom by the base slab. (See Fig. 177, p. 416.) The main wall is also subjected to the reactions of the superstructure resting upon it and due to its own weight.



(a) *Wall Freely Supported at Bottom* (b) *Wall Fixed at Bottom*

FIG. 177.—Abutment. Wall Supported at Top. (*See p.* 417.)

The following formulas may be used in determining bending moments in the wall due to earth pressures represented by a trapezoid.

Let p = unit pressure at top of wall, pounds per vertical foot of wall.
p_1 = unit pressure at bottom of wall, pounds per vertical foot of wall.
h = height of wall, feet.

Then

Both Ends of Vertical Wall Simply Supported. (See Fig. 177 (*a*).)
Reactions:

$$R_u = \tfrac{1}{6}(p_1 + 2p)h \quad \text{Top} \quad (1) \qquad R_l = \tfrac{1}{6}(2p_1 + p)h \quad \text{Bottom} \quad (2)$$

Point of Zero Shear and Maximum Bending Moment, from Top:

$$x_1 = \frac{p}{p_1 - p}\left[-1 + \sqrt{1 + \frac{1}{3}\left(\frac{p_1}{p} + 2\right)\left(\frac{p_1}{p} - 1\right)}\right]h \qquad (3)$$

Maximum Bending Moment:

$$M_{\text{max.}} = \frac{1}{18} \frac{\frac{x_1}{h}\left(3\frac{p}{p_1 - p} + 2\frac{x_1}{h}\right)}{2\frac{p}{p_1 - p} + \frac{x_1}{h}} (p_1 + 2p)h^2 \qquad (4)$$

Vertical Wall Fixed at Bottom and Supported at Top. (See Fig. 177 (*b*).)

Reactions:

$R_u = \frac{1}{40}(4p_1 + 11p)h$ Top (5) $R_l = \frac{1}{40}(16p_1 + 9p)h$ Bottom (6)

Points of Zero Shear and Maximum Bending Moment, from Top:

$$x_1 = \frac{p}{p_1 - p}\left[-1 + \sqrt{1 + \frac{1}{20}\left(11 + 4\frac{p_1}{p}\right)\left(\frac{p_1}{p} - 1\right)}\right]h \qquad (7)$$

Maximum Positive Bending Moment:

$$M_{\text{max.}} = \frac{1}{120} \frac{\frac{x_1}{h}\left(3\frac{p}{p_1 - p} + 2\frac{x_1}{h}\right)}{2\frac{p}{p_1 - p} + \frac{x_1}{h}} (11p + 4p_1)h^2 \qquad (8)$$

Maximum Negative Bending Moment:

$$M_1 = -\tfrac{1}{120}(7p + 8p_1)h^2$$

Point of Contraflexure, from Bottom:

$$x_2 = \frac{p}{p_1 - p}\left[\left(5 + \frac{p_1}{p}\right) - 6\sqrt{1 + \frac{1}{240}\left(11 + 4\frac{p_1}{p}\right)\left(\frac{p_1}{p} - 1\right)}\right]h \qquad (9)$$

The arrangement of the main reinforcement of the wall is shown in Fig. 177, p. 416.

Footing for Main Wall of Abutment. — The footing slab of the main wall for an abutment where the main wall is supported at the top and bottom is subjected only to vertical reactions of the superstructure, the weight of the wall, and the weight of the fill resting directly upon the slab. The horizontal reaction of the earth pressures at the bottom is resisted by the resistance of the base to sliding.

When the vertical wall is fixed at the bottom, in addition to the vertical reactions mentioned above, the footing is subjected to the bending moment M_1 transferred to it by the vertical wall. This bending moment must be taken into account when proportioning the base.

Wing Wall. — In this case, wing walls are designed in the same manner as retaining walls. The vertical walls are cantilevers free at the top, fixed at the bottom, and loaded by the horizontal earth pressures. The main reinforcement consists of vertical bars placed near the inside face of the wall. These bars must be fully anchored in the foundation slab either directly or through dowels.

When the wing walls are separated from the main wall by vertical expansion joints, each part of the abutment acts separately, and no force is transferred from the wing walls to the front wall. Therefore each part is designed separately to resist the forces acting upon it.

When the wing walls are connected with the main wall, there is a section of the wall at the juncture of the front wall and the wing wall where the conditions of one part affect the other part, even when the walls are on line, because the wing wall acts as a cantilever and the front wall as a slab supported at top and bottom. When the wing walls are deflected, they exert upon the main wall a restraining effect similar to that discussed on p. 411 in connection with T-abutments not connected at the top. Here again the main wall, in addition to being supported at the top and bottom, is restrained at the sides. The degree of the restraint depends upon the ratio of the height of the wall to the horizontal span between the wing walls, and upon the angle of deflection of the wing walls in respect to the main wall.

It is obvious that in addition to the vertical reinforcement it is necessary to provide horizontal reinforcement, either throughout the whole length of the main wall as explained on p. 414 or only at the junctures of the walls.

Footings for Wing Walls. — Footings for wing walls are the same as for retaining walls.

Temperature Reinforcement for T-type Abutments. — To prevent cracks in the abutment due to temperature changes and shrinkage, horizontal bars are placed near both faces of the wall. These are in addition to the corner reinforcement at the junctures of the wing walls with the main wall.

ABUTMENTS WITH COUNTERFORTS

Where the height of an abutment is more than 18 ft., its cost may be reduced by using a design with counterforts instead of the inverted T-design described in the previous pages. An additional important advantage of the counterforts is that they eliminate the possibility of noticeable deflection of the top of the abutment due to earth pressure.

An abutment with counterforts consists of a vertical main wall; wing walls; counterforts for main wall and wing walls; foundation slab;

bridge seat; and in some designs parapet walls back of the bridge seat. (See Fig. 178, below.)

Spacing of Counterforts. — Usually counterforts are spaced so as to divide the wall into a number of equal panels; and the panel length depends upon economy. Usually, the choice of spacings is rather limited. It would be of advantage to place the counterforts under the bearings of the girders. This, however, is seldom possible without an appreciable complication of the design of the wall and of the foundation

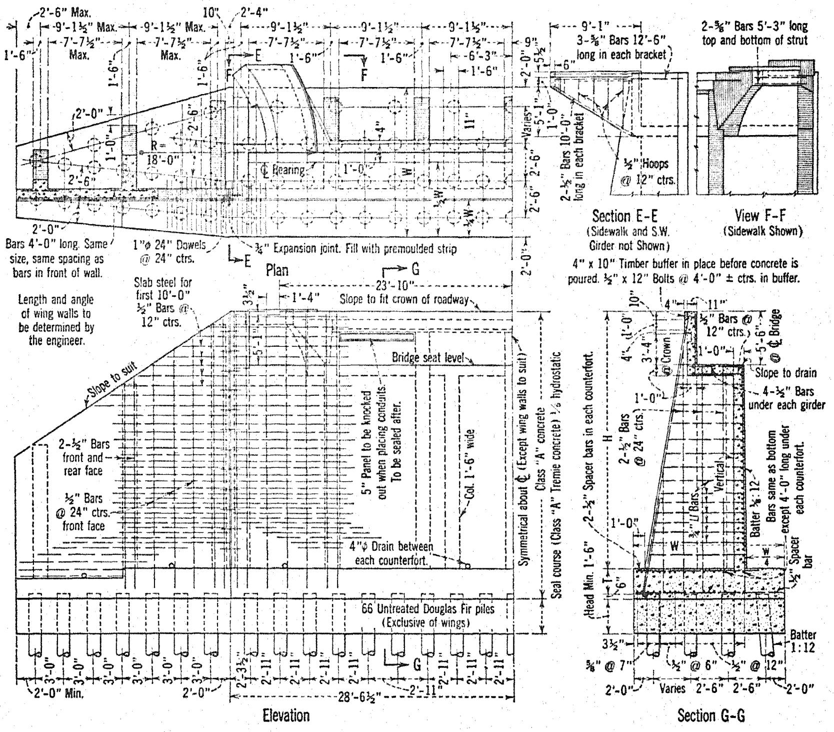


FIG. 178.—Counterfort Abutment, San Dieguito River, California. (*See p.* 419.)

slab. Where girder reactions are large, special columns, not connected with the walls, therefore, not affecting their design, may be used to carry them. In Fig. 178, p. 419, the outside girders rest on counterforts and the interior girders on special columns.

Vertical Walls. — Vertical walls are considered in design as slabs spanning between the counterforts and loaded by the horizontal earth pressures. Where the slab extends over several supports, it must be considered as a continuous slab. Main reinforcement of the vertical

wall consists of horizontal bars placed near its outside face to resist positive bending moments in the central portions of the spans, and of bars placed near the back of the wall to resist negative bending moments at the counterforts.

Since the horizontal pressure acting on the wall is largest at the bottom and decreases toward the top, the spacing of the horizontal bars is smallest at the bottom and may increase toward the top.

At the bottom, the vertical slab may be built monolithic with the foundation slab, as shown in Fig. 178, p. 419; or a construction joint with a key may be provided at the top of the foundation slab. In both cases, the vertical slab is prevented from deflecting horizontally by the foundation slab, and, therefore, no horizontal reinforcement is needed in the portion of the slab next to the bottom. In the design shown in Fig. 178, secondary bending moments are developed at the bottom, which should be provided for by short vertical bars placed near the back of the wall. U-shaped dowels may be used, one prong of which is near the inside and the other near the outside face of the wall.

Parapets. — The arrangement of parapets is shown in Fig. 178, p. 419. In Fig. 176, p. 410, no parapets are used.

Counterforts. — Counterforts for abutments are designed in the same manner as counterforts for retaining walls.[1] In resisting the earth pressures transferred to them by the vertical slabs, the counterforts act as cantilevers fixed at the foundation slab. Their main reinforcement consists of bars placed near the inside face of the counterforts and anchored in the foundation slab. Horizontal stirrups bind the vertical walls to the counterforts; vertical stirrups bind the counterforts to the foundation slab.

Wing Walls. — Wing walls are designed in the same manner as the main walls. The wing walls placed at right angles to the main wall serve there as end counterforts. In Fig. 178, p. 419, the vertical slab of the wing wall acts as a continuous slab.

In Fig. 178, p. 419, the wing walls are separated from the main wall by means of vertical expansion joints which extend from the top of the wall to the top of the foundation slab, and are filled with a $\frac{3}{4}$-in. premolded strip. To keep the walls in alignment, slip joints are used, which consist of 1-in. dowels extending 12 in. into the wing walls and also 12 in. into a pipe sleeve imbedded in the main wall.

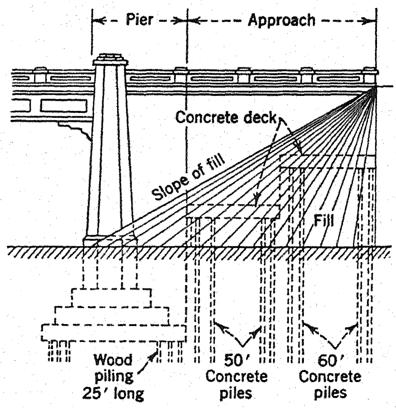
Foundation Slab. — The horizontal dimensions of the foundation slab are determined by the requirement that the unit pressure on the foundation must not exceed the allowable unit pressures.

The design of the foundation slab is the same as for a reinforced-

[1] See "Concrete, Plain and Reinforced," Vol. I, p. 867.

concrete retaining wall with counterforts. The slab in front of the wall is a cantilever resisting the upward reactions of the ground. The slab back of the wall is a continuous slab resisting either the downward weight of the earth, or the upward reactions of the ground.[2]

Drainage. — The abutment must be properly drained. An effective drainage of a counterfort abutment consists of 6-in. tile outlets in the vertical walls, one in each panel. The outlets are placed not less than 6 in. above the bed stream nor less than 6 in. below low water. To facilitate drainage, two horizontal 10 by 10 in. drain boxes filled with coarse bank run gravel are placed one about 1 ft. above the bridge seat and the other at the level of the drains. The two drain boxes are connected by means of vertical drain boxes.



From Engineering News-Record.

FIG. 179.—Abutment for St. Joseph River Crossing, Grand Trunk & Western Railroad. (*See p.* 421.)

Method of Construction. — It is advisable to specify or show on the plans the sequence in which the various parts of the counterfort abutment should be poured, and to show the locations of the construction points.

Example of Counterfort Abutments. — A counterfort abutment, shown in Fig. 178, p. 419, was used by the California Highway Commission in the bridge across the San Dieguito River, the superstructure for which is shown in Fig. 37, p. 91. Attention is called to the seal course used under the foundation slab to overcome the hydrostatic head. Another special feature of this design are the columns supporting two of the four girders. Each of the two outside girders has a bearing over the second counterfort from the expansion joint.

A $\frac{3}{4}$-in. vertical expansion joint is provided between the main wall and each wing wall. The separated parts are kept in alignment by 1-in. dowels spaced 2 ft. on centers, one end of which was imbedded in concrete, and the other in a pipe, thereby forming a slip joint.

Abutment of Special Design. — In Fig. 179, above, a high abutment for a railroad bridge is replaced by a pier and a hollow reinforced-concrete approach which with the pier gives the appearance of a U-abutment. The two parts are separated by a vertical expansion joint.

[2] See "Concrete, Plain and Reinforced," Vol. I, p. 866.

BURIED ABUTMENT

Buried abutments are abutments which are entirely concealed in the fill. Their function is mainly to carry the reactions of the superstructure to the foundation. The fill does not exert any pressure upon the abutment because it is allowed to assume a natural slope on all sides of the abutment.

The designs shown in Figs. 186 and 187, p. 432, and 36, p. 90, may serve as examples of buried abutments. In each case, the abutment consists of piles or other vertical supports extending to solid foundation. Shallow wing walls are usually provided to retain the embankment, and these may be a part of either the superstructure or of the substructure.

There are no difficulties so far as the design of such abutments is concerned because the forces acting upon them can be easily determined.

INTERMEDIATE VERTICAL SUPPORTS

The main purposes of intermediate vertical supports of a bridge are: (1) to transfer all vertical reactions to the foundation; (2) to resist all horizontal longitudinal and transverse forces acting upon the bridge. The stresses in the vertical supports must be investigated, and also the pressures upon the foundation must be computed.

In this section it is assumed that the superstructure is not rigidly connected with the piers.

Forces Acting upon Vertical Supports. — Vertical supports of a bridge may be subjected to the following forces, which may act either separately or simultaneously: (1) vertical reactions of the superstructure due to dead load, live load, and impact; (2) dead load of the vertical support, which for high, solid piers may form a considerable part of the total load; (3) longitudinal traction forces; (4) for bridges on a curve, centrifugal forces; (5) transverse wind pressure acting upon the superstructure, the vehicles on the bridge, and the vertical supports; (6) transverse pressure upon the piers due to the flow of water; (7) transverse pressure upon the pier due to ice and drift; (8) uplift due to hydrostatic action of the water; (9) temperature changes and shrinkage.

The impact ratio to be used in design of vertical supports depends upon conditions. For massive piers, the impact does not need to be considered below the bridge seat. For slender columns acting as vertical supports, the same impact formula should be used in the design of the columns as is specified for the superstructure, computed of course on the basis of the length of the structure to be loaded to get maximum reactions.

The traction forces are specified on pp. 13 and 405. In railroad structures carrying two tracks, the traction force on both tracks should be assumed to act in the same direction. The traction forces should apply at the fixed bearings; at the expansion bearings only a force sufficient to overcome the friction needs to be considered.

Transverse wind pressures are specified on p. 13.

Transverse pressures due to the flow of water and the pressure due to ice depend upon the speed of the water, and other local conditions which must be thoroughly investigated. Examples of successful piers built in the vicinity of the proposed structure should be studied.

Uplift due to hydrostatic pressure of the water should be taken into account only when its effect upon the stability of the pier is unfavorable; but it should not be considered as reducing the maximum pressures upon the foundation. The proportion of the total hydrostatic pressure to be considered as the active uplift depends upon conditions. Where the pier rests upon bedrock without seams and is well cemented to it, very little water can find its way under the pier base to produce uplift. The condition is different where the pier rests on piles. In such case, a considerable part of the total hydrostatic pressure may be exerted upward upon the base of the pier. In many designs seals are used below the piers to counteract the uplift. (See Fig. 37, p. 91.)

The effect of temperature changes and shrinkage needs to be considered only when the superstructure is connected with the supports. Where expansion bearings are used, forces due to temperature changes to be provided for need not exceed the frictional resistance of the bearings.

Bending Moments Produced by Horizontal Pressures. — When vertical supports of a bridge are rigidly connected with the superstructure, the whole structure should be treated in design as a rigid frame, using the formulas in the chapters on rigid frames.

When a pier is not connected with the superstructure, so far as horizontal forces are concerned it must be considered as a cantilever supported at the base and free at the top. Any horizontal pressure produces at any section of the pier below the point of application of the pressure a bending moment equal to the pressure multiplied by its vertical distance from the section.

Maximum bending moments act at the base of each pier, and these must be used to determine the pressure upon the foundation. Since some horizontal forces act in transverse and others in longitudinal direction, the bending moments also act in two directions.

To find the pressure on the foundation or stresses at any section of the pier, proceed as follows: Find the uniformly distributed pressures

(or stresses) for the central vertical load; then find pressures (or stresses) due to bending moments, separately for the longitudinal bending moments and for the transverse bending moments; finally add the appropriate pressures (or stresses). The maximum pressures or stresses act in one corner of the horizontal cross section of the pier, and the minimum stresses in the opposite corner. When plotted on the section, the stresses form an irregular wedge.

A design is satisfactory when the maximum stress is within working limits. For plain concrete piers, no tension should be allowed.

RIVER PIERS

General Description. — In its simplest form, a river pier consists of a vertical shaft resting upon a base course. The top of the pier may be finished with a coping.

In a horizontal cross section, the pier may be rectangular or it may have rounded ends. The width and length of the pier at the top must be large enough to accommodate all bearings of the superstructure. It is obvious that if a pier carries two simply supported spans, which require two bearings per each line of girders, its width must be larger than when it carries a continuous girder with only one bearing for each line of girders. When the slab of the cross section of the bridge is cantilevered out at the sides, the length of the pier is governed not by the width of the bridge but by the distance between the outside faces of the exterior girders. (See Fig. 37, p. 91.)

Usually, the dimensions at the bottom of the pier required for stability are larger than the top dimensions, therefore the sides of the pier are appropriately battered. Usually, the batter varies from $\frac{1}{2}$ to 1 in. to the foot.

When the pier is of masonry or of plain concrete, the stresses produced at any section by the vertical loads must be larger than the tensile stresses produced by the bending moments. When, for any condition of loading, tensile stresses are developed in the shaft of the pier, they must be provided for by reinforcement placed near the face of the pier and anchored in the base. Reinforcement is often used in piers to prevent cracking of concrete even when it is not required by the stresses. Such reinforcement consists of horizontal and vertical bars. (See Fig. 180, p. 426, and Fig. 186, p. 434.)

Starlings for Piers. — River piers are usually provided with starlings below the level of high water for the purpose of reducing the obstruction to the flow of water, and as a protection against ice and drift. Starlings are needed only at the upstream end of the piers, as shown in Fig. 186, p. 434, but for the sake of symmetry they are often used at both ends.

Where appreciable ice floes and drifts are expected, the upstream starlings of the piers are protected by steel nosings. In northern rivers, special ice breakers at the piers are used to protect the structure from ice.

In cross section, the starlings may be triangular (see Fig. 69, p. 158), with a right angle at their apex. A more pleasing effect is obtained by using for the sides of the starlings two segments of a circle, the diameter of which is larger than one-half of the pier width. The segments are tangent to the pier surface and form a point at the apex of the starling. (See Fig. 186, p. 434.)

In some designs, the starlings are extended upward above the level of the bridge seat. This affords protection to the superstructure and also gives a pleasing finish to the pier. (See Fig. 62, p. 145.)

Foundation of River Piers. — The vertical shaft of a pier rests upon a footing slab the width and length of which are greater than the dimensions of the shaft at the bottom. The dimensions of the footings depend upon the allowable unit pressure upon the ground, and, in case of pile foundations, upon the number of piles to be used.

The unit pressures upon the foundation are found by combining the unit pressures for the vertical loads with the unit pressures produced by the bending moments due to transverse and longitudinal horizontal forces. These are computed as explained under the heading "Bending Moments," on p. 423.

The projections of the base beyond the face of the shaft act as cantilevers loaded by the upward reactions of the ground, or of the piles. The projections must be deep enough to keep the tensile stresses in concrete within working limits permitted for plain concrete; or they must be reinforced to resist tensile stresses.

The base of the foundation must be located below the frost line; and, also, it must be deep enough to be out of danger from underscouring, which is the most frequent cause of pier failures. Underscouring is often guarded against by surrounding the pier with riprap. (See Fig. 63, p. 147.) Often, the sheet piles used in the cofferdam are left in place as a protection of the foundation. These should be driven at least 3 ft. below the bottom of the foundation and should extend some distance above the top of the footing course. (See Figs. 61, p. 144, and 63, p. 147.)

Where possible, particularly for statically indeterminate structures, piers should be carried to rock. This is usually done when the rock is not more than 20 ft. below the river bed. Satisfactory foundation may be obtained on hardpan, gravel, and other firm ground by adopting proper allowable unit pressures. When firm ground is too deep below the river bed, wood or concrete piles may be used. Wood piles must be cut below low water level.

For allowable unit pressure on the soil, see "Concrete, Plain and Reinforced," Vol. I, p. 470; and for allowable unit pressure on piles see same volume, p. 542.

Special designs of foundation were used in Fig. 37, p. 91; Fig. 60, p. 142; and Fig. 181, p. 427. The piers of the new Oswald Bridge in Glasgow, Fig. 66, p. 155, are supported by cylindrical caissons driven to rock. Similar foundation was used for the bridge across the Loire; see p. 143. The discussion of difficult foundation work is outside the scope of this book; reference may be made to Jacoby and Davis, "Foundations of Bridges and Buildings."

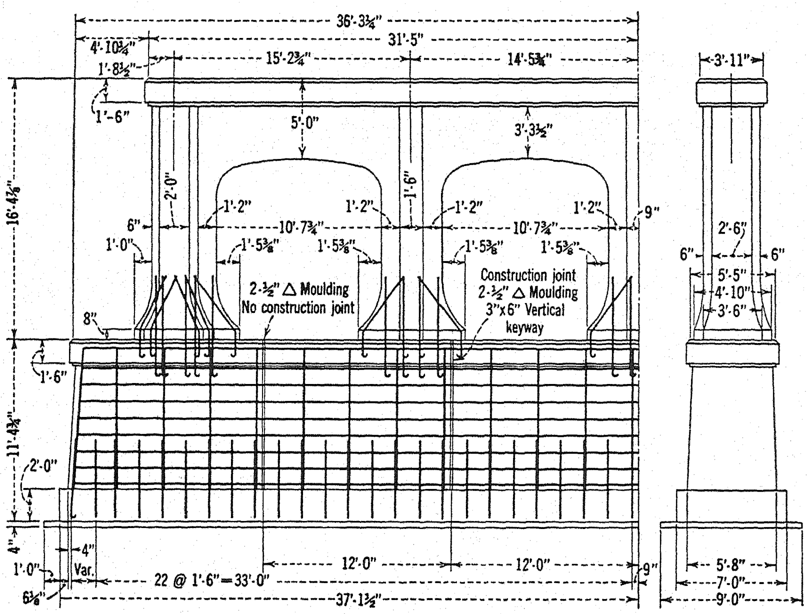


Fig. 180.—Pier for Grand River Crossing at Grandville, Mich. (*See p.* 427.)

Examples of Pier Design. — Solid concrete piers, as used for a continuous bridge, are shown in Fig. 69, p. 158. They are provided with triangular starlings on the upstream ends only. Other designs of plain concrete piers are shown in Fig. 63, p. 147, and Fig. 67, p. 156.

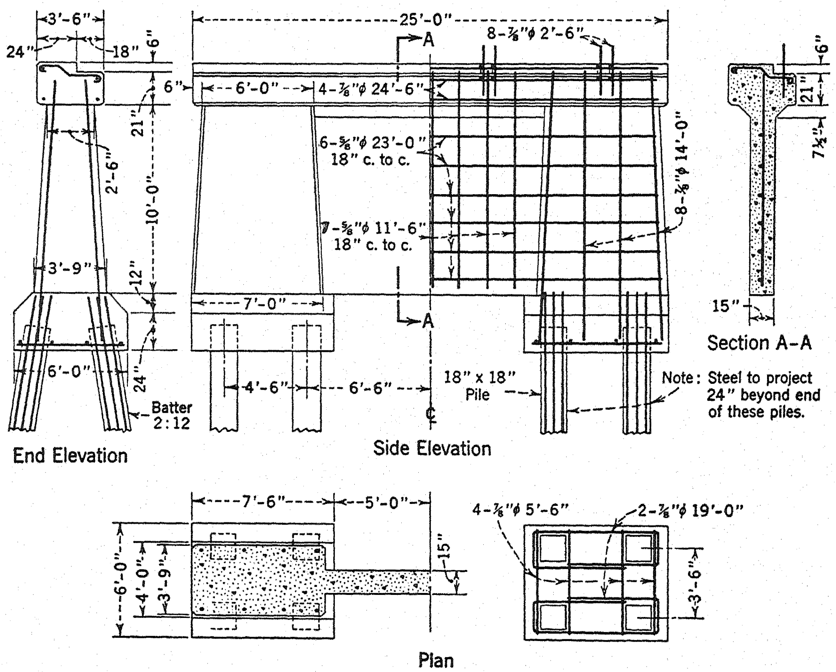
Piers with wide footings resting on hard clay are shown in Fig. 64, p. 153. Their footings are reinforced. Piers of similar design, but founded on rock, are shown in Fig. 69, p. 158.

Piers resting on piles, used for a railroad trestle, are shown in Fig. 186, p. 434. They are provided with segmental starlings on one side only. The other end of the pier is rounded.

Piers with pockets for reinforced-concrete rockers are shown in Figs. 62, p. 145, and 68, p. 157.

A number of examples of pier design are shown in connection with illustrations of continuous bridges and of cantilever bridges.

Piers of Special Designs. — In many cases, it is economical to replace solid piers by special designs, thus saving a great amount of material. The piers for the Grand River Crossing at Grandville, Michigan, shown in Fig. 180, p. 426, may serve as one example. In this design a solid pier



S. B. Slack, Bridge Engineer.

FIG. 181.—Pier Supporting Through-Spans. Savannah Delta Trestle. (*See p.* 427.)

was used in the lower portion; but, above the high water level, the solid pier is replaced by five reinforced-concrete pillars connected at the top by a cross beam.

In the design shown in Fig. 37, p. 91, separate pillars are used, resting on a common foundation under each girder. To produce the effect of a solid pier, the pillars are connected by side walls.

In Fig. 181, p. 427, piers consist of pillars connected by a wall. In this way stability was combined with economy.

INTERMEDIATE VERTICAL SUPPORTS OTHER THAN RIVER PIERS

When the horizontal forces exerted by water do not need to be considered, intermediate supports may consist of comparatively thin walls, of independent columns or of piles, as in pile trestles. Vertical supports for pile trestles are treated on p. 429.

When the columns are rigidly connected with the superstructure so as to form rigid frames, the bending moments and reactions there are computed as a part of the rigid-frame computations. The columns are then proportioned so as to resist the bending moments and direct pressures acting upon them.

When the vertical supports and the superstructures are separated by expansion bearings, the vertical supports must be stable by themselves without relying on any assistance from the superstructure. In some designs, the structure is supported by isolated columns, each strong enough to resist all longitudinal and transverse horizontal forces to which the structure may be exposed. In resisting horizontal forces, the columns act as cantilevers fixed at the bottom and free at the top. The footing must be designed to resist not only the vertical reactions but also the bending moments acting at the bottom of the columns and produced by the horizontal forces.

When it is desired to increase the stability of the columns in the transverse direction, they may be connected at the top by a horizontal cap; and when their height is large, by one or more intermediate struts. The columns then form a transverse bent which acts as a unit in resisting transverse forces such as wind forces. In this manner, the stability of the vertical supports is appreciably increased.

The number of columns in a bent may be the same as the number of the girder lines in the superstructure, and each column may support one girder line directly. When, however, the number of columns is smaller, the cap must be made strong enough to carry as a beam the reactions of the intermediate girder lines.

When large longitudinal forces must be resisted by the vertical supports, a transverse bent may be connected by longitudinal struts with the adjoining bent to form a tower. Horizontal forces then produce positive and negative reactions in the columns and shears in each member of the tower. Since there are no diagonals, the shears produce bending moments both in the columns and in the struts. These should be properly provided for.

VERTICAL SUPPORTS FOR PILE AND PIER TRESTLES

Description of Pile and Pier Trestles. — Trestles here described consist of decks supported by pile bents or by piers built on piles. They

are used extensively for highway as well as railroad crossings over swamps, stagnant waters, and shallow lakes. Often they are of considerable lengths, as, for example, the 24 922-ft. crossing through Lake Pontchartrain in Louisiana and the 37 183-ft. San Francisco Toll Bridge.

A general idea of pile trestles may be had from Fig. 182, p. 429, showing the Baltimore and Ohio Railroad trestle near Medora, Indiana.

Pile Bents. — A pile bent consists of a number of precast concrete piles driven into the ground and connected at the top by a reinforced-concrete cap.

The piles are either octagonal in cross section or rectangular with chamfered edges. To facilitate driving, they may be pointed at one

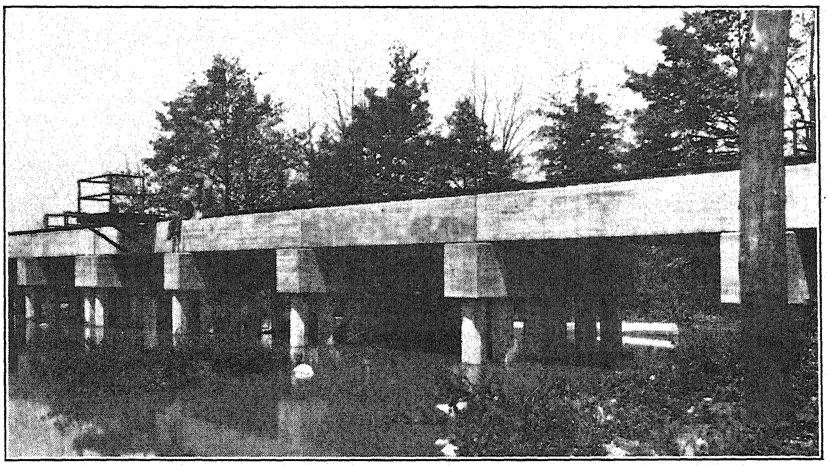


FIG. 182.—View of Railroad Pile Trestle. (*See p.* 429.)

end. The main reinforcement of a pile consists of longitudinal bars, the area of which should be equal to not less than 1 per cent of the cross section of the pile. Hoops should be provided as for a reinforced-concrete column. The point and the top should be strengthened by closely spaced hoops to prevent injury during driving. Details of piles used in several structures are shown in Fig. 183, p. 430.

Since the piles are precast, it is important to investigate the condition at the site to make possible a close estimate of their lengths.

The dimensions of the pile depend not only upon the load to be carried but also upon the unsupported length. For instance, in the crossing over the Savannah Delta in Georgia, piles carrying from 23 to 27 tons, each, were made of the following dimensions with reinforcement

depending upon their lengths. The details of these piles are shown in Fig. 183 (*b*), p. 430.

Cross section	Reinforcement	Length of piles
14 in. sq.	4–$\frac{7}{8}$ in. rd.	26 ft.
16 in. sq.	8–$\frac{3}{4}$ in. rd.	27–36 ft.
18 in. sq.	8–$\frac{7}{8}$ in. rd.	Over 36 ft.

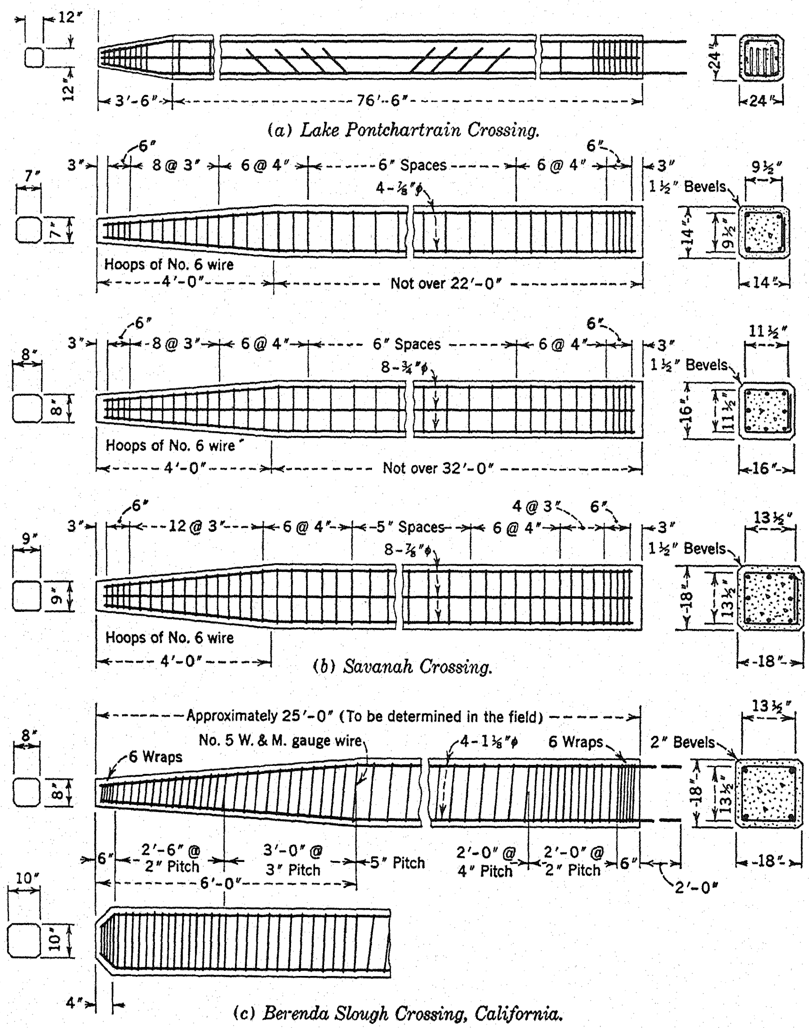


FIG. 183.—Details of Precast Concrete Piles. (*See p.* 430.)

In the San Francisco Toll Bridge, the piles varied in length from 45 to 105 ft. and in cross section from 16 in. square to 24 in. square. In the bridge across Lake Pontchartrain, the piles varied in lengths from 55 to 85 ft. with an average length of 75 ft., and they were 25 in. in cross section.

The standards of the Canadian National Railways base the size of piles upon the distance from base of rail to a point 2 ft. below the ground level. They require 12 in. square piles when the distance is 10 ft. or under; 14 in. square for 15 ft.; 16 in. square for 20 ft., and 18 in. square for 25 ft.

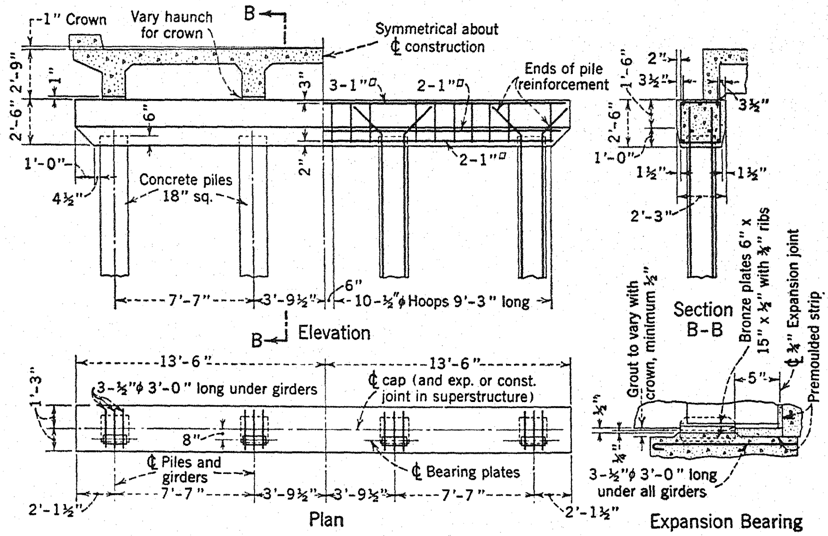


FIG. 184.—Pile Bent, Berenda Slough Crossing, California. (See p. 431.)

Lateral Spacing of Piles in Bent. — In highway bridges with roadways up to 30 ft. in width, four piles are used per bent, and for greater widths five piles. All piles may be vertical, or, to give stability to the bent, the outside piles may be battered. (See Fig. 184, p. 431.)

In railroad trestles the number of piles in a bent per track depends upon the loads to be carried. The standards of the Canadian National Railway specify, for bents spaced 15 ft. on centers and for Cooper E 60 loading, five piles per bent for a single-track trestle, and nine piles for a double-track.

In the double-track trestle for the Baltimore and Ohio Railroad shown in Fig. 185, p. 432, with an 18-ft. spacing of bents and Cooper E 60 loading, eleven piles were used per bent.

In recent years, the Missouri Pacific Railroad has found it economi-

cal to use three 24-in. square piles per track, instead of the five or six 16-in. square piles previously used. A 24-in. pile drives as fast as a 16-in. pile; and tests show that the load-carrying capacity of a 24-in. piles is more than twice that of the smaller pile.

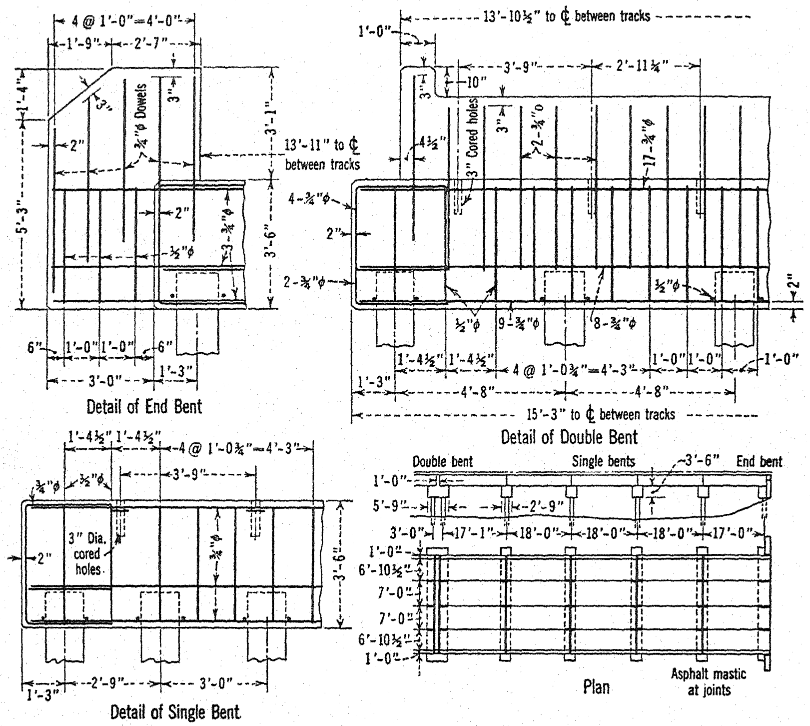


FIG. 185.—Typical Concrete Trestle. Baltimore and Ohio Railroad. (*See p.* 431.)

Longitudinal Spacing of Pile Bents. — The bents in a pile trestle should be spaced so as to make the total cost of the bents and of the superstructure a minimum. It is obvious that, where long piles are needed and the driving is difficult, the economical spacing is larger than for short piles.

The spacing may also be affected by the design of the superstructure. If, for instance, it is desired to use simple slabs for superstructure, the spacing of bents should not exceed 20 ft. The method of construction may affect the spacing of bents. Thus, if it is desired to use precast deck units, the spacing may depend upon the available facilities for handling the units, and the maximum weight of the units may govern the span lengths.

In railroad construction, the deck of the trestles usually consists of precast slabs; therefore, the spacing of bents is smaller than in highway trestles and ranges from 15 to 25 ft. A typical design of a railroad trestle is shown in Fig. 185, p. 432, as used by the Baltimore and Ohio Railroad. It was designed for Cooper E 60 loading.

Caps. — Piles composing a bent are connected at the top by a reinforced-concrete cap which is cast in place after the piles are driven and cut off to proper grade. Vertical pile reinforcement usually extends from the pile into the cap. The piles are allowed to extend into the cap from 6 to 12 in., thus stiffening the connection.

The width of the cap must be sufficient to accommodate the bearings of two adjacent spans. It is good practice to make the cap at least 10 in. wider than the pile.

When the deck construction consists of slabs, the cap must be considered as a continuous beam supported by the piles and loaded by the slab reactions. Since the cap also acts as a strut connecting the piles and resisting the lateral forces acting upon the trestle, its cross section should be larger than required by its function as a beam.

When the deck construction consists of girder and slab designs, each girder is usually placed directly over a pile; and the vertical reactions, being transferred directly to the pile, do not produce any bending moments in the cap. Where, however, the lateral spacing of girders is different from the spacing of the piles, bending moments and shears are produced in the caps by the girder reactions.

Double Bents. — In long trestles, the construction is often strengthened longitudinally by double bents. In the typical design shown in Fig. 185, p. 432, every fifth bent is a double bent.

In the James River highway trestle,[4] double bents are introduced every fifteenth span.

Pier Bents. — Pier bents are often used when the water is too deep for pile bents, or when it is desired to get a greater lateral stiffness than can be obtained with pile bents. The piers may rest on one, two, or three lateral rows of piles. In the design shown in Fig. 186, p. 434, illustrating a trestle used by the C. B. & Q. R.R., the piers are of considerable heights and each of them is supported by three rows of piles.

End Bents. — The design of an end bent and of the embankment around it, shown in Fig. 187, p. 434, was used in the trestle across the Savannah Delta in Georgia. This bent carries a deck of the girder and slab type shown in Fig. 35, p. 89. The end bent is provided with parapet walls to retain the shoulders. In other respects it does not differ materially from an interior bent.

[4] See *Engineering News-Record*, Jan. 10, 1929, p. 57.

It is important that the piles in end bents should not be driven until after the fill is completed. When the piles are driven first, the piles may be broken or pushed out of line by the placing of the fill.

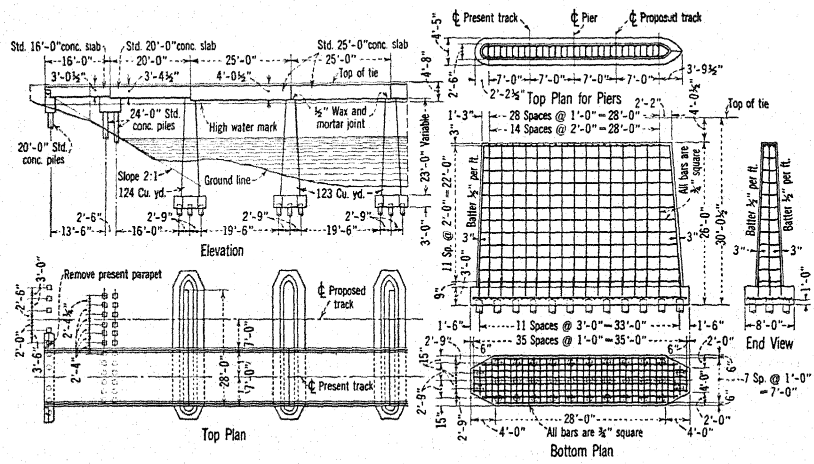
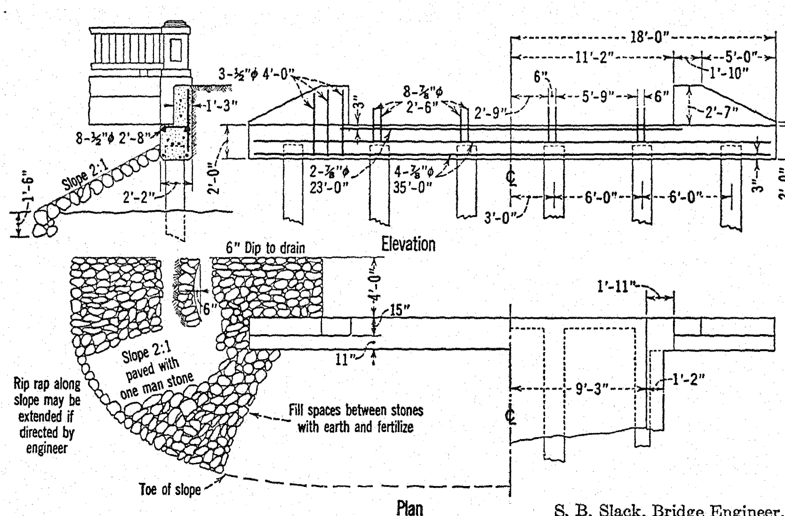


FIG. 186.—Railroad Pier Trestle. C. B. & Q. R. R. (*See p.* 433.)



S. B. Slack, Bridge Engineer.

FIG. 187.—End Bent. Savannah Delta Trestle. (*See p.* 433.)

FOOTINGS FOR ISOLATED COLUMNS

Columns supporting girder bridges or frames may rest on isolated footings, or all columns in a transverse row may be placed on a combined footing. Sometimes, when footings rest on piles, additional stability in

the longitudinal direction is obtained by connecting adjacent footings by longitudinal braces as described on p. 372.

Dimensions of Base of Footings. — The area of the base of the footings must be proportioned so that under the most unfavorable loading conditions the maximum unit pressure on the soil, or the maximum pressure per pile, does not exceed the allowable unit pressures. To avoid any appreciable settlement of the structure, particularly when it is statically indeterminate, the allowable unit pressures must be carefully selected. The base of each footing must be proportioned according to the loads to be carried. It is not permissible, for the sake of uniformity, to provide footings of the same dimensions for columns for which the loads to be carried are appreciably different, as this might cause unequal settlement of the structure. The base of the footing may be either concentric with the column or eccentric, whichever design gives the best distribution of the reactions upon the foundation.

If possible, the footings should be carried to rock or to hardpan, particularly if the structure is statically indeterminate. However, satisfactory results may be obtained with foundations on gravel and hard clay, provided that proper working unit pressures are used. When it is not practicable to reach firm ground, piles may be used to support the footings. Caisson piles may also be used as shown in Fig. 161, p. 371, and described on p. 372.

Forces Acting upon Footings. — Footings may be subjected to the following forces and bending moments.

1. Vertical forces acting at the center of the columns.
2. Bending moments transferred from the columns to the footings.
3. Horizontal thrusts transferred from the columns at the top of the footings and the horizontal reactions resisting these thrusts.

Vertical forces are the reactions of the dead and live loads, and also the vertical reactions caused by wind pressure and other horizontal forces.

Bending moments are transferred from the columns to the footings in rigid frames, and also when the columns resist horizontal forces as cantilevers. Eccentric loadings of columns also produce bending moments in footings.

Horizontal thrusts are produced when the columns are rigidly connected with the girders; they are determined by rigid-frame formulas. Horizontal forces also produce horizontal reaction on the top of the footings. All horizontal forces acting at the top of a footing are resisted either by friction acting at the bottom of the footing, or by passive earth pressure acting against the side of the footing. In either case, the resisting force is below the point of application of the active force; and

therefore the two forces form a couple, which must be taken into consideration in designing the footing.

Interior Footings of Rigid Frames. — Bending moments at the bottom of the columns in rigid frames are produced only when the columns are fixed at the bottom. No bending moments are developed when column ends are hinged. When the spans at both sides of a column are about equal, no appreciable bending moments are produced by the dead load; but live loads produce either positive or negative bending moments, depending upon the position of the loaded panel. Where the difference between the lengths of spans is appreciable, bending moments are developed even for dead load.

Exterior Footings of Rigid Frames. — Exterior footings of rigid frames with fixed ends are subjected to bending moments for live and dead loads. For vertical loads, the bending moments transferred by the columns to the footings are always positive; and the horizontal thrusts always negative, i.e., acting at the left end columns from left to right. The bending moments and thrusts produced in the footings by vertical loads on cantilevers are of opposite sign to those for vertical loads on the main spans.

For earth pressure acting symmetrically at both sides of the structure, the bending moments and horizontal thrusts act in the opposite direction to those for vertical loads. For one-sided earth pressure the bending moments and thrusts at the loaded side are of the same signs as for symmetrical earth pressures, and at the other end they are of opposite signs.

Independent Footings. — Each independent footing may be of a solid heavy concrete or masonry block without reinforcement. If the allowable bearing stresses on the foundation block are small, the base of the column is enlarged, as shown in Fig. 116 (*b*), p. 269. Independent footings may also be of reinforced concrete. The design of independent footings for bridges is somewhat complicated when bending moments are transferred by the columns to the footings.

For interior columns, the footings are usually made concentric with the columns. In determining the unit pressures on the foundation, the reactions and bending moments are combined so as to get the most unfavorable results. The resultant on the footing, then, will be eccentric, and the consequent pressure on the foundation will not be uniform. The maximum pressure governs the size of the base. The maximum reaction of the ground must be used to determine the bending moments in the footing, for which the depth of footing and the amount of reinforcement are computed.

For exterior footings, it may be advisable to make the center of the

footing coincide with the position of the resultant obtained by combining the vertical forces with the bending moments. In such case, the footing would be eccentric as far as the column center is concerned.

If an independent footing consists of a reinforced concrete slab, it is reinforced in two directions. The bending moments in the footing may be found as for footings carrying central loads, explained in "Concrete, Plain and Reinforced," Vol. I, p. 495, using, of course, for each part the largest upward reactions of the ground.

Combined Footings. — Combined footings are designed as beams or slabs loaded by the reactions of the ground and supported by the columns. As in independent footings, for interior columns the center line of the footing usually coincides with the center line of the columns; for exterior columns it may coincide with the center line of the resultant pressures. The pressure and the reactions being known, the bending moments in the footing may be easily computed.

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